JOHN VENN'S PLURALISM REGARDING LOGICAL FORMS

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Introduction

The context of the emergence of modern symbolic logic in Britain, with Boole's *The Laws of Thought* (1854) as cornerstone, has been the focus of increased attention by historians and philosophers in recent years. Volker Peckhaus (e. g., 1999) emphasized that this period was marked by an interplay between philosophy and mathematics, and discerned three main lines of development: the tradition of Aristotelian logic together with attempts at extending it, most notably by Hamilton and De Morgan; the inductive logic mainly associated with the name of Mill; and mathematical developments around symbolical algebra and the calculus of operations, which paved the way for Boole's importing of symbolic methods in logic.[1]

A particularly interesting position in this landscape is that of John Venn (1834–1923), who has been relatively neglected by scholars, at least until recent work by Lukas Verburgt.[2] As may be typical of such transitional, or even revolutionary, periods of intellectual developments, one

[1] See also Durand-Richard (2000) for a discussion of these developments.

[2] See Verburgt (2020; 2021; 2022a). We had not been able to consult Verburgt (2022b) when first writing this paper, but have added references to it where it usefully complements our account.

cannot fail but notice a general sentiment of rivalry and antagonism between many of the protagonists. It is here that Venn stands out among his contemporaries. Initially trained in mathematics and embedded in the philosophical tradition in Cambridge, he showed an interest in uncovering the underlying assumptions of competing positions and in mediating between them. He certainly had views on the main debates of the day: he was a strong proponent, and in fact the chief British expositor, of Boole's logic; philosophically, we shall see that he had long-standing sympathies for Mill's views on logic. Nevertheless, he strove to craft a broad compromise, both between Mill's account of logic and that of the more traditional Aristotelian logicians, and between the verbal methods of the philosophers and the heavily symbolic, mathematical ones of Boole.

Venn thus attempted a subtle balancing act. On the whole, his aim was to disseminate and defend Boole's symbolic logic. But while he was convinced that symbolic logic was clearly superior for some purposes (in particular because it provided a general method that could deal even with intricate logical problems) and that it shed much light on the nature of the subject, he did not frame his account of it as directed against traditional logic. In fact, he was at pains to emphasize that, in contrast to what was often assumed, Boole's system did *not* stand in opposition to more traditional approaches to the subject, but that there was room for both. Moreover, he insisted that Boole's methods were *independent* of substantial philosophical commitments as to the nature of logic, so that everyone could embrace them. Which logic to use, for Venn, would ultimately depend on the particular aims one wanted to pursue.

This balancing act led Venn to a remarkably original account of *logical form*, to which our paper is devoted. He recognized that the different approaches to logic on offer led to different analyses of the 'forms of Propositions', but denied that one had to choose one of them as the 'right' account: different, equally legitimate analyses of propositions could coexist, while being best suited for different purposes. Thus, we could call his account of logical form *pluralist*, *instrumentalist*, or even—following the broader analysis of Verburgt (2021)—*pragmatist*. As such, Venn's account should be of interest not just to historians of logic, but to philoso-

phers as well, particularly given the contemporary vogue of various forms of logical pluralism.

Section 1 sets the stage by describing the confusing state of logic when Venn started writing on the subject, with a particular focus on Venn's own description of it. Section 2 then describes Venn's discussion of the forms of propositions, while Section 3 examines the broader expressions of Venn's pluralism and instrumentalism with respect to logical methods. Section 4 focuses on the tensions and difficulties lurking below Venn's ecumenical approach. Finally, Section 5 attempts to situate Venn's position within the coordinates of today's debates.

1 The state of logic and Venn's view of it

Here is how Venn begins his first paper on logic,[3] in 1876, in the inaugural issue of the journal *Mind*:

> It would not be going too far to say that the principal difficulty in the way of a student of Logic at the present day (at any rate in England) consists not so much in the fact that the chief writers upon the subject contradict one another upon many points, for an opportunity of contradiction implies agreement up to a certain stage, as in the fact that over a large region they really hardly get fairly within reach of one another at all. [...] Much of the consequent confusion can, we are convinced, be easily allayed by a simple process of intercomparison, provided only the various systems be referred to their leading principles of distinction. (Venn 1876, 43)

This could be a fit manifesto for Venn's entire work on logic: providing an overview of different positions and then assessing their faults and merits is characteristic of his approach, and is prominent in our main topic below—his discussion of logical form. But before we turn to this, let us say a few words about the context Venn is gesturing at here.

[3] Venn's work before the middle of the 1870s was mostly concerned with theology and probability; for an overview of Venn's career, see Verburgt (2022a).

The 'confusion' in question goes back to the developments of British logic in the preceding half-century. The subject was revived in Britain by the publication of Richard Whately's immensely popular *Elements of Logic* (1826, and many subsequent editions), described 25 years later as 'one of the most important and influential logical publications of modern times' (Blakey 1851, 454; see also, e. g., Van Evra 1984). Whately's logic was in a broadly Aristotelian tradition: logical inferences were presented in the form of syllogisms, which relate two premises and a conclusion; each premise and conclusion was 'a sentence affirming or denying one thing of another' (Aristotle, *Prior Analytics*, 24a16), such as 'All A are B' and 'No B is C'. His book spurred much work on the subject in relatively quick succession, by authors such as Bentham, Mill, Hamilton, De Morgan, Boole, etc. (see Peckhaus 1999), offering substantive proposals for reform as well as debating, philosophically, the nature and significance of the subject.

Among proposals for reform, the best-known are, on the one hand, Hamilton and De Morgan's 'quantification of the predicate' (which gave rise to a well-known priority dispute between the two authors; see Heinemann 2015), and, on the other hand, the introduction of *symbols*. Regarding the latter, after the nearly simultaneous publications by De Morgan (1847) and Boole (1847) attracted very little attention, it was George Boole's (1815–1864) *An Investigation of the Laws of Thought* (1854) that exerted the most profound influence on the further development of symbolic logic. Boole presented logic as an algebra whose expressions looked very much like arithmetical ones and whose inferences also resembled the familiar manipulations of algebraic equations. Using lowercase letters to stand for classes and the usual arithmetical symbols, such as '+' and '×', to stand for ways of combining classes, Boole would, for example, represent the statement 'Wealth consists of things transferable, limited in supply, and either productive of pleasure or preventive of pain' by the equation $w = st(p+r)$ (Boole 1854, 60). Using '1' to stand for the universe of discourse, Boole would represent the complement (or negation) of x by '$1-x$'. With the device of 'indefinite' class symbols (say, v), 'vx' would stand for 'some x', and Boole was able to represent 'All men are mortal' by '$y = vx$', symbolizing that the class of men is equal to a subclass of

the class of mortals. In this way, Boole meant to show that the valid syllogisms could be proved in his logic (though his treatment of syllogisms involving particular propositions would generally be found unsatisfactory), but that the latter allowed for an effective treatment of propositions much more complex than those involved in the traditional syllogisms.[4]

Even among proponents of symbolic logic, there was much dissent. While most symbolic logicians after Boole built on his work in one way or another, most of them criticized it for various reasons and proposed their own systems of logic that differed from his. W. S. Jevons (1835–1882), for example, advocated a more combinatorial approach to symbolic logic and rejected Boole's emphasis on the similarity between logic and arithmetic. In particular, some of the operations that Boole employed in his calculus, such as division, did not seem to have an obvious logical meaning, which made some of Boole's expressions appear meaningless. Boole's understanding of $x + y$ as being defined only in the case that x and y are disjoint classes was also frequently criticized.

On the philosophical side, there was controversy as to the very nature of logic. Here is Venn's account, in 1876, of a salient point of contention. He assumed that there was a modicum of consensus on the nature of propositions:[5] 'Every one, it is to be presumed, will admit that a proposition is a statement in words of a judgment about things' (Venn 1876, 44). However, this characterization leads to 'three alternative views on the general nature of logic', depending on whether one puts the emphasis on words, judgments, or things. Venn quickly dismissed as too implausible the view that words are the fundamental constituents of logic, which he ascribed to Whately: 'no clear thinker', he wrote, could adhere to it, and even Whately found himself unable to follow it consistently. This left him with two main conceptions of logic: on the one hand, the *conceptualist view* (which, in his 1876 paper, he attributed explicitly to Hamilton and Mansel, but which can be taken to underlie all traditional

[4] For a brief introduction to Boole's conception of logic and to his methods, see for instance Waszek and Schlimm 2021.

[5] In a footnote Venn qualifies this assumption: 'The reader is reminded that we are confining our attention, not entirely to English logicians, but to those who may be considered as influential here. No Hegelian, I presume, would consider what we have taken as our starting point to be in any way deserving of such a name' (Venn 1876, 44).

formal logic as well as the work of De Morgan, Boole, and Jevons[6]), according to which logic is about concepts and judgments, and language is seen as a 'medium of thought'; on the other hand, the *materialist view* (which he associated with Mill), according to which logic is about things, and language is seen as 'having reference to facts'. Venn formulated reasons in favor and against both views, but he showed more sympathy for Mill's materialist or 'objective' view. Nevertheless, it is a telling example of Venn's open-mindedness that he ended his review of Sigwart's *Logik* (1878), which followed a conceptual approach to logic, by 'strongly recommending' it, because 'the entire cast of thought and mode of treatment are so unlike anything to which we are accustomed here, that the study of the work is unusually instructive and suggestive' (Venn 1879b, 431). As we shall see in Section 3.3, Venn would eventually craft a compromise, carving out a place for Boolean symbolic logic in a broadly materialist framework.

Venn's work on logic—which mostly took place from the mid-1870s to the early 1880s, culminating in his *Symbolic Logic* (1881; second edition 1894)—is largely a defense of symbolic logic in general (against proponents of traditional Aristotelian logic or possibly its reformed variants, who did not understand or see the point of introducing symbols), and of Boole's system in particular (against criticisms internal to the symbolic tradition, such as Jevons's). His specific approach, however, consisted in building a framework in which all systems could be seen as compatible and could find their proper place. The centerpiece of this endeavor is his pluralism about the forms of propositions, to which we now turn.

2 Venn on logical form

While preparing his monograph *Symbolic Logic* (1881), Venn published several parts in advance (Venn 1880b;c;d;e;f), including his reflections on logical form (Venn 1880b). The starting point for these is a dissatisfaction

[6]For further discussion, see Verburgt (2022b, 165–167).

with the heterogeneous situation in logic similar to the one he voiced in 1876.[7] His paper 'On the forms of logical propositions' begins as follows:

> Logicians have been much exercised in the attempt to determine the number and arrangement of the simple forms of proposition, and hardly any two who have reconsidered the question for themselves seem to have agreed in their decision. (Venn 1880b, 336)

Venn distinguishes three different accounts of 'the import of a proposition', namely the ordinary or *predication* view, the *class inclusion and exclusion* view, and the *compartmental* view.[8] In order to better understand them, Venn proposes a systematic comparison:

> We propose to inquire what are the prominent characteristics of each of these distinct, but not hostile, views. What are their relative advantages and disadvantages; to what arrangement and division of propositional forms do they respectively lead; and which of them must be adopted if we wish to carry out the design of securing the widest extension possible of our logical processes by the aid of symbols? (Venn 1880b, 336–337)

Such a study would not only provide an overview and an assessment of their respective advantages and disadvantages, but could also be useful to guide the future development of logic. In fact, Venn warns that the lack of such a study has previously led to 'error and confusion', in particular in combination with a too hasty commitment to a single account:

[7] In a similar vein, Venn's paper on the different notations used in logic begins with the following remark: 'Most logicians must be well aware of the general fact of the perplexing variety of symbolic forms which have been proposed from time to time by various writers, but probably few persons have any adequate conception of the extent to which this license of invention has been carried' (Venn 1880e, 36).

[8] Mill's *System of Logic* (1843) also has a chapter on 'Of the Import of Propositions', in which four proposals for the meanings of propositions are discussed. Mill rejects the analyses of propositions as relating two ideas, two names, and two classes, and suggests a five-fold classification of matters of fact.

> Logicians have been too much in the habit of considering that there could be only one account given of the import of propositions. [...] And the very useful question as to the fittest view for this or that purpose has been lost in the too summary decision that one view was right and the others wrong. (Venn 1880b, 337)

We shall now present these three different accounts of logical form and Venn's discussion of them.

2.1 The predication view of common logic

Consider the sentence 'All whales are mammals'. The traditional *predication view* follows the grammar of English and identifies a *subject* (whales) and a *predicate* (being a mammal). For Venn, the distinguishing feature of this view is that it introduces an asymmetry between subject and predicate. He accepted the traditional doctrine according to which terms, for instance 'human', have both a *denotation* (or, in modern terms, *extension*)—e. g., the class of humans—and a *connotation* (or *intension*), made up of attributes—e. g., 'animal', 'rational', and the like. In terms of this distinction, the forms of traditional logic, he wrote, 'generally and primarily regard the predicate in the light of an attribute and the subject in that of a class (whole or part)' (Venn 1880b, 337 = Venn 1881, 3), the question being whether the whole, or a part, of the *class* corresponding to the subject does or does not fall under the *attribute* corresponding to the predicate. Hence the four traditional Aristotelian forms: A ('All S are P'), E ('No S are P'), I ('Some S are P'), and O ('Some S are not P'). Because of the asymmetry between subject and predicate—and in contrast to the other views discussed below—the traditional forms, adds Venn, 'do not tell us whether any other things besides the whole or partial class in question possess the assigned attribute' (Venn 1880b, 337 = Venn 1881, 3–4). On the basis of the four Aristotelian forms we can express patterns of valid inferences, the so-called syllogisms, as well as rules for converting a form into logically equivalent ones, such as certain kinds of conversion and contraposition.

According to Venn, these forms 'certainly seem to represent the most primitive and natural modes in which thought begins to express itself with accuracy', although he is careful to remark that this analysis is relative to the particular language under consideration and that it might be different in 'non-inflectional languages' (Venn 1880b, 337–338).[9] The fact that the Aristotelian forms had been identified and used for the systematization of inferences two millennia earlier and had remained an object of study since then speaks in their favor, according to him, and provides 'very strong reasons for not disturbing them from the position they have so long occupied' (Venn 1880b, 338).

The only reasons for going so far as to reject them, writes Venn, would be that they are 'actually insufficient to express what we require to express' or that they are based on a 'wrong interpretation of the import of a proposition'. For him, however, neither of these objections applies. Regarding the first, he insists ('as no one would deny') that 'a combination of two or more of these forms will express almost anything in the way of a definite statement'. Regarding the second objection, he writes:

> [...] the point of this essay is to show that we are not necessarily tied down to one exclusive view as to the import of a proposition. I should say, therefore, that whatever other view we may find it convenient to adopt for special purposes, either of sensible illustration or with a view to solving intricate combinations of statements, there is no valid reason for not retaining the old forms as well. They may not be the most suitable materials for very complicated reasonings, but for the expression and improvement of ordinary thought and speech they are not likely to be surpassed. (Venn 1880b, 338)

Thus, we clearly see here how Venn relativizes his assessment of logical forms to the particular aims one might have in using them: while he admits that the traditional forms might well not be best suited for giving an intuitive illustration of the relations in question or for solving complex

[9] See also Venn's discussion that different forms might be better suited to different languages in (Venn 1888).

reasoning tasks, he emphasizes that for the 'expression and improvement of ordinary thought and speech they are not likely to be surpassed'.

This fits well with his repeated insistence—whether expressed out of genuine conviction or as a way to appease the opponents of symbolic methods—that the traditional logic remains the best for teaching. As he puts it in the introduction to his book:

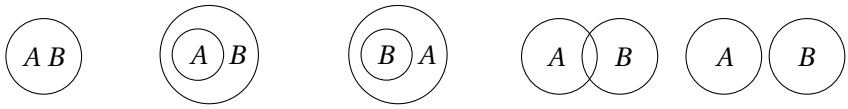
> No one can feel more strongly than I do the merits of [the traditional Logic] as an educational study. [...] [T]he forms of proposition in the ordinary logic are just those of common life with the least degree of modification consistent with securing accuracy of meaning. Common Logic should in fact be no more regarded as superseded by the generalizations of the Symbolic System than is Euclid by those of Analytical Geometry. And the grounds for retaining in each case the more elementary study seem to be identical. The narrower system has its peculiar advantage [...] being by comparison more concrete, it is easier for a beginner to understand [...]. I think then that the Common Logic is best studied on the old lines [...]. (Venn 1881, xxv–xxvii)

2.2 The class inclusion/exclusion view

Instead of following the grammatical subject–predicate structure, it is also possible to interpret the statement 'All whales are mammals' as expressing a relation between two classes—to wit, that the class of whales is included in the class of mammals. Indeed, Venn writes, '[i]t will hardly be disputed that every proposition can be so interpreted' (Venn 1880b, 338).

How many different forms does this understanding of propositions yield? In other words, in how many different ways can two classes A and B relate to each other? In this case, there are not four but *five* possibilities, which following Grattan-Guinness (1977) are often called the 'Gergonne

Class inclusion/exclusion view (diagrammatic and verbal forms):



All A is all B | All A is some B | Some A is all B | Some A is some B | No A is any B

Predication view (common logic forms):

$\begin{cases} \text{All } A \text{ is } B \\ \text{All } B \text{ is } A \end{cases}$ $\begin{cases} \text{All } A \text{ is } B \\ \text{Some } B \text{ is not } A \end{cases}$ $\begin{cases} \text{All } B \text{ is } A \\ \text{Some } A \text{ is not } B \end{cases}$ $\begin{cases} \text{Some } A \text{ is } B \\ \text{Some } A \text{ is not } B \\ \text{Some } B \text{ is not } A \end{cases}$ No A is B

Compartmental view (Boolean symbolic forms):

$\begin{cases} A\overline{B} = 0 \\ \overline{A}B = 0 \end{cases}$ $\begin{cases} A\overline{B} = 0 \\ \overline{A}B = v \end{cases}$ $\begin{cases} \overline{A}B = 0 \\ A\overline{B} = v \end{cases}$ $\begin{cases} AB = v \\ A\overline{B} = v \\ \overline{A}B = v \end{cases}$ $AB = 0$

Table 1: Translations of the five basic logical forms according to the class inclusion/exclusion view (represented diagrammatically as well as using Venn's proposed verbal equivalents) into traditional Aristotelian forms (corresponding to the predication view) and into Boolean symbolic forms (corresponding to the compartmental view). This table is adapted from (Venn 1881, 30); the braces should be read as *conjunctions*.

relations';[10] Venn illustrates them diagrammatically as shown in Table 1 (Venn 1880b, 339).[11]

The question then arises of how these five forms are related to the four (A, E, I, O) of the predication view. The answer is shown in Table 1. As can be seen, the different schemes do not correspond one-to-one: some forms of the predication view correspond to more than one form of the class inclusion/exclusion view, and vice versa. More precisely, we could say that most inclusion/exclusion forms can only be translated as *con-*

[10] This is in reference to Gergonne (1816), quoted by Venn himself (1881, 6).

[11] Note that distinct forms in this list can collapse into one unless all the portions of classes that are referred to are non-empty. Venn discussed this question at length elsewhere; see Sections 3.3 and 4.2 below.

junctions of predication forms (as indicated by the braces in Table 1), while most predication forms correspond to *disjunctions* of inclusion/-exclusion forms.

To recover a correspondence with verbal forms—that is, to express the new forms unambiguously 'in ordinary speech' (Venn 1881, 7) rather than through diagrams—Venn suggests using the five forms shown just below the diagrams in Table 1, where 'some' has to be interpreted as 'some, not all' and where the predicate is quantified. In this way, each form of the class inclusion/exclusion view corresponds unambiguously to a unique verbal form.

As Venn hastens to add, these modified predication forms seem similar to those of William Hamilton, who—in connection with his doctrine of the quantification of the predicate—had proposed to add the quantifiers 'all' and 'some' also to the predicates of the Aristotelian forms, thus doubling their number to eight (see, e. g., Hamilton 1860, 277):

Aristotelian forms		Hamilton's forms	
A	All A is B.	All A is all B.	All A is some B.
E	No A is B.	No A is any B.	No A is some B.*
I	Some A is B.	Some A is all B.	Some A is some B.
O	Some A is not B.	Some A is not any B.*	Some A is not some B.*

Indeed, Venn's five modified forms are among those of Hamilton, but the latter include three more (marked with an asterisk in the above table), which Venn criticizes as redundant:

> The Hamiltonian scheme has, no doubt, a specious look of completeness and symmetry about it. [. . .] But on subjecting [Hamilton's forms] to criticism, by inquiring what they really say, we see that this completeness is illusory. Regard them as expressing the relations of class inclusion and exclusion (and this I strongly hold to be the right way of regarding them) and we only need, or can find place for, *five*. Regard them as expressing to some extent our uncertainty about these class relations, and we want more than 8. This exact group of 8 seems merely the outcome of an exaggerated love of verbal symmetry. (Venn 1880b, 343)

Let us return to the class inclusion/exclusion view of statements. That the five basic forms can be straightforwardly depicted by diagrams, which yield visual representations of the relations between the classes in question, is considered by Venn an unmistakable advantage of this approach. After all, the diagrams can be easily apprehended and allow for relations between statements to be investigated almost empirically as relations between diagrams. Venn explains this by giving an illustrative example (the diagram was inserted by us into the quote):

> The advantages of this form of propositional statement, if few, are at any rate palpable and unmistakable. Each form has a corresponding diagram which illustrates its exact signification with the demonstrative power of an actual experiment. If any sluggish imagination did not at once realise that from 'All A is some B,' 'No B is any C,' we could infer that 'No A is any C,' he has only to trace the circles,
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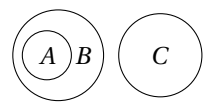
> 
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> and he sees it as clearly as any one sees the results of a physical experiment. And most imaginations, if the truth were told, are sluggish enough to avail themselves now and then of such a help with advantage. (Venn 1880b, 343–344)

Despite the diagrammatic representation of the class relations, the class inclusion/exclusion view of propositions has serious limitations with regard to its use:

> In spite of its merit of transparent clearness of illustration of a certain number of forms, it is far from answering our purpose as the basis of an *extension of Logic*. It soon becomes cumbrous and unsymmetrical, and has no flexibility or generality about it. (Venn 1880b, 345)

These limitations, according to Venn, are due to the restriction that the relations between only *two* classes are being represented. To achieve

greater flexibility and complete generality, an indefinite number of classes should be considered and relations between them that go beyond simple inclusion and exclusion:

> Any system which merely exhibits the mutual relations of two classes to one another is not extensive enough. We must provide a place and a notation for the various combinations which arise from considering three, four, or more classes; in fact we must be prepared for a complete generalisation. When we do this we shall soon see that the whole way of looking at the question which rests upon the mutual relation of classes, as regards exclusion and inclusion, will not suffice. There is a fatal cumbrousness and want of symmetry, about it which renders it quite inappropriate for any but the simplest cases. (Venn 1880b, 345)

These considerations lead to the third view regarding the import of propositions that Venn discusses, the compartmental view.

2.3 The compartmental view of symbolic logic

The 'compartmental view' is Venn's label for the analysis of the logical structure of propositions that underlies Boole's approach, where propositions are interpreted as 'indicating the *occupation or non-occupation of compartments*' (Venn 1880b, 345).[12] This can be considered an extension of the class view, since it allows for the treatment of any number of class terms and the expression of any relations between them. Thus, if an 'extended Symbolic Logic' is sought, this view is clearly superior than the ones previously discussed, but it is nevertheless 'just as simple and natural as either of them' (Venn 1880b, 345, 347).

For Venn, Boole's symbolic language is a 'simple and effective' language for representing all possible relations between classes. For example, given two class terms x and y, we can designate the four compartments into which they divide the universe as xy, $\overline{x}y$, $x\overline{y}$, $\overline{x}\,\overline{y}$ (where $\overline{x}$

[12] Heinemann argues that Jevons's approach also falls under this view, which she characterizes as a 'methodological principle' (Heinemann 2013, 43).

stands for the complement of x). A particular relation that is expressed by a proposition is then characterized by a specification of those compartments that are occupied and unoccupied. With 4 compartments, this yields 16 different possibilities, although Venn counts only 15, since he leaves out the case in which all compartments are empty (this would imply the emptiness of the entire universe of discourse, a possibility Venn rejects).[13] It should be noted here that Venn came up with his famous diagrams, in which empty compartments are represented by shaded areas, precisely to give an appropriate diagrammatic representation of Boolean forms, though he only discusses his diagrams elsewhere (in Venn 1880c and in Chapter V of his 1881 book).

Venn regards those propositions as the simplest that leave only one compartment empty. In Boole's notation they are expressed as

$$x\,y = 0, \qquad \overline{x}\,y = 0, \qquad x\,\overline{y} = 0, \qquad \text{and} \qquad \overline{x}\,\overline{y} = 0,$$

and are rendered in English by Venn as

'No x is y', 'All y is x', 'All x is y', and 'Everything is either x or y',

respectively. This comparison gives Venn the opportunity to remark on the fact that simplicity of a propositional form is a relative notion. In the compartmental view, the fourth form listed above

> is just as simple as any of the others; but in the traditional arrangement it would probably get in only as a disjunctive, since that arrangement dislikes the double negation 'No not-x is not-y'. (Venn 1880b, 347)

In contrast, the relatively simple expression 'All X is all Y' corresponds in Boole's system to the combination (in fact, conjunction) of two equations: $x\,\overline{y} = 0$, $\overline{x}\,y = 0$. Venn is well aware that the compartmental view might seem overly complicated when only two terms are considered, but argues that its advantages become obvious when several terms are taken into account at once, an issue we shall come back to in Section 4.

[13] In the article, Venn mentions 14 cases (Venn 1880b, 346), but this is an error that is corrected in the book (Venn 1881, 24).

Strikingly, while Venn presents Boole's account as a generalization of the old logic, the compartmental propositions as discussed so far (following the article version of Venn's discussion) appear insufficient to translate the forms of the other views. The most glaring example of this are the particular propositions of Aristotle, 'Some X is Y' and 'Some X is not Y', which plainly cannot be interpreted as asserting the emptiness of any compartments, but assert instead that some compartment is *not* empty.

We shall come back to this at greater length in Section 4, but let us note here that Venn briefly fills in the blanks in his *Symbolic Logic* (1881): while Chapter I of it reproduces his article (Venn 1880b) almost verbatim, it also contains a few additions, the most consequential of which are two new pages that flesh out the compartmental view beyond what we have mentioned so far. There, Venn introduces the indeterminate class symbol v, which can take any value *strictly between* 0 and 1 (this is, in fact, one of the places where Venn corrects Boole, as we shall see below), so that 'Some X is Y' can be translated as $xy = v$. Writing $xy = 1$ would mean, instead, that the compartment xy contains all elements of the universe of discourse, i. e., that *all other* compartments are empty. Thus, assuming that the universe of discourse (UD) is not empty, as Venn does, the following table sums up the information that a simple equation containing xy would give about the compartment in question and the other compartments:

	Compartment xy	Other compartments
$xy = 0$	empty	all elements of UD
$xy = v$	some elements of UD	some elements of UD
$xy = 1$	all elements of UD	empty

Using the indeterminate class symbol v, Venn then maintains that the compartmental view yields *twelve* basic propositional forms involving two terms x and y, namely

$$xy = w, \qquad x\overline{y} = w, \qquad \overline{x}y = w, \qquad \overline{x}\,\overline{y} = w,$$

where w can take any of the values 0, v, and 1 (Venn 1881, 31, 170, 358). How these can be used to express the forms of the class inclusion/exclusion view is shown, following Venn, in the last row of Table 1.

2.4 Assessment of the approaches to logical form

Let us now summarize Venn's discussion of the three accounts of logical form. The advantages of the predication view, also referred to as the 'old view' by Venn (1880b, 349), are that it is very simple and bears a close relation to ordinary language; in (Venn 1888, 415), he also argues that this form 'is of universal applicability' and best suited 'for the expression of new acquisitions or experience'. Venn also thinks that 'for ordinary educational purposes [it] will probably never be superseded' and that 'it possess a fine heritage of accurate technical terms and rules of application'. However, it lacks a 'correspondent diagrammatic system of illustration' and its 'want of symmetry forbids its successful extension and generalisation' (Venn 1880b, 349). The class inclusion/exclusion view has a 'transparent clearness of illustration' though its diagrammatic representation, which allows to '*intuit* the proposition'. However, just like the predication view, it lacks 'symmetry' and 'consequent adaptability of generalisation', and it is 'considerably removed from popular forms of expression'. Moreover, in Hamilton's version, the approach is actually deemed to be 'inconsistent' by Venn. Finally, the compartmental view strikes one as an 'artificial scheme', 'couched in too technical form' and 'too far removed from the language of common life'. However, in regard to 'symmetry [...] and the power that comes with it, nothing can well be put into competition with it' (Venn 1880b, 349). As becomes clear in the book, this power is exhibited particularly in its ability to provide general methods of solving intricate logical problems.

With regard to the number of basic propositional forms, we have seen that the predication view of common logic has four, the class inclusion/exclusion view has five (or eight, in Hamilton's version), and the compartmental view twelve (when only two terms are considered). This is in keeping with Venn's conviction that 'there can be no absolute arrangement of propositional forms. The number and grouping of our forms must depend upon the fundamental view we take as to what should be the import of a proposition' (Venn 1881, 171).

2.5 The approach Venn missed: Frege

A contemporary reader might be surprised by the absence, in Venn's discussion, of any reference to Frege's analysis of propositions. Although Frege's well-known paper 'Function und Begriff' was published only in 1891, his analysis of propositions in terms of functions and arguments is presented already in § 9 of his *Begriffsschrift* (1879), which Venn reviewed in *Mind* (Venn 1880a) just around the time he was working on his book on symbolic logic. Nevertheless, Venn did not appreciate the innovative character of Frege's approach. He notes that Frege does not refer to Boole and thus might have developed his system independently, but ultimately concludes: 'it does not seem to me that Dr. Frege's scheme can for a moment compare with that of Boole' (Venn 1880a, 297). For Venn, what Frege claims to be novel in his approach 'is common to every symbolic method' and well known to British logicians:

> [Frege] calls attention to the fact that, on his scheme, the distinction, so important in grammar and on the predication view of ordinary logic, between subject and predicate loses all its significance, [...] and so on; all these being points which must have forced themselves upon the attention of those who have studied this development of Logic. (Venn 1880a, 297)

Our analysis above helps us make sense of Venn's reaction. Standard historical narratives typically credit Frege with the overthrow of the subject–predicate analysis of propositions, in keeping with Frege's own remarks at the beginning of the *Begriffsschrift*. For Venn, however, such a move was implicit throughout Boolean logic (and even in the simpler class inclusion/exclusion view), making Frege's claims sound like reinventions of the wheel. Meanwhile, the power of Frege's system to tackle relations and nested quantifications would not have been immediately apparent to Venn.

Instead, it seems that it was mainly the two-dimensional character of Frege's notation that caught Venn's attention. And while Venn notes the danger of bias when assessing unfamiliar systems of logic, such that 'they will almost necessarily appear cumbrous and inconvenient to those who have been accustomed to make use of some different system', he con-

cludes his brief review by writing: 'I have not made myself sufficiently familiar with Dr. Frege's system to attempt to work out problems by help of it, but I must confess that it seems to me cumbrous and inconvenient' (Venn 1880a, 297).[14]

Venn noted, as a drawback of Boole's system, that its appearance was unfamiliar; this criticism would certainly also apply to Frege's. However, given its explicit symbolization of quantifiers that makes it more similar to ordinary language, its two-dimensional presentation that exhibits the hierarchical logical structure visually, and its expressive power, it might seem that according to Venn's own criteria, summarized in Section 2.4, he should have held Frege's system in high esteem, had he taken the time to better understand it.

More realistically, though, the change of point of view between Venn's logical framework—which, despite the shift from the old logic to Boole's, is still mostly concerned with simple relations between classes—and Frege's new approach is so profound that it is probably unsurprising that the point of Frege's work largely escaped Venn. Schröder, the most prominent Boolean logician in Germany, similarly reviewed Frege's work and utterly misunderstood it, likewise stating that everything of interest in it was already in Boole (see Peckhaus 2004a;b).

3 Venn's broader views on logic

3.1 Pluralism

Let us now return to the problem referred to in the opening passage of Venn's 'On the forms of logical propositions' (quoted at the beginning of Section 2, above), namely the determination of 'the number and arrangement of the simple forms of proposition'. For Venn, the disagreement among logicians on this issue is due to a more fundamental disagreement about what propositions are in the first place. Only once we have agreed on a particular view of propositions does it make sense to determine the number of basic logical forms. However, unlike many of

[14] See Schlimm (2018) for a more detailed assessment of Frege's notation and for other reactions to it by his contemporaries.

his contemporaries, Venn does not think that there is a unique answer to the more fundamental problem. He writes:

> There is no occasion whatever to tie ourselves down to one view only, as if the import of propositions was fixed and invariable. Very likely other views might be introduced in addition to the three which have been thus examined [...]. Each of these three stands upon its own basis, yields its appropriate number of fundamentally distinct propositions, and possesses its own merits and defects. (Venn 1880b, 348–349 = 1881, 28)

Almost a decade later, Venn again reiterated this pluralist attitude towards logical forms. In his *The Principles of Empirical or Inductive Logic* (1889), he writes:

> In speaking, as we have been doing, of three distinct renderings of the import of a proposition, and the consequent distinct schedules of propositional forms which have to be drawn up, the reader must be on his guard against a possible misunderstanding. There is no question here of right or wrong; we are not now deciding between the claims of hostile theories. Nothing more serious is at stake than a question of convenience and of efficiency of method. There has been far too much of a disposition on the part of logicians to consider that there must necessarily be some one correct view as to the import of propositions, and that therefore in deciding for one they must reject others. They have always retained something of the theologian's spirit. (Venn 1889, 230)

A key term in this quote is that of 'theories'; notice that Venn, perhaps surprisingly, does not regard the choice of one single theory as the task of logicians. Thus, to better understand his view on logic, we need to look a closer look at Venn's understanding of logical theories.

3.2 Instrumentalism: Theories vs. methods

The 'hostility' that Venn mentioned in the previous quotation between different theories is due to fact that he considers a theory to be 'the *true*' and 'most fundamental' account of a subject matter, which he contrasts with the 'methods' that are employed in the study of a subject matter. With regard to logic, he writes:

> If we were constructing a complete theory of Logic we should have to ask what is the *true* account, by which we should understand the most fundamental account, of the nature and import of a proposition, and on this point different accounts would be in direct hostility to one another. But when we are discussing methods rather than theories, this is not necessary. (Venn 1880b, 336)

On what constitutes the fundamental character of theories of logic for Venn, the following quote gives us an indication:

> It may fairly be maintained that one of these views must be more fundamental than the others, or possess a better psychological warrant, but it cannot be denied that they are all three tenable views; that is, that we may, if we please, interpret a proposition in accordance with any one of the three. (Venn 1880b, 336)

Notice Venn's insertion of 'or possess a better psychological warrant' when mentioning the claim that one view must be 'more fundamental' than the others. Thus, at least with regard to logical theories, Venn considered the psychological adequacy as an important criterion for determining the true account. Indeed, when discussing the class inclusion/exclusion view, he wrote:

> [...] this interpretation may not be the most fundamental in a psychological sense; but when, as here, we are concerned with logical methods merely, this does not matter. For the justification of a method it is clearly not necessary that it should spring directly from an ultimate analysis of the phenomena;

> it is sufficient that the analysis should be a correct one. (Venn 1880b, 338)

Thus, the only necessary condition for a method, according to Venn, is its 'correctness', by which he might mean that it leads to the correct conclusions as to what follows from what. (Indeed, Venn never seems to doubt that there are right and wrong answers to such questions, in contrast to today's logical pluralists; in contemporary terms, he does not seem to be a pluralist about logical consequence, a point we shall come back to in Section 5.) But, as we have seen in his discussion of the advantages and disadvantages of the various views of logical propositions in Section 2, different methods might, for instance, be most useful for beginners to discern what follows from what in simple cases, or for more advanced logicians to solve complex problems. It seems therefore appropriate to consider Venn an *instrumentalist* with regard to accounts of logical form.

3.3 Conventions and the objectivity of logic

Instrumentalist themes also emerge in Venn's general attitude towards logic.[15] In 'The difficulties of material logic' (1879a), Venn referred to the distinction between the 'Science of Logic' and the 'Theory of Reasoning' made in Herbert Spencer's *The Principles of Psychology* (1872); he quoted the following passage as 'the best exposition perhaps' of the objective view of logic, which he considered to be the 'essentially sound view' (Venn 1879a, 36):[16]

> The distinction is, in brief, this, that Logic formulates the most general laws of correlation among existences consid-

[15] Our focus here is on Venn's writings from around 1880, at the time he devised his account of logical form. However, some of the ideas we present go back to earlier work, and were further articulated in Venn (1889). For a broader discussion, see Verburgt (2022a, Chapter 11).

[16] This passage from Spencer is also quoted in Read's *On the Theory of Logic* (1878, 11) as the best characterization of the materialist view of logic, which Read takes from Mill and Bain. Read's book was reviewed by Venn before writing the paper under discussion (Venn 1878). It might well be that this review prompted Venn to write on 'The difficulties of material logic' (1879a) in the first place.

> ered as objective; while an account of the process of Reasoning, formulates the most general laws of correlation among the ideas corresponding to those existences. (Spencer 1872, 87)

Venn immediately clarifies that 'the objectivity here referred to does not in any way imply acquaintance with more than phenomena'—it does not refer to 'things in themselves' as opposed to 'things as presented to us'.[17] Rather, one should properly call 'objective' the ideal, perfected state of human knowledge:

> My knowledge of the 'thing' is very inaccurate and defective; this imperfect presentation of it is my conception or idea of it, and we term it subjective. But suppose this knowledge, always within the range of phenomena, developed and perfected to the utmost attainable degree; let it be determined with all the accuracy which present or future methods of measurement may invent; let this knowledge receive the final and general assent of mankind,—and we should then have obtained what we may call objective knowledge. [...] [T]his knowledge thus rendered final and general *is*, for all practical and speculative purposes, the same thing as the sum-total of "existences considered as objective" which, according to the above extract, is to be regarded as the subject-matter of Logic.[18]

However, Venn wrote, 'any such attainment as this of objective knowledge is at present indefinitely remote'. So adhering strictly to Spencer's distinction is impossible; while an objective logic should be the goal, we can only achieve an approximation of it that relies on (subjective) 'assumptions or conventions' (Venn 1879a, 47).

An example will clarify the kind of conventions Venn has in mind. Should the proposition 'All A is B' be taken to imply that there exist As and Bs? From an objective point of view, writes Venn, the answer should be yes,

[17] Venn (1879a, 36–37).
[18] Venn (1879a, 37).

> for otherwise the proposition would not be a true one; or rather, by not saying that existence is implied, we should be losing our hold of that distinction between truth and falsehood [...] which it is the main prerogative of an objective Logic to keep clearly before us. (Venn 1879a, 41)

Indeed, insofar as the proposition is taken to be about actual things (rather than about our limited knowledge of things), if there are no *A*s in the first place, then 'All *A* is *B*' and 'No *A* is *B*' become one and the same. Likewise, 'No *C* is *D*' implies that there exist *C*s. However, from an objective point of view, it *does not* imply that there exist *D*s, 'for take the proposition "No object possesses a temperate below 280°C". The very meaning of the proposition denies the existence of its predicate.'[19]

The problem with such a strictly objective point of view is that it is incompatible with standard rules, including *conversion* (which allows transforming 'No *C* is *D*' into 'No *D* is *C*', hence would force the existence of *D*s also) and *contraposition* (which allows transforming 'All *A* is *B*' into 'No not-*B* is *A*' and conversely, hence forcing the existence of not-*B*s also); accordingly, it would lead one to sacrifice much of the simplicity of traditional logic:

> If we chose to adhere to our strict logical view with punctilious accuracy, we should have to lay down our rules somewhat as follows:—In an Affirmative Proposition the subject and predicate distinctly imply the existence of their objects; but, as we must appeal to experience to make sure of the existence of their contradictories, we have no right without due inquiry to contraposit such a proposition. In a Negative Proposition the subject must exist, but not necessarily the predicate (for negation does not carry existence with it). Accordingly we have no right without due examination even to convert a negative proposition. (Venn 1879a, 41–42)

Venn's conclusion is that what logic actually does is to make 'assumptions or conventions' (such as that of taking *all* terms mentioned to exist)

[19] Venn (1879a, 41).

that are not justified by a strictly objective point of view, but that allow for a workable system:

> It is clear therefore that what we really do is take a licence or make a convention for convenience sake. (Venn 1879a, 41)

In sum,

> it seems better not to claim an objectivity unattainable at present, but to admit frankly that our processes and results in Logic are conditioned on every side by subjective or relative considerations. Our logical machinery and technical phraseology can only be interpreted by the help of numerous assumptions or conventions; relative, not merely to human intelligence in general but, more narrowly, to the amount and distribution of the knowledge of the persons who have to use the Logic. (Venn 1879a, 46–47)

This is what Verburgt (2022b) calls Venn's 'conventionalism', further developed in Venn (1889) and in an unpublished lecture delivered in 1889 (Verburgt 2021, 87).

However, objectivity remained for Venn an ideal to be strived for. Regarding the existential import of propositions in particular, he believed that Boolean symbolic logic suggested a more satisfactory set of conventions than traditional logic. We shall come back to this in Section 4.2.

4 Venn's balancing act

In his defense of symbolic logic, and in particular of Boole's system, Venn is at pains to show that it does not stand in opposition to traditional methods but is wholly compatible with them; his statement from the introduction to his *Symbolic Logic* is that 'the Common Logic is best studied on the old lines' and that 'the Symbolic Logic should be regarded as a Development or Generalization of it' (Venn 1881, xxvii). Yet he also wants to emphasize how profoundly the new methods should transform our view of the entire subject; for instance, in his book he prefaces his discussion of the forms of proposition by asserting 'that the System of Logic which this

work is intended to expound is not merely an extension of the ordinary methods—though this is perhaps its principal characteristic—but that it also involves a considerable change from the ordinary point of view' (Venn 1881, 1). His general introduction also shows his efforts at striking a balance between these two claims:

> A thorough generalization assumes sometimes an entirely unfamiliar aspect to those who were previously acquainted only with some very specialized form of the generalized process [...]. In such cases the realization of the generalization may amount almost to the acquisition of a new conception, rather than to the mere extension of one with which we were already intimate. (Venn 1881, xxi)

Our aim here is to show that Venn's account of logical form is a crucial piece in his efforts to strike a balance between these two claims. We shall also show that this perspective sheds light on a knotty part of Venn's account, namely his somewhat inconsistent treatment of particular propositions.

4.1 Equivalent but revolutionary: the new logic between radical innovation and compatibility with tradition

On the one hand, Venn seems to say that the new logical forms of Boole's logic do not offer more expressive power that the old, Aristotelian ones:

> [B]y a little management [the traditional four forms] can be made to express nearly all the simple forms of assertion or denial which the human mind can well want to express. [...] By combining two or more of them together they can readily be made equivalent to much more complicated forms. Thus, by combining 'All X is Y' with 'All Y is X,' we obtain the expression 'All X is all Y,' or 'X and Y are coextensive,' and so forth. [...] [A] combination of two or more of these forms will express almost anything in the way of a non-numerical statement. (Venn 1881, 3–5 = 1880b, 337-338.)

On the other hand, he seems to contradict himself a few pages later, apparently asserting that the new forms are indispensable for cases involving several terms at once:

> The full merits of this [compartmental] way of regarding and expressing the logical proposition are not very obvious when only two terms are introduced, but it will readily be seen that *some such method is indispensable* if many terms are to be taken into account. (Venn 1881, 26, our emphasis = 1880b, 347, with 'real merits' instead of 'full merits'.)

Reading further, however, clarifies how Venn resolves this apparent contradiction:

> Let us introduce three terms, x, y and z; and suppose we want to express the fact that there is nothing in existence which combines the properties of all these three terms, that is that there is no such thing as xyz. If we had to put this into the old forms we should find ourselves confronted with six alternative statements, all of them tainted with the flaw of unsymmetry; viz. No x is yz, No y is xz, No z is xy, and also the three converse [...] No reason could be shown for selecting one rather than another of them; and if we attempted to work with the symmetrical form 'There is no xyz,' we should find that we had no supply of rules at hand to connect it with propositions which had only x, y, or z, for subject or predicate. (Venn 1881, 26 = 1880b, 347.)

In other words, the deficiency of the old logic does not lie in any *inability* to express propositions involving many terms, but in the fact that it could only express them in too complicated a way and, more importantly, that it would lack *methods* appropriate to handle them. In sum, there is a sense in which the old and the new forms are equivalent, while leaving room for the new ones to ground a much more powerful logic.

The rest of the book illustrates how, despite the expressive equivalence, Boole's system can transform one's entire view of logic. In particular, Chapter XXVII, 'Generalizations of the Common Logic', shows how the

main concepts and methods of the old logic are altered beyond recognition when examined from the new perspective: of course, as already mentioned, the 'schedule of propositional forms' will be transformed; the concepts of contrariety and contradiction will, basically, have to be replaced by complementation; and instead of the syllogism and conversion, the main methods become elimination (a general version of the syllogistic elimination of the middle term) and a general method of 'inference', consisting in the algebraic transformation of the form of premises. A deeper exploration of these issues goes beyond the scope of this paper.

4.2 A tension in Venn's account: existential import and particular propositions

One example of a change in viewpoint afforded by the new logic deserves further discussion, both because it is given pride of place by Venn as clarifying an intractable difficulty of the old logic, and because it highlights the tension in his efforts to insist on the equivalence of the different accounts of propositions while defending the contributions of the symbolic approach. The example in question is Venn's theory of the 'existential import' of propositions, and, more pointedly, his account of the traditional 'particular' propositions (i. e., 'Some A is B' or 'Some A is not B').

The question at issue is whether propositions mentioning a certain term, say A, should be taken to imply that the corresponding class is non-empty, that is, that there are As in the universe of discourse. As mentioned above (Section 3.3), Venn argued that in the old logic this question does not admit of a satisfactory solution; he discussed this at length in Chapter VI of Venn (1881). Asserting that propositions *always* or *never* imply that the classes mentioned are non-empty is implausible, he wrote. Moreover, the accepted rules of traditional logic make it impossible to find a middle ground: on top of conversion and contraposition, discussed above, the *Darapti* syllogism allows us to infer 'Some X is Z' from 'All Y is Z' and 'All Z is X', so that we will have to admit existential assumptions for universal propositions if we do so for particular propositions.

Venn argued that Boole's logic lends itself to a neat solution. In keeping with our discussion in Section 3.3, a solution here means a set of conventions determined by considerations of 'convenience and consis-

tency in the working out of the Symbolic or Generalized Logic'.[20] Moreover, once more, Venn insisted that his solution, though suggested by the Boolean framework, is not intrinsically tied to it: 'once realized it will be found to apply also to the ordinary interpretation of the proposition'.[21]

Venn's account is based on the fact that the central procedures in Boole relied on reducing propositions to lists of what Venn sometimes called 'elementary denials' (i. e., in the language of the above, to assertions that some compartment is empty). As we have seen, the proposition 'All X is Y', for instance, just says that the compartment $x\overline{y}$ is empty. Viewed in this light, propositions never imply the existence of anything, except possibly in the limit, if all compartments save one have been deemed empty (for Venn always assumes that the universe of discourse cannot be empty). Any further existential assumptions will have to be made explicit and treated on a par with particular propositions, which do assert existence and require a separate symbolic treatment using the special-purpose indeterminate class symbol v, which can take any value *save 0 or 1*, i. e., can refer to any class except the empty one and the universe of discourse. 'Some X is Y' will then be written $xy = v$. Inferring 'Some X is not Y' from 'No X is Y' then requires an additional existential assumption that should be made explicit, namely the assumption that there is some X; in other words, such an inference is only valid if 'No X is Y' is implicitly taken to mean $xy = 0$ together with $x\overline{y} = v$ (Venn 1881, 360).

However, such subtleties are omitted from Venn's discussion of logical forms. The problem is that the use of the symbol v, and hence the treatment of particular propositions in general, is a rather uneasy addition to Boole's system. Boole's treatment of particular propositions had problems, as he used indeterminate class symbols like v in an ambiguous manner, without clearly specifying whether $v = 0$ was allowed or not; he has often been criticized for this.[22] Venn admitted as much and corrected Boole on this point by requiring v to be a non-empty (and non-universal) class.[23] This restriction, however, meant that v could no longer

[20] Venn (1881, 141)

[21] Venn (1881, 141).

[22] See, e. g., Dummett (2000, 79–80).

[23] See in particular Chapter VII of Venn (1881).

be freely manipulated; as Venn wrote, the use of v is only fine as long as one is careful only to use a limited subset of Boole's methods (excluding the most general and powerful, since these rely on the development of propositions into elementary denials, which particular propositions are not capable of).[24] Surprisingly, Venn's solution then seems to be to banish particular propositions from logic as much as possible:

> For the purposes of the higher generalizations it does not seem to me as if any theory were yet proposed, which would answer except for a dichotomous scheme, represented symbolically by 1 and 0 [i. e., with no v] [...] Into such a dichotomous scheme truly particular propositions will not apparently fit, and they have accordingly to be rejected from all the higher generalizations. If such propositions were of real scientific importance, or forced themselves into many of our familiar problems, this inability to grapple effectually with them would certainly be a blemish in the Symbolic Logic. As it is, we can afford to part with them without much sense of loss. (Venn 1880c, 361)

Venn returned to the question of particular propositions in a 1883 review of the work of the Peircean school, and in particular of Christine Ladd, who attempted a full treatment of the syllogism that included particular propositions. He maintained that particular propositions did not admit of a satisfactory general treatment. Asking 'whether any perfectly general treatment of [particular propositions] is available, that is, corresponding in generality and brevity to those which Boole has given and which have been simplified in their practical employment by a succession of writers', he answered: 'I am inclined to think that it is not' (Venn 1883, 598). A further detail brings home how little regard Venn, from his Boolean vantage point, had for particular propositions: the diagrams he designed to represent compartmental propositions (the famous 'Venn diagrams') initially did not include a device for representing existential claims: both his first article on the topic (Venn 1880c) and Chapter V of his book (Venn 1881) just covered universal propositions, and it is only in the 1883 review just

[24] Venn (See 1880c, 360–361).

discussed that he first hinted, incidentally, at a way of extending his diagrams to particular propositions (Venn 1883, 599–600). (For further discussion of Venn's successive versions of his diagrams, including for particular propositions, see Verburgt 2022b, 187–194.)

The tension underlying Venn's rhetorical strategy is here made apparent. He seemingly thought that symbolic logic, if followed to its logical conclusions, would lead one to eliminate particular propositions as much as possible. Yet this is in conflict with his efforts to paint the various accounts of propositions as equivalent. This tension, we suggest, helps explain the peculiarities of his expositions of the forms of propositions. As remarked above, in the paper version of his discussion (Venn 1880b), Venn's account of the 'compartmental' view is limited to universal claims, thus presenting a view that is faithful to the new logic, but inadequate to express a good half of the traditional view (though he did not mention this). In his book, he did introduce the symbol v, thus obtaining a 'schedule of propositional forms' that was expressively equivalent to that of the old logic; but he only did so fleetingly at the end of the chapter, without insisting upon the difficulties v created for Boole's general methods.

5 Situating Venn's pluralism philosophically

Let us now attempt to situate, with respect to today's philosophical debates, Venn's pluralism regarding logical forms. As we shall see, his is a peculiar pluralism by contemporary standards: it is not about consequence or about what should count as 'logical', but about ways of regimenting sentences for the purposes of logical investigations.

We begin with a few remarks about what Venn's position is not. While his 'conventionalism' (as discussed in Section 3.3) might in principle open the door to it, Venn did not frontally discuss the possibility of pluralism about *logical consequence*—about what follows from what. In this, he is far removed from the positions usually called logical pluralism today, the best-known of which is Beall and Restall's,[25] as well as from the pluralism

[25] Beall and Restall (2006). See for instance Shapiro (2014) and Hjortland (2017) for a discussion of a broader variety of views.

sometimes ascribed to Venn's contemporary Hugh MacColl.[26] He did not seem concerned, either, with demarcating the domain of the logical; in the context of his discussion on the forms of propositions, he seems to take it for granted.

Instead, one may initially be tempted to compare Venn's discussions with contemporary debates about the nature of propositions. Today, we might argue about whether propositions are sets of possible worlds, or some kind of structured entities (say Fregean or Russellian propositions). Different views on the nature of propositions lead to different ways of *individuating* them: for instance, if propositions are sets of possible worlds, then all sentences that are true in all possible worlds express *the same* proposition, whereas they will usually express different Fregean or Russellian propositions. Likewise, the views Venn compares seem to be about what propositions are: specifically, whether they are relations between a class and a predicate, relations between two classes, or assertions of the emptiness or non-emptiness of some of the 'compartments' defined by some classes. These positions lead to different ways of individuating propositions. For instance, on the traditional (predication) view, 'No whales are birds' and 'No birds are whales' are different, while on the two other views, they express the same proposition.

The analogy stops there, however. Contemporary views that identify propositions with sets of possible worlds, say, do not necessarily come with specific ways of representing propositions verbally or symbolically. In contrast, Venn is only interested in answers to the fundamental question of the nature of propositions insofar as they provide general principles for *regimenting* sentences using specific technical forms.

The 'logical forms' one might describe Venn as a pluralist about are precisely such technical forms used to regiment sentences; his argument is that several choices of such forms can be made, each best suited for specific purposes. This may bring to mind Carnap's 'principle of tolerance', much discussed in recent years by logical pluralists (see, e. g., Restall 2002): indeed, this principle implies that questions of logical form are (at least in part) a matter of which language one chooses in order to recon-

[26] On what has been described as Hugh MacColl's instrumentalism and pluralism about logic, see Grattan-Guinness (1999) and Rahman and Redmond (2008).

struct some inferential practice, and that ultimately, choices of language can only be made on pragmatic grounds.[27] But once more, Venn's position is less radical. His pluralism about the regimentation and individuation of propositions is not linked to a pluralism about what counts as logical, or about which relations of consequence hold between sentences. It does, however, foreground finer-grained pragmatic consequences of language choice.

6 Conclusion

Venn's work on logic, culminating in his *Symbolic Logic* (1881, 2nd ed. 1894), was first and foremost an exposition and defense of Boole's symbolic approach; indeed, Venn is possibly the only prominent author who endorsed Boole's system in all its details, keeping such features as the exclusive interpretation of logical addition or Boole's division of class terms. However, whether out of strategy or genuine conviction, Venn insisted that the symbolic logic was not opposed to traditional Aristotelian methods, but could co-exist side-by-side with them, each being best suited for its own ends—with the old logic particularly appropriate for teaching beginners. Since he recognized that these different logics offered different analyses of the 'forms of propositions', he was led to a striking kind of instrumentalist pluralism with respect to the different ways we might choose to regiment sentences of natural language for logical purposes. From a contemporary perspective, this account is surprising and original, for at least three reasons.

First, Venn's view on the forms of propositions is historically surprising because it is formulated within the confines of an old-fashioned logic of terms and classes, whereas we typically associate the very idea of a 'logical form' of sentences—which would underlie their use in *reasoning* and may well be different from their *grammatical* structure—with the work of Frege and Russell. Admittedly, if one looks closely enough, the Aristotelian tradition too is in the business of describing covert logical structures. For instance, as Pietroski (2021) emphasizes, it was customary to

[27] On Carnap's principle of tolerance, see Leitgeb and Carus (2020, in particular supplements H and D); on its connection with logical form, see Pietroski (2021, section 6).

fit sentences like 'Every star twinkles' into the predication mold by paraphrasing them as 'Every star is a twinkling thing'; still, the divergence with natural language 'was held to be relatively minor' (Pietroski 2021, section 2)—in Venn's own words, '[i]t is often, as we know, difficult to say what is a grammatical and what a logical question, owing to the fact that the forms of proposition in the ordinary logic are just those of common life with the least degree of modification consistent with securing accuracy of meaning' (Venn 1881, xxvi). Instead, the idea of substantial and systematic discrepancies between logic and language is commonly associated today with examples like Russell's (1905) analysis of 'The present King of France is bald' as '$\exists x\,(Fx \wedge \forall y\,(Fy \rightarrow y = x) \wedge Gx)$', or Frege's famous distinction between the 'is' of identity and the 'is' of predication, e. g., in the verbally similar but logically very different sentences 'The Morning Star is Venus' and 'The Morning Star is a planet' (Frege 1892, 194 = Frege 1984, 183). The interesting point here is that Venn drew an analogous lesson—that one can fruitfully offer *logical* analyses of propositions that diverge significantly from Aristotelian ones and from natural language—but on the basis of Boole's arguably much less thoroughgoing reconception of logic. Incidentally, this helps us understand why Frege's claims of having replaced the subject–predicate analysis of propositions struck Venn as unoriginal.

Second, Venn's pluralism with respect to logical form, in stark contrast to positions normally called 'logical pluralism' today, does not involve intuitionistic or other non-classical logics; it does not involve pluralism about logical consequence or about the boundaries of the logical. Nowhere does Venn suggest that adopting Boole's 'compartmental' view of propositions instead of the traditional 'predication' view might change what can correctly be deduced from given premises; he also explicitly says, as we have seen, that by combining several Aristotelian forms one can express everything one might ever need—in other words, that the different systems have the same expressive power. While in principle, his idea that logic relies on conventions—discussed in Section 3.3—could open the door to a more thorough-going pluralism, such a possibility does not play a role in his discussion of the forms of propositions.

In this sense, one might be tempted to say that the differences between the views examined by Venn are 'merely pragmatic'. But—and this is our third point—what Venn offers is essentially an extended argument that such pragmatic differences *do* matter: by allowing to systematically express propositions involving an arbitrary number of terms in a way that lends itself to a general, methodical treatment, Boole's system offers a new systematization of logic that sheds a new light on the subject. It yields a different way of individuating propositions; instead of contrariety and contradiction, *complementation* becomes salient; instead of conversion and the syllogism, *elimination* and Boole's algebraic solution method take center stage; and it lends itself to a clear, explicit treatment of the existential import of propositions. This, then, is what Venn's account of logical form—despite its restriction to very simple class logics, and its lack of discussion of any broader pluralism about logical consequence—can bring to contemporary discussions of the variety of possible logical systems: it highlights the way seemingly 'pragmatic' differences, even among logics that are in some sense *equivalent*, can end up substantially transforming our view of the subject.

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