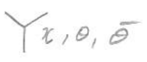
Renormalization and supersymmetry

Renormalization is under better control in a supersymmetric theory. Some supersymmetric theories are finite and do not require any renormalization at all.

Constraints on renormalization → non-renormalization theorems.
Historically shown using supergraph techniques.



$\int d^4x \qquad\qquad\qquad \int d^4x\, d^2\theta\, d^2\bar\theta$

propagators become superpropagators.

Main result

In a theory of chiral and vector multiplets the most general term generated by loop diagrams is of type

$$\int \underbrace{d^4x_1 \dots d^4x_n}_{n \text{ space integrals}} \underbrace{d^2\theta\, d^2\bar\theta}_{\text{only one superspace}} F_1(x_1,\theta,\bar\theta)\dots F_n(x_n,\theta,\bar\theta)\, \underbrace{G(x_1,\dots,x_n)}_{\text{translational invariant}}$$

It follows immediately that this can not modify the superpotential. In other words, the superpotential is tree-level exact (not renormalized at loop level). There can be non-perturbative corrections to the superpotential.

In contrast, the Kähler potential is renormalized and in fact receives corrections to all orders in perturbation theory and non-perturbatively.

Examples

* $\int d^4x\, d^2\theta\, d^2\bar\theta\ \bar\Phi\phi \quad \Rightarrow \quad Z_\phi \int d^4x\, d^2\theta\, d^2\bar\theta\ \bar\Phi\phi$ where Z_ϕ is a real "wave-function" renormalization

$\Rightarrow \quad \phi \to Z_\phi^{1/2}\phi \;, \quad \bar\Phi \to Z_\phi^{1/2}\bar\Phi$

Since the superpotential is unrenormalized, we have

$m\phi^2 \;\to\; Z_m Z_\phi\, m\phi^2 \qquad \Rightarrow \qquad Z_m Z_\phi = 1$

$\lambda\phi^3 \;\to\; Z_\lambda Z_\phi^{3/2}\, \lambda\phi^3 \qquad \Rightarrow \qquad Z_\lambda^2 Z_\phi^3 = 1$

* $\int d^4x\, d^2\theta\, d^2\bar\theta\ \bar\Phi e^{2V}\phi$

If V were to renormalize in a non-trivial way, $V \to Z_V^{1/2} V$, then the gauge transformations would have to change

$e^{2V'} = e^{-i\bar\Lambda} e^{2V} e^{i\Lambda} \quad \longrightarrow \quad e^{2Z_V^{1/2}V'} = e^{-i\bar\Lambda} e^{2Z_V^{1/2}V} e^{i\Lambda}$

In components, this can be seen by looking at the field strength

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] \qquad \Rightarrow \qquad (\partial_\mu A_\nu - \partial_\nu A_\mu) Z_A^{1/2} - i[A_\mu, A_\nu] Z_A$

$A_\mu \to Z_A^{1/2} A_\mu$

and the gauge transformation properties are changed.

The kinetic term

$$\mathrm{Im}\left(\tau \int d^4x\, d^2\theta\, \mathrm{tr}(W^\alpha W_\alpha)\right) \quad , \quad \tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

naively looks like an F-term (integral over half the superspace) so one might expect it not to be renormalized. However, this is incorrect since it is possible to write this chiral integral as a full superspace integral. Said differently, one can generate a full superspace integral which then can be brought to this chiral form. This can not be done for the superpotential in the Wess-Zumino model.

In conclusion, there are two renormalizations Z_ϕ and Z_τ (logarithmically divergent at one loop).

Holomorphy and non-renormalization

Idea: Each complex coupling can be thought as the lowest component of a very heavy chiral superfield. This imposes restrictions on the physics.

Example Wess-Zumino model

$$W_{tree} = \frac{1}{2} m \phi^2 + \frac{1}{3}\lambda \phi^3$$

What is W_{eff} which includes quantum corrections?

- promote m and λ to spurion fields
- this new theory has enlarged symmetry (before m, λ were constants, now we allow ourselves to transform them). The new symmetries are an $U(1)_R$ symmetry and a flavor $U(1)$ symmetry with charges

	$U(1)_R$	$U(1)$
ϕ	1	1
m	0	-2
λ	-1	-3

W_{tree} has $U(1)_R$ charge 2 and flavor $U(1)$ charge 0.

- the most general function of ϕ, m, λ with correct $U(1)_R$ charge and invariant under $U(1)$ flavor is

$$W_{eff} = \frac{1}{2} m \phi^2 f\left(\frac{\lambda\phi}{m}\right) \qquad \text{where} \quad f_{tree}(x) = 1 + \frac{2x}{3}$$

- take the weak coupling limit

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$\Rightarrow \quad a_0 = 1, \quad a_1 = \frac{2}{3}$$

The general term is $\frac{1}{2} m \phi^2 a_n \frac{\lambda^n \phi^n}{m^n} = \frac{1}{2} a_n \lambda^n \phi^{n+2} m^{1-n}$

- If we impose that the limit $m \to 0$, $\lambda \to 0$ so that $\frac{m}{\lambda} \to 0$ exists $\Rightarrow a_{n\geq 2} = 0$

$\Rightarrow W_{eff} = W_{tree}$