Supersymmetric standard Model

The Standard Model is a gauge theory with gauge group $SU(3)\times SU(2)\times U(1)$ (actually a $\mathbb{Z}_6$ subgroup leaves all the particles invariant so the right gauge group is a $\mathbb{Z}_6$ subgroup).

The gauge bosons corresponding to the three gauge are denoted by A^a_μ where $a=1,2,3$

The matter fields are

- quarks } three generations.
- leptons }
- Higgs

We consider separately the left and the right parts of quarks and leptons since they transform under different representations of the gauge groups. ($I=1,2,3$ labels the generation)

quarks	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_{em}$
$q^I_L=\begin{pmatrix}u^I_L\\ d^I_L\end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$
u^I_R	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
d^I_R	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$

leptons	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_{em}$
$l^I_L=\begin{pmatrix}\nu^I_L\\ e^I_L\end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix}0\\ -1\end{pmatrix}$
e^I_R	1	1	1	1

Higgs	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_{em}$
$h=\begin{pmatrix}h^0\\ h^-\end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix}0\\ -1\end{pmatrix}$

The Standard Model Lagrangian is, schematically

$$\mathcal{L}_{SM} = \frac{1}{4}\sum_{a=1,2,3}(F^a)^2 \qquad \leftarrow \text{gauge field kinetic terms}$$

$$+\left(D_\mu \bar{h}\right)\left(D^\mu h\right) \qquad \leftarrow \text{Higgs field kinetic terms}$$

$$+\sum_{I=1,2,3}\left(i\,\bar{q}^I_L \not{D} q^I_L + i\,\bar{u}^I_R \not{D} u^I_R + i\,\bar{d}^I_R \not{D} d^I_R + i\,\bar{l}^I_L \not{D} l^I_L + i\,\bar{e}^I_R \not{D} e^I_R\right)$$

fermionic kinetic terms

$$+\sum_{IJ=1,2,3}\left((Y_u)_{IJ}\,\bar{h}\,q^I_L u^J_R + (Y_d)_{IJ}\,h\,q^I_L d^J_R + (Y_e)_{IJ}\,h\,l^I_L e^J_R + h.c.\right)$$

Yukawa terms

$$-V(h,\bar{h}) \qquad \text{Higgs potential}$$

Where Y_u, Y_d, Y_e are 3×3 matrices

Check gauge invariance of some of the interactions

$$\bar{h}\, q_L^I\, u_R^J \quad \rightarrow \quad (1,2)_{\frac{1}{2}} \otimes (3,2)_{1/6} \otimes (\bar{3},1)_{-2/3} \; = \; \text{irreps}$$

This decomposition contains a $(1,1)_0$ representation

The potential V also has to be gauge invariant

$$V(h,\bar{h}) = \mu^2 h \bar{h} + \lambda (h\bar{h})^2$$

- stability $\Rightarrow$ $\lambda > 0$
- spontaneous breaking $\mu^2 < 0$

Supersymmetric extension

quarks & leptons $\longrightarrow$ chiral multiplets (squarks $\tilde{q}_L$, $\tilde{u}_R$, $\tilde{d}_R$, sleptons $\tilde{\ell}_L$, $\tilde{e}_R$)
gauge bosons $\longrightarrow$ vector multiplets (gauginos $\tilde{G}$, $\tilde{W}$, $\tilde{B}$)
Higgs boson $\longrightarrow$ chiral multiplet. (higgsino)

Actually, one needs to add a second Higgs multiplet since otherwise one can not write dow interactions which are both supersymmetric and an extension of the ones in the Standard Model.

super multiplet	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_{em}$
$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
U_R^I	$\bar{3}$	1	$-2/3$	$-2/3$
D_R^I	$\bar{3}$	1	$\frac{1}{3}$	$1/3$
$L_L^I = \begin{pmatrix} N_L^I \\ E_L^I \end{pmatrix}$	1	2	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
E_R^I	1	1	1	1
$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	1	2	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	1	2	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The superpotential is

$$W = \sum_{IJ} \Big((Y_u)_{IJ}\, H_u\, Q_L^I\, U_R^J + (Y_d)_{IJ}\, H_d\, Q_L^I\, D_R^J + (Y_e)_{IJ}\, H_d\, L_L^I\, E_R^J \Big) + \mu H_u H_d$$

Notice that in the Yukawa interactions of the Standard Model we used an $\bar{h}$, but we can't do this here since the superpotential is holomorphic in the superfields.

Check gauge invariance

$H_u Q_L^I U_R^J$ transforms as $(1,2)_{1/2} \otimes (3,2)_{1/6} \otimes (\bar{3},1)_{-2/3}$

contains $(1,1)_0$ which is the trivial repr

There are more possibilities for the superpotential

$$W^{bad} = \sum_{I,J,K}\left(\alpha_{IJK}\, Q_L^I\, L_L^J\, D_R^K + \beta_{IJK}\, L_L^I L_L^J E_R^K + \gamma_{IJK}\, D_R^I D_R^J U_R^K\right) + \sum_I \delta_I\, L_L^I H_u$$

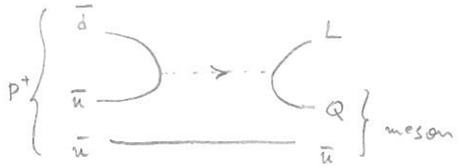
$$Q_L L_L D_R \rightarrow (3,2)_{1/6} \otimes (1,2)_{-1/2} \otimes (\bar{3},1)_{1/3} \ni (1,1)_0$$

$$L_L L_L E_R \rightarrow (1,2)_{-1/2} \otimes (1,2)_{-1/2} \otimes (1,1)_1 \ni (1,1)_0$$

$$D_R D_R U_R \rightarrow (\bar{3},1)_{1/3} \otimes (\bar{3},1)_{1/3} \otimes (\bar{3},1)_{-2/3} \ni (1,1)_0$$

$$L_L H_u \rightarrow (1,2)_{-1/2} \otimes (1,2)_{1/2} \ni (1,1)_0$$

Such terms would lead to lepton and baryon number violation. The proton can decay into a lepton and a meson



R - parity

To avoid the contributions in W^{bad}, we impose a discrete symmetry called R-parity.

The action on Q, U, D, L, E superfields is

$$F(x,\theta) \rightarrow -F(x,-\theta)$$

The action on H_u, H_d and the vector superfields is $\theta \rightarrow -\theta$

This can be also formulated as (observed particle) $\rightarrow$ (observed particle)
(superpartner) $\rightarrow$ (superpartner)

Under the action of R-parity we have

$$W^{bad} \rightarrow -W^{bad}\big|_{\theta\rightarrow-\theta} \text{ and } W \rightarrow W\big|_{\theta\rightarrow-\theta}$$

Clearly then W^{bad} is forbidden if we impose R-parity.

Phenomenological consequences

- in colliders the superpartners are produced in pairs
- the lightest supersymmetric partner (LSP) is stable. If electrically neutral, it is a dark matter candidate.