Gauge - matter interactions

To couple matter to gauge fields, the matter fields need to transform in a representation of the gauge group. All the fields in a SUSY multiplet need to transform in the same way, otherwise supersymmetry is broken.

Consider the case of a chiral superfield Φ transforming in a representation R of the gauge group G.

A gauge transformation acts as

$$\Phi \rightarrow \Phi' = R(g)\,\Phi \qquad \text{for } g \in G$$

a local gauge transformation, so it has to be a chiral superfield

(we could make the gauge transformation be a field, independent on the odd coordinates, but we know from the study of the gauge multiplet that the gauge parameter is a chiral superfield)

Such gauge transformations in superspace are called super-gauge transf.

If $r(g) = e^{i\Lambda}$,

$$\Phi \rightarrow \Phi' = e^{i\Lambda}\Phi$$

$$\bar{\Phi} \rightarrow \bar{\Phi}' = \bar{\Phi}\, e^{-i\bar{\Lambda}}$$

then the combination $\bar{\Phi} e^{2V} \Phi$ is gauge invariant.

$$\bar{\Phi} e^{2V} \Phi \rightarrow \bar{\Phi}' e^{2V'} \Phi' = \bar{\Phi} e^{-i\bar{\Lambda}} \left(e^{i\bar{\Lambda}} e^{2V} e^{-i\Lambda}\right) e^{i\Lambda} \Phi = \bar{\Phi} e^{2V} \Phi$$

We can expand the exponential

$$\bar{\Phi} e^{2V} \Phi = \underset{\uparrow \text{ usual Kähler potential}}{\bar{\Phi}\Phi} + \underbrace{2\bar{\Phi} V \Phi + 2\bar{\Phi} V^2 \Phi}_{\text{interaction terms.}} + \dots$$

Recall that in the WZ gauge $V^3 = 0$ so the expansion stops at second order in V.

$$2\bar{\Phi} V \Phi\big|_{\theta^2\bar{\theta}^2} = i\bar{\varphi} v^\mu \partial_\mu \varphi - i(\partial_\mu \bar{\varphi}) v^\mu \varphi - \bar{\psi}\bar{\sigma}^\mu v_\mu \psi$$
$$+ i\sqrt{2}\,\bar{\varphi}\lambda\psi - i\sqrt{2}\,\bar{\psi}\bar{\lambda}\varphi + \bar{\varphi} D \varphi$$

where ψ is the fermion in the chiral multiplet Φ

λ ——— gauge multiplet V.

$$2\bar{\Phi} V^2 \Phi\big|_{\theta^2\bar{\theta}^2} = \bar{\varphi} v^\mu v_\mu \varphi$$

It follows that

$$\bar{\Phi} e^{2V} \Phi\big|_{\theta^2\bar{\theta}^2} = \left(\overline{D_\mu \varphi}\right)(D^\mu \varphi) - i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi + \bar{F}F + i\sqrt{2}\,\bar{\varphi}\lambda\psi - i\sqrt{2}\,\bar{\psi}\bar{\lambda}\varphi$$
$$+ \bar{\varphi}\underset{\uparrow\ D\text{-field not derivative}}{D}\varphi.$$

where

$D_\mu = \partial_\mu - i v_\mu$ is the gauge covariant derivative.

Clearly the field V is not chiral so it can not appear in the superpotential.
(the field W_α is holomorphic, but the superpotential is restricted to not depend on derivatives of the superfields)

However, the superpotential has to preserve the gauge symmetry. For example, if we have some chiral superfields $\Phi_{(1)}, \dots, \Phi_{(\ell)}$ transforming under the representations $r_1, \dots, r_\ell$ of the gauge group, then their product transforms under the tensor product $r_1 \otimes \dots \otimes r_\ell$ which can be decomposed as a sum of irreducible representations, If there is no trivial (singlet) representation in the decomposition, then such a term can not be made gauge invariant and therefore can not appear in the superpotential.

Example: If the gauge group is $SU(3)$, then $3 \otimes 3 \otimes 3 = 1 \oplus \dots$
ϕ_i transforms as a triplet $i = 1, 2, 3$. Then $\varepsilon_{ijk} \phi_i \phi_j \phi_k$ is invariant under $SU(3)$ transformations. Such a term can appear in the superpotential.
In contrast, $3 \otimes 3 = \bar{3} \oplus 6$ so no ϕ^2 (mass like term) are possible.
To get a mass term, we can start with ϕ_i transforming as 3 and $\tilde{\phi}^i$ transforming as a $\bar{3}$. Then $\tilde{\phi}^i \phi_i$ can appear in the superpotential.

Fayet - Iliopulous terms

For Abelian ($U(1)$) terms in the gauge group one can also have supersymmetric terms of the form

$$\mathcal{L}_{FI} = \xi \int d^2\theta \, d^2\bar{\theta} \, V = \frac{1}{2} \xi D \leftarrow \text{D-term of } V.$$

This is manifestly SUSY invariant. It is also gauge invariant since
$\delta V = -i(\Lambda - \bar{\Lambda})$ which vanishes upon integration over superspace.