## Introduction

A non-abelian vector superfield $V$ has gauge transformations

$$e^{2V'} = e^{i\bar{\Lambda}} e^{2V} e^{-i\Lambda}, \tag{1}$$

where $\Lambda$ is chiral and $\bar{\Lambda}$ is anti-chiral.

We can choose the Wess-Zumino gauge

$$V_{WZ} = (\theta\sigma^\mu\bar{\theta})v_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \tag{2}$$

## Questions

1. Find the gauge transformations of $W_\alpha = -\frac{1}{4}\bar{D}^2(e^{-2V}D_\alpha e^{2V})$.

2. Show that in the Wess-Zumino gauge

$$W_\alpha = -\frac{1}{2}\bar{D}^2(D_\alpha V) - \frac{1}{2}\bar{D}^2([D_\alpha V, V]). \tag{3}$$

3. Show that

$$[D_\alpha V, V] = (\sigma^{\mu\nu}\theta)_\alpha\bar{\theta}^2[v_\mu, v_\nu] + i\theta^2\bar{\theta}^2[v_\mu, (\sigma^\mu\bar{\lambda})_\alpha]. \tag{4}$$

4. We have shown in class that

$$-\frac{1}{4}\bar{D}^2(D_\alpha V) = -i\lambda_\alpha + \theta_\alpha D + i(\sigma^{\mu\nu}\theta)_\alpha(\partial_\mu v_\nu - \partial_\nu v_\mu) - \theta^2(\sigma^\mu\partial_\mu\bar{\lambda})_\alpha. \tag{5}$$

   Show that adding the quadratic term in $V$ makes the derivatives into covariant derivatives, i.e.

$$\partial_\mu v_\nu - \partial_\nu v_\mu \to \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu], \qquad (\sigma^\mu\partial_\mu\bar{\lambda})_\alpha \to [\partial_\mu - iv_\mu, (\sigma^\mu\bar{\lambda})_\alpha]. \tag{6}$$

5. There remains a term depending on $\bar{\theta}$ in the formula for $W_\alpha$. Show that this term can be absorbed into a translation $x^\mu \to y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. After doing this translation the superfield $W_\alpha$ is expressed in terms of variables $(y, \theta)$, as befits a chiral superfield.

6. Let us introduce a chiral superfield with gauge transformations

$$\Phi' = e^{i\Lambda}\Phi, \qquad \bar{\Phi}' = \bar{\Phi}e^{-i\bar{\Lambda}}. \tag{7}$$

   One can write a gauge-invariant Kähler potential $\bar{\Phi}e^{2V}\Phi$. Compute the $D$-terms (i.e. the terms containing the auxiliary field $D$) arising from the Kähler potential and from the gauge kinetic terms

$$\int d^4x d^2\theta d^2\bar{\theta}\bar{\Phi}e^{2V}\Phi + \frac{1}{32\pi}\Im\int d^4x d^2\theta\, \tau\, \mathrm{tr}(W^\alpha W_\alpha), \tag{8}$$

   where $\tau = \frac{4\pi i}{g^2}$. What scalar potential is produced by integrating out $D$?