Introduction

Take Φ to be a chiral superfield $\bar{D}_{\dot{\alpha}}\Phi = 0$. A superspace action for this superfield is given by

$$S = \int d^4x d^2\theta d^2\bar{\theta} K(\bar{\Phi}, \Phi) + \int d^4x d^2\theta W(\Phi) + \int d^4x d^2\bar{\theta} \bar{W}(\bar{\Phi}), \tag{1}$$

where K is a real function called the Kähler potential and W is a holomorphic function called superpotential

We will take the expansion of these superfields to be

$$\Phi = \phi + \theta\psi + \theta^2 F, \tag{2}$$

$$\bar{\Phi} = \bar{\phi} + \bar{\theta}\bar{\psi} + \bar{\theta}^2 \bar{F}, \tag{3}$$

where ϕ is a complex scalar field, ψ is a Weyl spinor and F is a complex auxiliary field.

Questions

1. What is the dimension of the integration measures $d^4xd^2\theta$ and $d^4xd^2\theta d^2\bar{\theta}$? What are the dimensions of ϕ, ψ and F? What is the dimension of K and of W?

2. Count the number of bosonic and fermionic degrees of freedom in the superfield Φ.

3. What is the most general form of the superpotential W which does not involve any non-renormalizable terms?

4. What happens to the action if we redefine W by an additive constant?

5. What is the simplest form of the Kähler potential which satisfies the dimensionality and reality constraints?

6. Show that the product of two chiral superfields is a chiral superfield. Is W a chiral superfield? What about K?

7. The variational principle in space-time works as follows. If we have a field ϕ such that for all its variations $\delta\phi$ we have that $\int d^4x \delta\phi G(\phi) = 0$, then $G(\phi) = 0$. If we have a chiral superfield Y, give examples of non-vanishing superfields G such that

$$\int d^4xd^2\theta d^2\bar{\theta}\ \delta Y G = 0, \tag{4}$$

for all variations δY of Y.

8. Show that if Y and G are *chiral* superfields and

$$\int d^4xd^2\theta\ \delta Y\, G = 0, \tag{5}$$

for all variations δY of Y, then we can conclude that $G = 0$.

9. Use the expressions for K and W found in the previous questions and take the variation with respect to Φ and $\bar{\Phi}$ to find the superspace version of the equations of motion (Hint: you will have to rewrite the action such that the integral is either $\int d^4x d^2\theta$ or $\int d^4x d^2\bar{\theta}$).

10. For a field Y we denote by $Y|$ the projection to the bottom component of the Grassmann expansion (i.e. the field we obtain by setting $\theta = 0$ and $\bar{\theta} = 0$). Write ϕ, ψ and F as projections of the chiral superfield Φ and its supersymmetry covariant derivatives.

11. By acting with supersymmetry covariant derivatives on the superspace equations of motion and taking the lowest components, i.e. $D\ \mathrm{eom}| = 0$, compute the equations of motion for the physical fields ϕ, ψ.

12. Compute the space-time action. You will need to do all the integrals (derivatives) with respect to θ and $\bar{\theta}$ and then eliminate the auxiliary field F by its equations of motion.

13. Can the scalar potential ever by negative?