1)

$$2V = 2(\theta\sigma^\mu\bar\theta)\, v_\mu - 2i\theta^2\bar\theta\bar\lambda - 2i\bar\theta^2\theta\lambda + \theta^2\bar\theta^2 D$$

$$V' = (\theta\sigma^\mu\bar\theta)\, v'_\mu + i\theta^2\bar\theta\bar\lambda' - i\bar\theta^2\theta\lambda' + \frac{1}{2}\theta^2\bar\theta^2 D'$$

$$\bar H_d\, e^{2V - V'} H_d \xrightarrow{\text{D-term}} h_d^+ D\, h_d - \frac{1}{2} h_d^+ h_d D'$$

$$\bar H_u\, e^{2V + V'} H_u \xrightarrow{\text{D-term}} h_u^+ D\, h_u + \frac{1}{2} h_u^+ h_u D'$$

(D is a 2x2 matrix
D' is a complex number)

$$\frac{1}{32\pi}\,\mathrm{Im}\left(2\frac{4\pi i}{g^2}\, \mathrm{tr}(W^\alpha W_\alpha)\right) \xrightarrow{\text{D-term}} \frac{1}{g^2}\,\mathrm{tr}(D^2)$$

$$W_\alpha = -2i\lambda_\alpha + 2\theta_\alpha D + \dots$$

$$\frac{1}{32\pi}\,\mathrm{Im}\left(\frac{4\pi i}{g'^2}\, W'^\alpha W'_\alpha\right) \xrightarrow{\text{D-term}} \frac{1}{2g'^2} D'^2$$

Integrate out D and D'

$$\Rightarrow \quad \frac{1}{4} g^2 \left(h_d^+ \tau^a h_d + h_u^+ \tau^a h_u\right)\cdot\left(h_d^+ \tau^a h_d + h_u^+ \tau^a h_u\right)$$

$$+ \frac{1}{8} g'^2 \left(h_d^+ h_d - h_u^+ h_u\right)^2 ,$$

where τ^a are generators of $SU(2)$ ($a = 1,2,3$) normalized such that $\mathrm{tr}(\tau^a\tau^b) = \delta^{ab}$

$$(\tau^a)_i{}^k (\tau^a)_j{}^l = \delta_j^k \delta_i^l - \frac{1}{2}\delta_i^k \delta_j^l$$

$$(v\tau^a w)(v'\tau^a w') = (w\cdot v')(v\cdot w') - \frac{1}{2}(v\cdot w)(v'\cdot w')$$

$$\left(h_d^+ \tau^a h_d + h_u^+ \tau^a h_u\right)^2 = \frac{1}{2}|h_d^+ h_d|^2 + \frac{1}{2}|h_u^+ h_u| + 2\left(h_d^+ \tau^a h_d\right)\left(h_u^+ \tau^a h_u\right)$$

$$= \frac{1}{2}|h_d^+ h_d|^2 + \frac{1}{2}|h_u^+ h_u| + 2\left(|h_u^+ h_d|^2 - \frac{1}{2}|h_u^+ h_u|\,|h_d^+ h_d|\right)$$

$$= \frac{1}{2}\left(h_d^+ h_d - h_u^+ h_u\right)^2 + 2|h_u^+ h_d|^2$$

$\Rightarrow$ D-term contribution to the potential

$$\boxed{\frac{1}{8}(g^2 + g'^2)\left(h_d^+ h_d - h_u^+ h_u\right)^2 + \frac{1}{2} g^2 |h_u^+ h_d|^2}$$

The F-terms are

$$\int d^4x \left(\bar{F}_d F_d + \bar{F}_u F_u\right) + \int d^4x\, \mu \Big(\underbrace{F_u^T \varepsilon h_d}_{-h_d^T \varepsilon F_u} + h_u^T \varepsilon F_d + c.c\Big)$$

equations of motion

$$\bar{F}_d + \mu h_u^T \varepsilon = 0 \qquad F_d - \mu^* \varepsilon h_u^* = 0$$

$$\bar{F}_u - \mu h_d^T \varepsilon = 0 \qquad F_u + \mu^* \varepsilon h_d^* = 0$$

Replace back the solution

$$\bar{F}_d F_d + \bar{F}_u F_u + \mu\left(-h_d^T \varepsilon F_u + h_u^T \varepsilon F_d + \bar{F}_u \varepsilon h_d^* - \bar{F}_d \varepsilon h_u^*\right)$$

$$= -|\mu|^2 \left(h_u^+ h_u + h_d^+ h_d\right)$$

$\Rightarrow$ the F-term contribution to the potential is

$$\boxed{|\mu|^2 \left(h_u^+ h_u + h_d^+ h_d\right)}$$

In addition, the soft terms contribute (the θ integrals are straightforward)

$$\boxed{m_u^2 h_u^+ h_u + m_d^2 h_d^+ h_d + \left(m_{ud}^2 h_u^T \varepsilon h_d + c.c\right)}$$

$$m^2 = \begin{pmatrix} a & b \\ \bar{b} & c \end{pmatrix} \qquad \text{where} \qquad \begin{aligned} a &= |\mu|^2 + m_u^2 \\ b &= -m_{ud}^2 \\ c &= |\mu|^2 + m_d^2 \end{aligned}$$

The eigenvalues of m^2 satisfy the equation

$$(a-\lambda)(c-\lambda) - |b|^2 = 0 \quad \Rightarrow \quad \lambda^2 - (a+c)\lambda + (ac - |b|^2) = 0$$

If $a > 0$ and $c > 0$ then we know that $\lambda_1 + \lambda_2 > 0$. If $ac - |b|^2 < 0$, then we know that one of λ_1 and $\lambda_2 < 0$.

Therefore, we have symmetry breaking if $|b|^2 > ac$

Replacing,

$$\boxed{|m_{ud}^2|^2 > (|\mu|^2 + m_u^2)(|\mu|^2 + m_d^2)}$$

Another condition arises from stability at large values of the fields. Suppose we take $h_d^0 = \overline{h_u^0}\, e^{-i\varphi}$, where φ is the phase of $m_{ud}^2 = |m_{ud}^2| e^{i\varphi}$

Then, the quartic terms vanish and the quadratic terms all become proportional to $|h_u^0|^2$ with proportionality coefficient

$$2|\mu|^2 + m_u^2 + m_d^2 - 2|m_{ud}^2|.$$

Therefore

$$\boxed{2|m_{ud}^2| \leq 2|\mu|^2 + m_u^2 + m_d^2}$$

If $m_u^2 = m_d^2$ then $|m_{ud}^2| = |\mu|^2 + m_u^2$ can take a single value. If m_u^2 and m_d^2 are close together, tight constraints on the possible values of $|m_{ud}^2|$.

<u>Scalar masses</u>

Take the vev's to be $\langle h_u^0 \rangle = \dfrac{v_u}{\sqrt{2}}$, $\langle h_d^0 \rangle = \dfrac{v_d}{\sqrt{2}}$

Notation $v^2 = v_u^2 + v_d^2$ $\quad (\simeq (246\,\text{GeV})^2)$

Shift fields by their vev. $h_u^0 \to \dfrac{v_u}{\sqrt{2}} + h_u^0$, $h_d^0 \to \dfrac{v_d}{\sqrt{2}} + h_d^0$, $v_u, v_d \in \mathbb{R}_+$ (after choosing the phases)

Relations between v_u, v_d and the parameters of the potential

$$\left.\frac{\partial \tilde{V}}{\partial h_u^0}\right|_{h_u^0 = \frac{v_u}{\sqrt{2}},\, h_d^0 = \frac{v_d}{\sqrt{2}}} = (|\mu|^2 + m_u^2)\frac{v_u}{\sqrt{2}} - m_{ud}^2 \frac{v_d}{\sqrt{2}} + \frac{1}{4}(g^2 + g'^2)\frac{v_u}{\sqrt{2}}\left(\frac{v_u^2}{2} - \frac{v_d^2}{2}\right)$$

$$\Rightarrow \boxed{|\mu|^2 + m_u^2 = m_{ud}^2 \frac{v_d}{v_u} - \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)}$$

$$\left.\frac{\partial \tilde{V}}{\partial h_d^0}\right|_{h_u^0 = \frac{v_u}{\sqrt{2}},\, h_d^0 = \frac{v_d}{\sqrt{2}}} = (|\mu|^2 + m_d^2)\frac{v_d}{\sqrt{2}} - m_{ud}^2 \frac{v_u}{\sqrt{2}} + \frac{(-1)}{4}(g^2 + g'^2)\frac{v_d}{\sqrt{2}}\left(\frac{v_u^2}{2} - \frac{v_d^2}{2}\right)$$

$$\boxed{|\mu|^2 + m_d^2 = m_{ud}^2 \frac{v_u}{v_d} + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)}$$

Higgs potential in the Supersymmetric Standard Model

$$V(h_u, h_d) = (|\mu|^2 + m_u^2)\, h_u^\dagger h_u + (|\mu|^2 + m_d^2)\, h_d^\dagger h_d$$
$$+ \left(m_{ud}^2\, h_u^T \varepsilon h_d + c.c. \right) \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$+ \frac{1}{2} g^2 \, |h_u^\dagger h_d|^2 + \frac{1}{8}(g^2 + g'^2)\left(h_u^\dagger h_u - h_d^\dagger h_d \right)^2$$

where $h_u = \begin{pmatrix} h_u^{(+)} \\ h_u^0 \end{pmatrix}$ plus, $h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$ Warning! Do not confuse (+) (plus) with $\dagger$ (dagger)

$m_u^2, m_d^2 \in \mathbb{R}$, $m_{ud}^2 \in \mathbb{C}$

By an $SU(2)$ gauge transformation, set $h_u^{(+)} = 0$.

$$\left.\frac{\partial V}{\partial h_u^{(+)}}\right|_{h_u^{(+)}=0} = m_{ud}^2\, h_d^- + \frac{1}{2} g^2\, \overline{h_d^0}\, h_d^-\, \overline{h_u^0}$$

overline is complex conjugation.

If $h_d^- \neq 0$, then the electromagnetic gauge symmetry will be broken and the photon will become massive. Prefer the solution $h_d^- = 0$. It could have been such that $h_d^- = 0$ is incompatible with $\frac{\partial V}{\partial h_u^{(+)}} = 0$ but fortunately this doesn't happen here.

Restrict to the neutral fields and study their values in the electro-weak breaking vacuum.

$$\tilde{V}(h_u^0, h_d^0) = V(h_u, h_d)\big|_{h_u^{(+)}=0,\ h_d^-=0}$$
$$= (|\mu|^2 + m_u^2)\, |h_u^0|^2 + (|\mu|^2 + m_d^2)\, |h_d^0|^2 - (m_{ud}^2\, h_u^0 h_d^0 + c.c.)$$
$$+ \frac{1}{8}(g^2 + g'^2)\left(|h_u^0|^2 - |h_d^0|^2 \right)^2$$

For the fields h_u^0, h_d^0 to get a non-zero vacuum expectation value, the matrix of second derivatives should have a negative eigenvalue.

The truncation of $\tilde{V}$ to second order in the h's when expanding around the origin $h_u^0 = h_d^0 = 0$ yields

$$(|\mu|^2 + m_u^2)\, |h_u^0|^2 + (|\mu|^2 + m_d^2)\, |h_d^0|^2 - \left(m_{ud}^2\, h_u^0 h_d^0 + c.c. \right)$$

$$= \begin{pmatrix} h_u^0 & \overline{h_d^0} \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_u^2 & -m_{ud}^2 \\ -\overline{m_{ud}^2} & |\mu|^2 + m_d^2 \end{pmatrix} \begin{pmatrix} \overline{h_u^0} \\ h_d^0 \end{pmatrix}$$

This is a complex mass term $m^2_{i\bar{\imath}}\, \phi^i \bar{\phi}^{\bar{\imath}}$ where $(m^2)_{i\bar{\imath}}$ is a hermitian matrix so it has real eigenvalues. The potential is unstable at the origin if at least one of these eigenvalues of $m^2_{i\bar{\imath}}$ is negative

$$V = (|\mu|^2 + m_u^2)(|h_u^0|^2 + |h_u^+|^2) + (|\mu|^2 + m_d^2)(|h_d^0|^2 + |h_d^-|^2)$$
$$+ m_{ud}^2 (h_u^+ h_d^- - h_u^0 h_d^0) + m_{ud}^2 (\bar{h}_u^+ \bar{h}_d^- - \bar{h}_u^0 \bar{h}_d^0)$$
$$+ \frac{1}{2} g^2 |h_u^+ \bar{h}_d^0 + h_u^0 \bar{h}_d^-|^2 + \frac{1}{8}(g^2 + g'^2)(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2)^2$$

A real mass term is of type $\frac{1}{2} m_{ij} \phi^i \phi^j$ where ϕ^i are real and m is symmetric.
A complex mass term is of type $m_i{}^j \bar{\phi}^i \phi_j$ where ϕ_i is complex and m is hermitian.
In our case we have a mix of the two types of masses so we will convert all of them to real and imaginary components. The fields are $\mathrm{Re}\, h_u^0$, $\mathrm{Im}\, h_u^0$, $\mathrm{Re}\, h_d^0$, $\mathrm{Im}\, h_d^0$, $\mathrm{Re}\, h_u^+$, $\mathrm{Im}\, h_u^+$, $\mathrm{Re}\, h_d^-$, $\mathrm{Im}\, h_d^-$
We will compute the matrix of double derivatives at $\mathrm{Re}\, h_u^0 = \frac{v_u}{\sqrt{2}}$, $\mathrm{Re}\, h_d^0 = \frac{v_d}{\sqrt{2}}$ and the rest $= 0$.

$$\frac{\partial V}{\partial \mathrm{Im}\, h_u^0} = 2(|\mu|^2 + m_u^2)\, \mathrm{Im}\, h_u^0 + 2 m_{ud}^2\, \mathrm{Im}\, h_d^0$$
$$+ \frac{1}{2} g^2 \left[(i \bar{h}_d^-)(\bar{h}_u^+ h_d^0 + \bar{h}_u^0 h_d^-) + (h_u^+ \bar{h}_d^0 + h_u^0 \bar{h}_d^-)(-i h_d^-) \right]$$
$$+ \frac{1}{4}(g^2 + g'^2)\, 2\, \mathrm{Im}\, h_u^0 \left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2 \right)$$

(can not contribute to $\frac{\partial^2 V}{\partial \mathrm{Im}\, h_u^0\, \partial *}$)

$$\frac{\partial^2 V}{\partial (\mathrm{Im}\, h_u^0)^2} = 2(|\mu|^2 + m_u^2) + \frac{1}{4}(g^2 + g'^2)(v_u^2 - v_d^2) = 2 m_{ud}^2 \frac{v_d}{v_u}$$

$$\frac{\partial^2 V}{\partial \mathrm{Im}\, h_u^0\, \partial \mathrm{Im}\, h_d^0} = 2 m_{ud}^2$$

$$\frac{\partial^2 V}{\partial (\mathrm{Im}\, h_d^0)^2} = 2 m_{ud}^2 \frac{v_u}{v_d}$$ (can be obtained from $\frac{\partial^2 V}{\partial (\mathrm{Im}\, h_u^0)^2}$ by $v_d \leftrightarrow v_u$)

$\Rightarrow$ 2×2 block in the mass matrix

$$\frac{1}{2} \frac{\partial^2 V}{\partial \cdot \partial \cdot} = m_{ud}^2 \begin{pmatrix} \frac{v_d}{v_u} & 1 \\ 1 & \frac{v_u}{v_d} \end{pmatrix}$$

The determinant $= 0$ $\Rightarrow$ one of the eigenvalues vanishes
The other can be obtained from the trace

$$\Rightarrow \begin{cases} M_{\pi^0}^2 = 0 \\ M_{A^0}^2 = m_{ud}^2 \dfrac{v^2}{v_u v_d} \end{cases} \qquad (v^2 = v_u^2 + v_d^2)$$

π^0 will combine with the diagonal broken generator in $SU(2) \times U(1)$ to form a massive gauge boson Z.

Next, look at the charged particles. These can be put in the form of a complex mass matrix, $(\bar{h}_u^+, h_d^-)\, M^2 \begin{pmatrix} h_u^+ \\ \bar{h}_d^- \end{pmatrix}$

In other words, we need to compute the derivatives $\frac{\partial^2 V}{\partial h_u^+ \partial \bar{h}_u^+}$, $\frac{\partial^2 V}{\partial h_u^+ \partial h_d^-}$, $\frac{\partial^2 V}{\partial \bar{h}^- \partial \bar{h}_u^+}$, $\frac{\partial^2 V}{\partial \bar{h}_d^- \partial h_d^-}$

We find

$$\frac{\partial V}{\partial h_u^+} = (|\mu|^2 + m_u^2)\,\overline{h_u^+} + m_{ud}^2\, h_d^- + \frac{1}{2} g^2\, \overline{h_d^0}\left(\overline{h_u^+} h_d^0 + \overline{h_u^0} h_d^-\right)$$

$$+ \frac{1}{4}(g^2+g'^2)\,\overline{h_u^+}\left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2\right)$$

$$\left.\frac{\partial^2 V}{\partial h_u^+ \partial \overline{h_u^+}}\right| = |\mu|^2 + m_u^2 + \frac{1}{2} g^2 \frac{v_d^2}{2} + \frac{1}{8}(g^2+g'^2)(v_u^2 - v_d^2)$$

$$= \boxed{\frac{1}{4} g^2 v_d^2 + m_{ud}^2 \frac{v_d}{v_u}}$$

$$\frac{\partial^2 V}{\partial h_u^+ \partial h_d^-} = \boxed{m_{ud}^2 + \frac{1}{4} g^2 v_u v_d} = \frac{\partial^2 V}{\partial \overline{h_u^+}\, \partial \overline{h_d^-}}$$

$$\frac{\partial V}{\partial h_v^- \partial \overline{h_v^-}} = \boxed{\frac{1}{4} g^2 v_u^2 + m_{ud}^2 \frac{v_u}{v_d}} \quad (\text{by replacing } h_u^+ \leftrightarrow h_v^-,\ v_u \leftrightarrow v_d)$$

Therefore, the complex mass matrix is

$$\begin{pmatrix} \frac{1}{4} g^2 v_d^2 + m_{ud}^2 \frac{v_d}{v_u} & m_{ud}^2 + \frac{1}{4} g^2 v_u v_d \\ m_{ud}^2 + \frac{1}{4} g^2 v_u v_d & \frac{1}{4} g^2 v_u^2 + m_{ud}^2 \frac{v_u}{v_d} \end{pmatrix}$$

Again, the determinant is zero so

$$M^2_{\pi^\pm} = 0$$ ($\pi^\pm$ combines with the non-diagonal generators in $SU(2)$ to form massive gauge bosons $W^\pm$.)

$$\boxed{M^2_{H^\pm} = \frac{1}{4} g^2 v^2 + m_{ud}^2 \frac{v}{v_u v_d}}$$

Finally, consider the real components $\mathrm{Re}\, h_u^0$, $\mathrm{Re}\, h_d^0$

$$\frac{\partial V}{\partial \mathrm{Re}\, h_u^0} = 2(|\mu|^2 + m_u^2)\,\mathrm{Re}\, h_u^0 + m_{ud}^2 \big(\underbrace{-h_d^0 - \overline{h_d^0}}_{-2\,\mathrm{Re}\, h_d^0}\big)$$

$$+ \frac{1}{2} g^2 \left(\overline{h_d^-}\left(\overline{h_u^+} h_d^0 + \overline{h_u^0} h_d^-\right) + \left(h_u^+ \overline{h_d^0} + h_u^0 \overline{h_d^-}\right) h_d^- \right)$$

$$+ \frac{1}{4}(g^2+g'^2)\, 2\,\mathrm{Re}\, h_u^0 \left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2\right)$$

$$\frac{\partial^2 V}{\partial (\mathrm{Re}\, h_u^0)^2} = 2(|\mu|^2 + m_u^2) + \frac{1}{4}(g^2+g'^2)(v_u^2 - v_d^2)$$

$$+ (g^2+g'^2)\frac{v_u^2}{2} = \boxed{2 m_{ud}^2 \frac{v_d}{v_u} + (g^2+g'^2)\frac{v_u^2}{2}}$$

$$\frac{\partial^2 V}{\partial \mathrm{Re}\, h_u^0\, \partial \mathrm{Re}\, h_d^0} = -2 m_{ud}^2 + \frac{1}{4}(g^2+g'^2)\, 2(\mathrm{Re}\, h_u^0)(-2\,\mathrm{Re}\, h_d^0)$$

$$= \boxed{-2 m_{ud}^2 - (g^2+g'^2)\frac{v_u v_d}{2}}$$

Similarly,
$$\frac{\partial^2 V}{\partial (\mathrm{Re}\, h_d^0)^2} = \boxed{2 m_{ud}^2 \frac{v_u}{v_d} + (g^2+g'^2)\frac{v_d^2}{2}}$$

In this case, the mass matrix is

$$\frac{1}{2}\frac{\partial^2 V}{\partial \cdot \partial \cdot} = \begin{pmatrix} m_{ud}^2 \frac{v_d}{v_u} + (g^2+g'^2)\frac{v_u^2}{4} & -m_{ud}^2 - (g^2+g'^2)\frac{v_u v_d}{4} \\ -m_{ud}^2 - (g^2+g'^2)\frac{v_u v_d}{4} & m_{ud}^2 \frac{v_u}{v_d} + (g^2+g'^2)\frac{v_d^2}{4} \end{pmatrix} = M^2$$

$$\det M^2 = \left(m_{ud}^2 \frac{v_d}{v_u} + (g^2+g'^2)\frac{v_u^2}{4}\right)\left(m_{ud}^2 \frac{v_u}{v_d} + (g^2+g'^2)\frac{v_d^2}{4}\right) - \left(m_{ud}^2 + (g^2+g'^2)\frac{v_u v_d}{4}\right)^2$$

$$= \cancel{m_{ud}^4} + \frac{1}{4} m_{ud}^2 (g^2+g'^2)\left(\frac{v_d^3}{v_u} + \frac{v_u^3}{v_d}\right) + \frac{1}{16}(g^2+g'^2)^2 v_u^2 v_d^2$$

$$- \cancel{m_{ud}^4} - \frac{1}{2} m_{ud}^2 (g^2+g'^2) v_u v_d - \frac{1}{16}(g^2+g'^2)^2 v_u^2 v_d^2$$

$$= \frac{1}{4} m_{ud}^2 (g^2+g'^2)\left(\frac{v_d^3}{v_u} - 2 v_u v_d + \frac{v_u^3}{v_d}\right)$$

$$= \frac{1}{4} m_{ud}^2 (g^2+g'^2)\frac{(v_u^2 - v_d^2)^2}{v_u v_d}$$

$$\operatorname{tr} M^2 = m_{ud}^2 \frac{v^2}{v_u v_d} + \frac{1}{4}(g^2+g'^2) v^2$$

Characteristic equation

$$\lambda^2 - \lambda\left(m_{ud}^2 \frac{v^2}{v_u v_d} + \frac{1}{4}(g^2+g'^2) v^2\right) + \frac{1}{4} m_{ud}^2 (g^2+g'^2)\frac{(v_u^2 - v_d^2)^2}{v_u v_d} = 0$$

$$\Rightarrow \quad m^2_{h^0, H^0} = \frac{1}{2}\left(m_{ud}^2 \frac{v^2}{v_u v_d} + \frac{1}{4}(g^2+g'^2) v^2 \pm \sqrt{\left(m_{ud}^2 \frac{v^2}{v_u v_d} + \frac{1}{4}(g^2+g'^2) v^2\right)^2 - m_{ud}^2 (g^2+g'^2)\frac{(v_u^2 - v_d^2)}{v_u v_d}}\right)$$

Conventionally, h^0 is the lighter mass.

In conclusion, there are 5 Higgs bosons $h^0, H^0, H^\pm, A^0$
↑ charged

The masses of $H^0, H^\pm, A^0$ can be made very large by sending $m_{ud}^2 \to \infty$.
However, the mass of h^0 remains bounded

$$m_{h^0}^2 \le \left(\frac{v_u^2 - v_d^2}{v^2}\right)^2 M_Z^2$$

where $M_Z^2 = \frac{1}{4}(g^2+g'^2) v^2$ is the mass of the Z boson.

However, this bound is excluded experimentally. One way out is to include radiative (loop) corrections since all the mass relations above only hold at tree level.