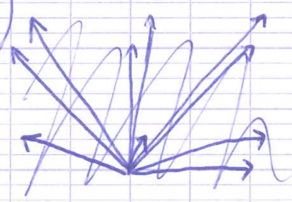
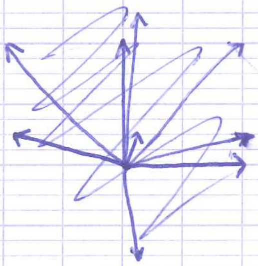
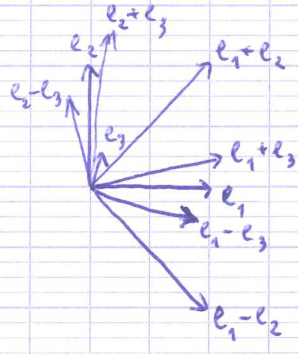
05/05/2015

~~$\mathfrak{so}(5,2)$~~ B_3

Racines positives:

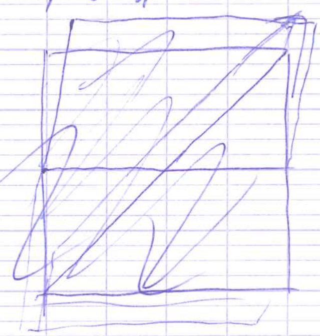




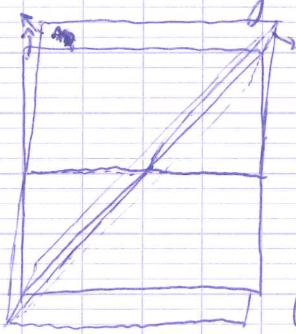


Racines simples: $e_1-e_2,\ e_2-e_3,\ e_3$

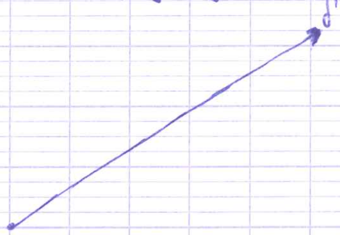
~~Chambre de Weyl:~~



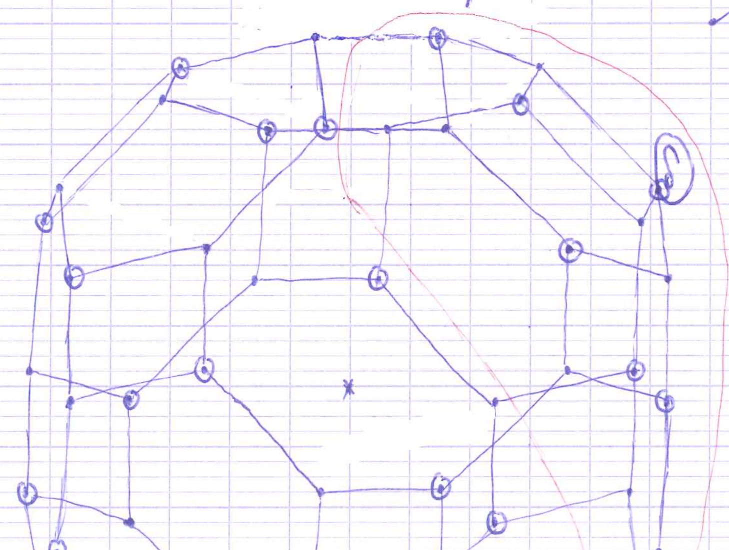
Chambre de Weyl:



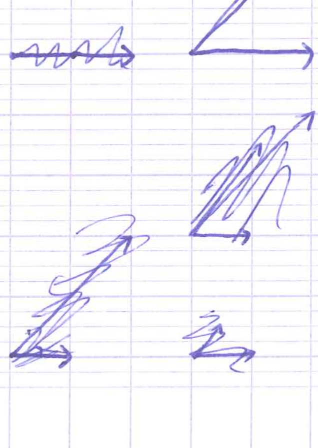
$$\delta = \frac{5}{2}e_1 + \frac{3}{2}e_2 + \frac{1}{2}e_3$$



Orbite de δ par W:



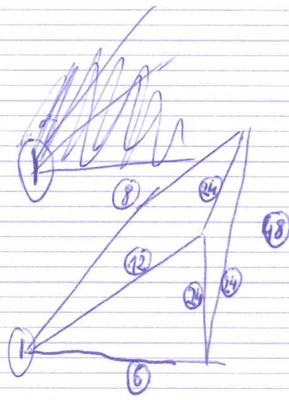
Poids fondamentaux:



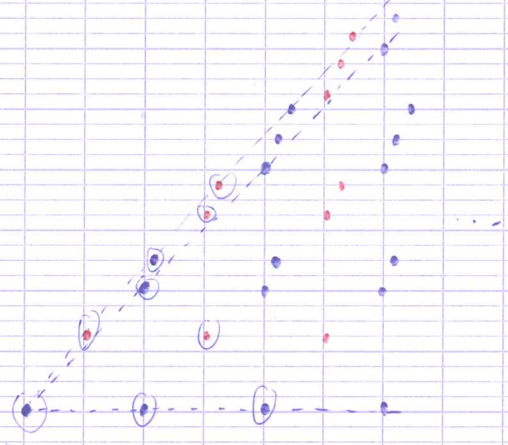
$$\frac{2\langle \varpi_j, \alpha_i\rangle}{\langle \alpha_i, \alpha_i\rangle} = \delta_{ij}$$

		$(1\ 0\ 0)$	$(0\ 1\ 0)$	$(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$
~~$(0\ 0\ 1)$~~	$(0\ 0\ 1)$	×	×	$\frac{1}{2}$
~~$(0\ 1\ -1)$~~	$(0\ 1\ -1)$	×	1	×
~~$(1\ -1\ 0)$~~	$(1\ -1\ 0)$	1	×	×

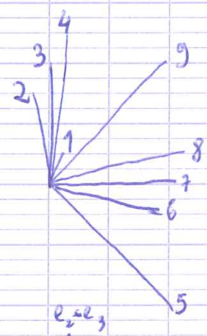
Taille des orbites

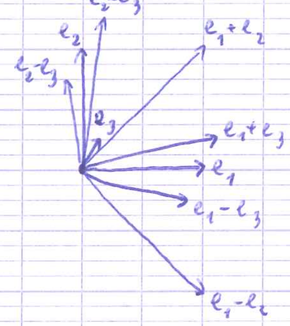


Poids contenus dans la chambre de Weyl.

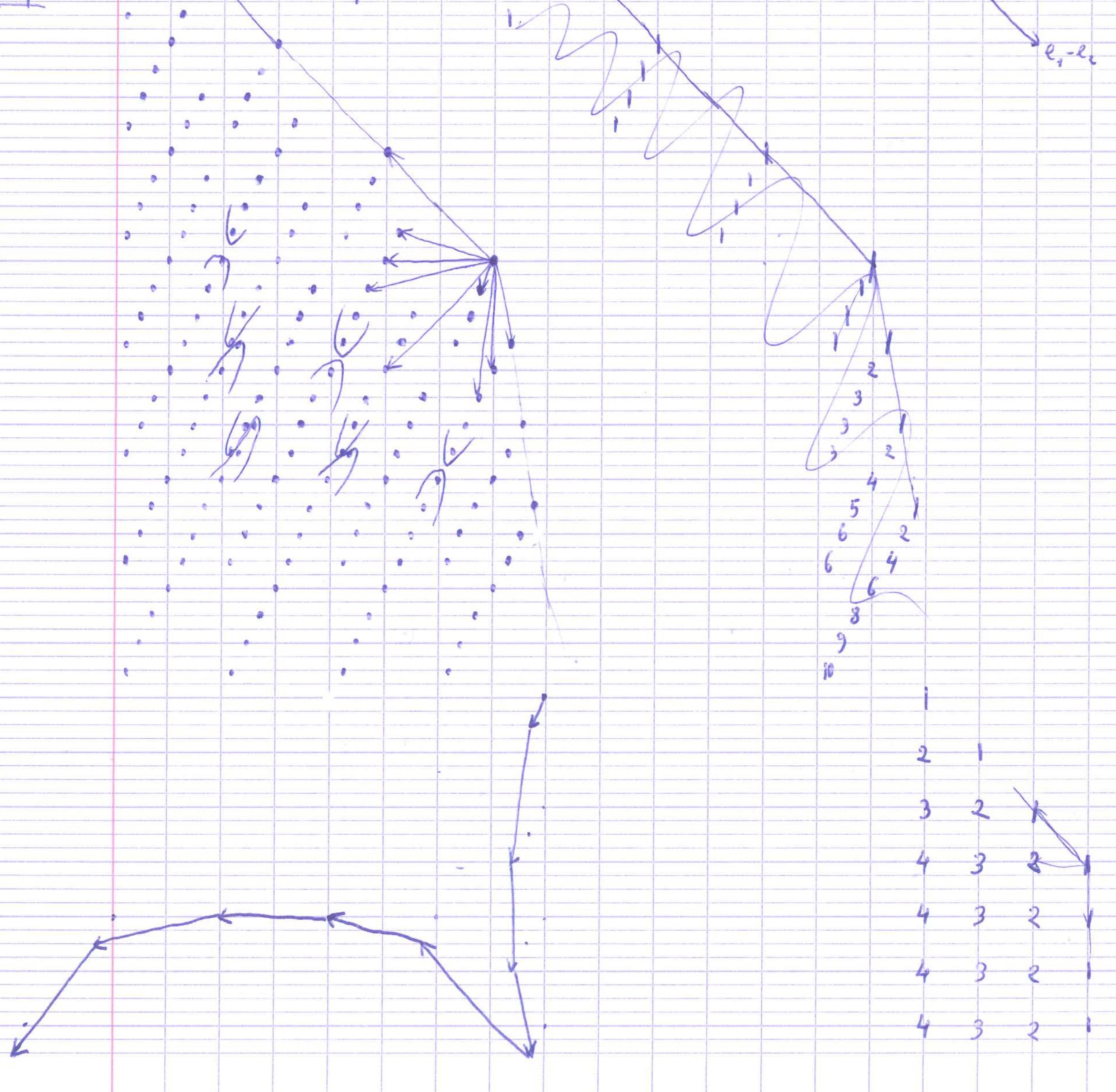


Ordre:





Fonction de partition de Kostant:



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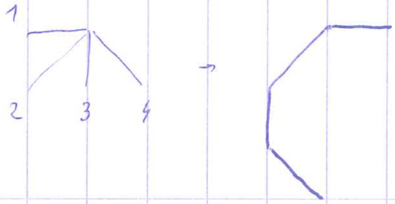

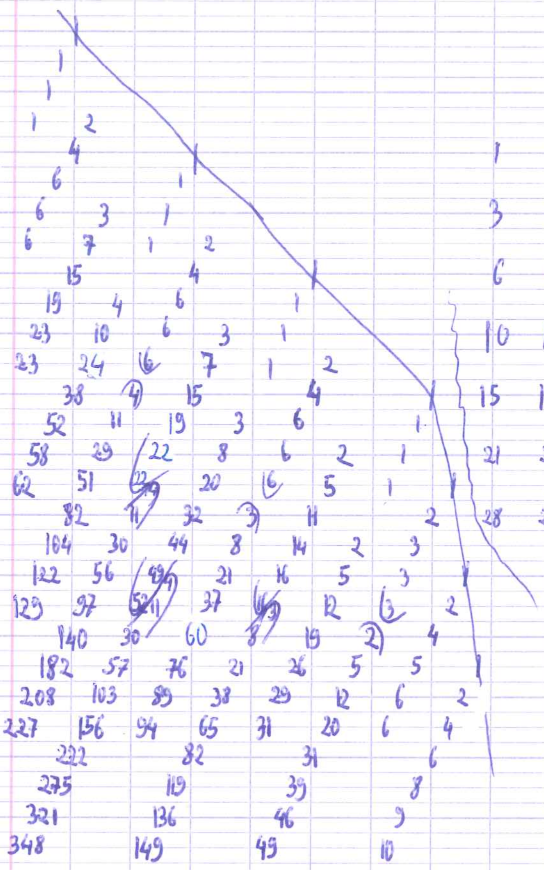

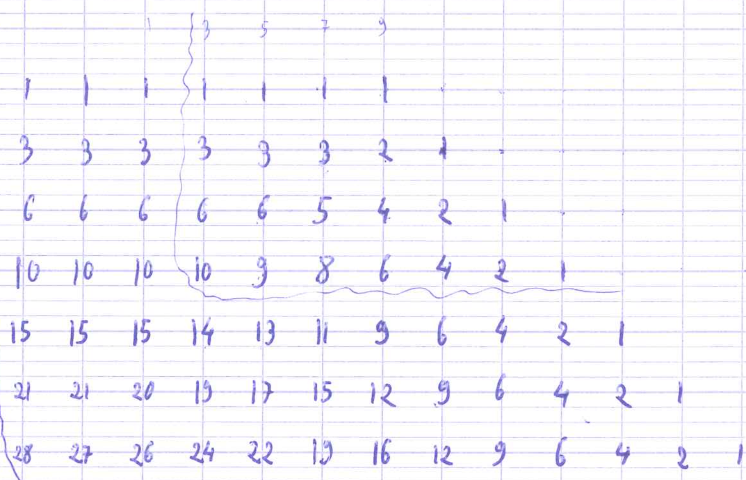

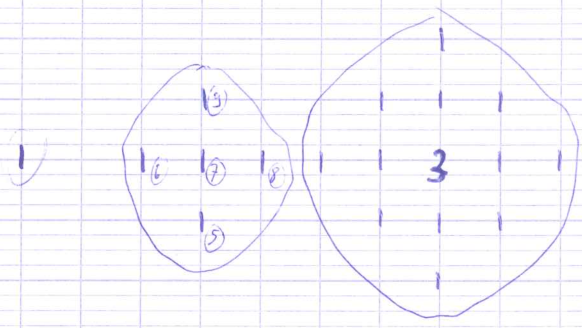

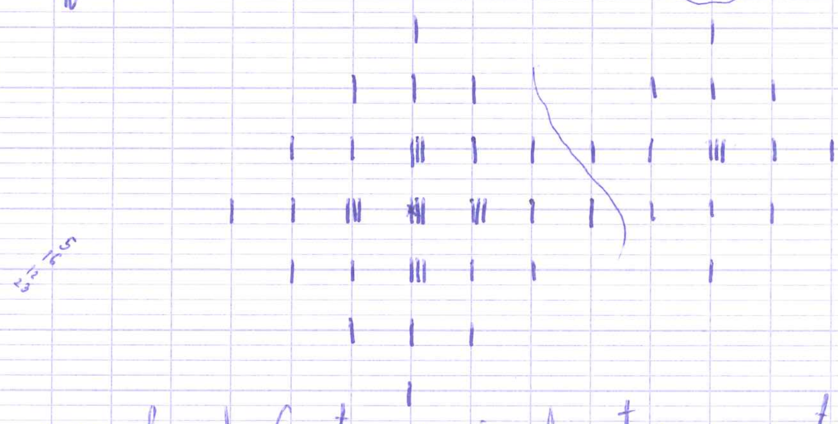

$\boxed{\mathfrak{so}(5,2)}$: dans une sous-alg. de Cartan maximalement non-compacte, les racines réelles positives sont e_2, $e_1 - e_2$, e_1 et $e_1 + e_2$

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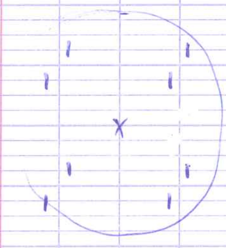
$\boxed{\mathfrak{so}(6,1)}$: dans une sous-alg. de Cartan maximalement non-compacte, ~~les racines~~ la seule rac. réelle positive est e_1.

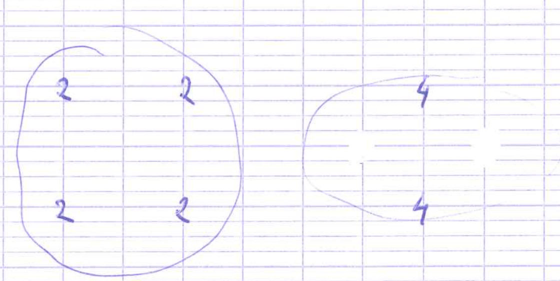
ca||6e

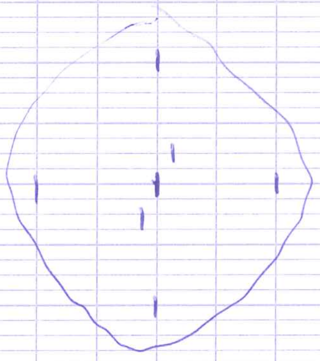
Représentations:

(1) Repr. triviale.

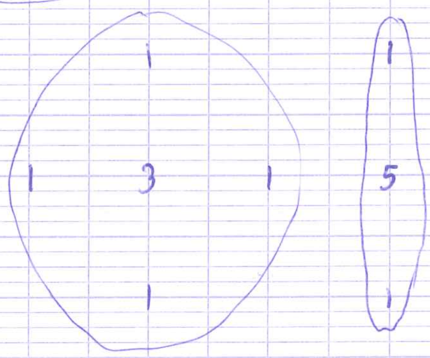


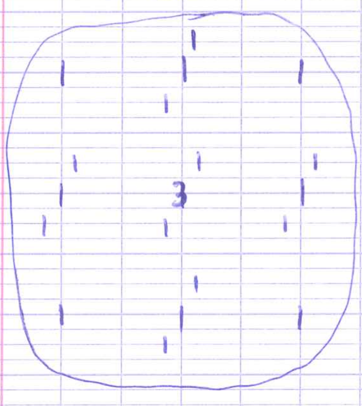
Repr. de spin. Projections:



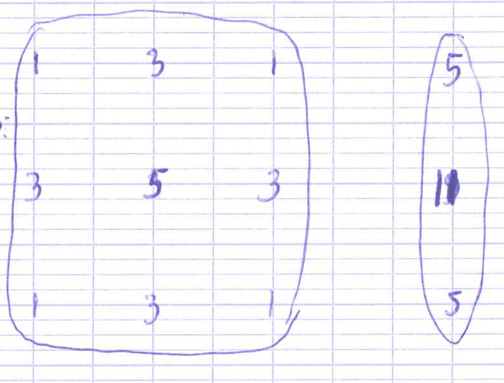


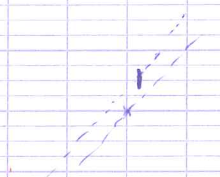
Repr. standard. Projections:



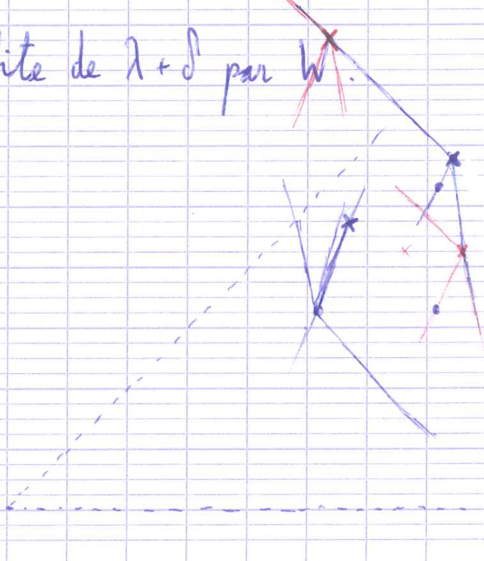


Projections:
Repr. adj.





Orbite de $\lambda + \delta$ par W



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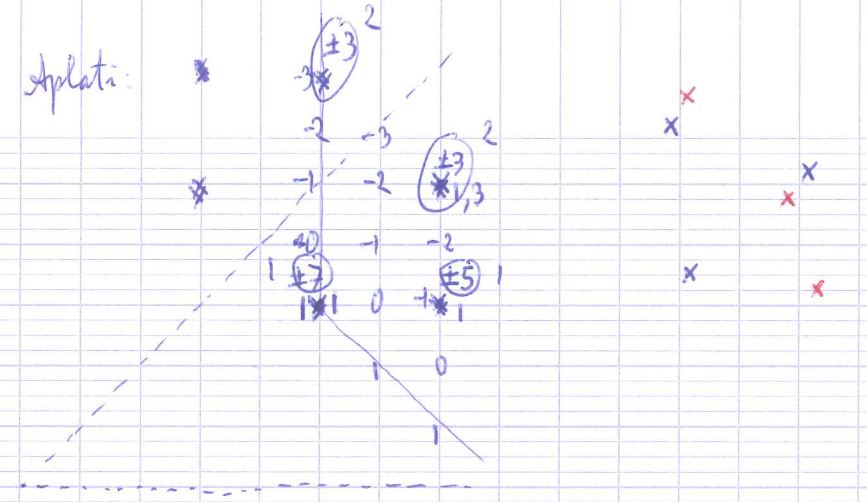
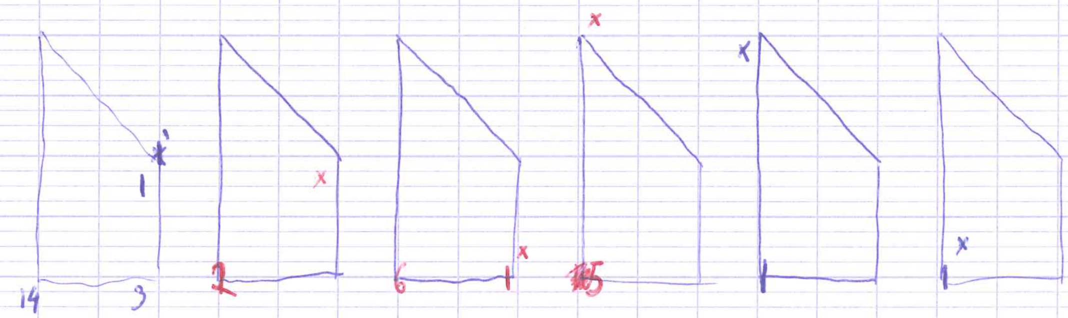
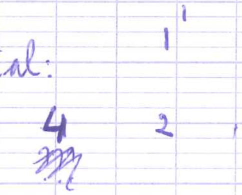
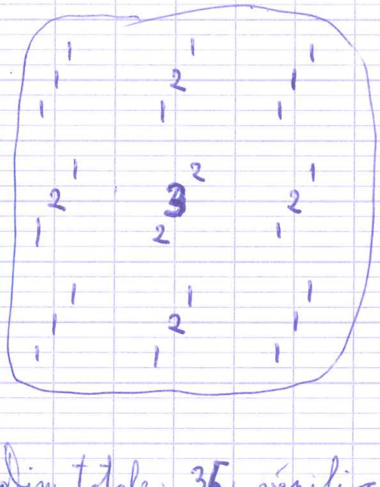
Aplati :



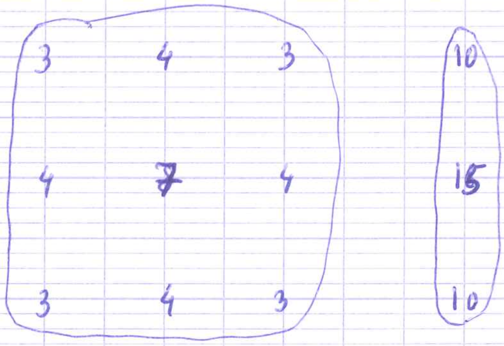


Total :





; projections.

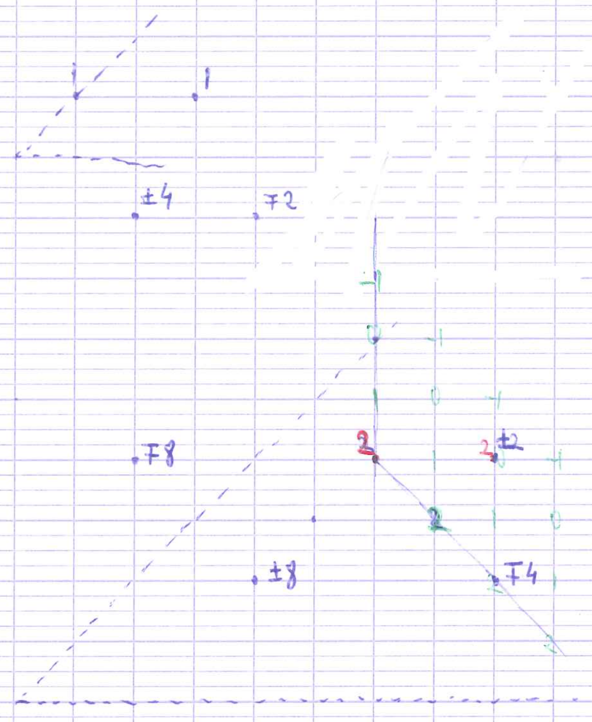


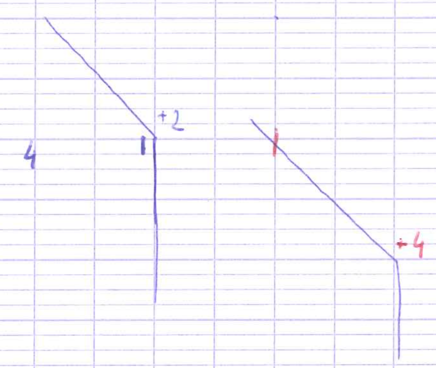
Dim totale : 35, vérification par la formule de dimension de Weyl :

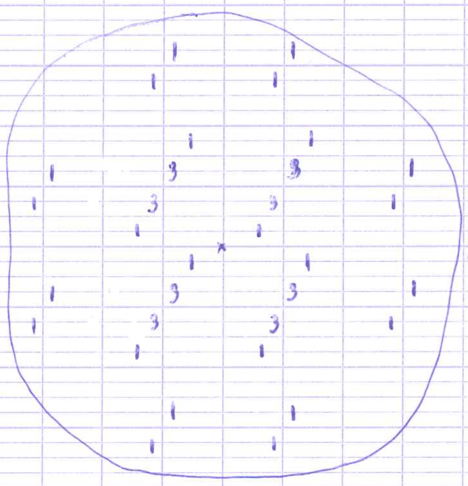
$$\prod_{\alpha\in\Delta^+}\frac{\frac{1}{2}\langle(7,5,3),\alpha\rangle}{\frac{1}{2}\langle(5,3,1),\alpha\rangle} = \frac{3}{1}\,\frac{2}{2}\,\frac{5}{3}\,\frac{8}{4}\,\frac{2}{2}\,\frac{4}{4}\,\frac{7}{5}\,\frac{10}{6}\,\frac{12}{8}$$

$$\frac{7\times10\times3}{6} = 35$$

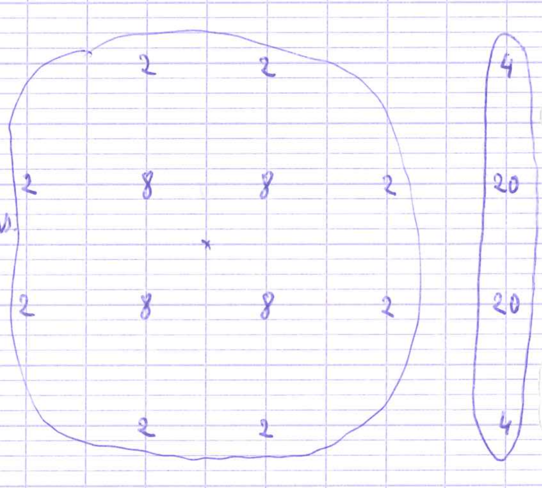
(0, 0, 2)	(2, 2, 0)	(0, 0, 1)	(1, -1, 0)
(0, 2, -2)	(2, 0, -2)	(0, 1, -1)	(1, 0, -1)
(0, 2, 0)	(2, 0, 0)	(0, 1, 0)	(1, 0, 0)
(0, 2, 2)	(2, 0, 2)	(0, 1, 1)	(1, 0, 1)
	(2, 2, 0)		(1, 1, 0)







; projections:



Dim. totale: 48, vérif. par la formule de Weyl:

$$\prod_{\alpha \in \Delta^+} \frac{\frac{1}{8}\langle (8,4,2), \alpha \rangle}{\frac{1}{8}\langle (5,3,1), \alpha \rangle} = \frac{2}{1}\,\frac{2}{2}\,\frac{4}{3}\,\frac{6}{4}\,\frac{4}{2}\,\frac{6}{4}\,\frac{8}{5}\,\frac{10}{6}\,\frac{12}{8} \quad \text{OK}$$

$(0,0,1)$ $(1,-1,0)$
$(0,1,-1)$ $(1,0,-1)$
$(0,1,0)$ $(1,0,0)$
$(0,1,1)$ $(1,0,1)$
$(1,1,0)$

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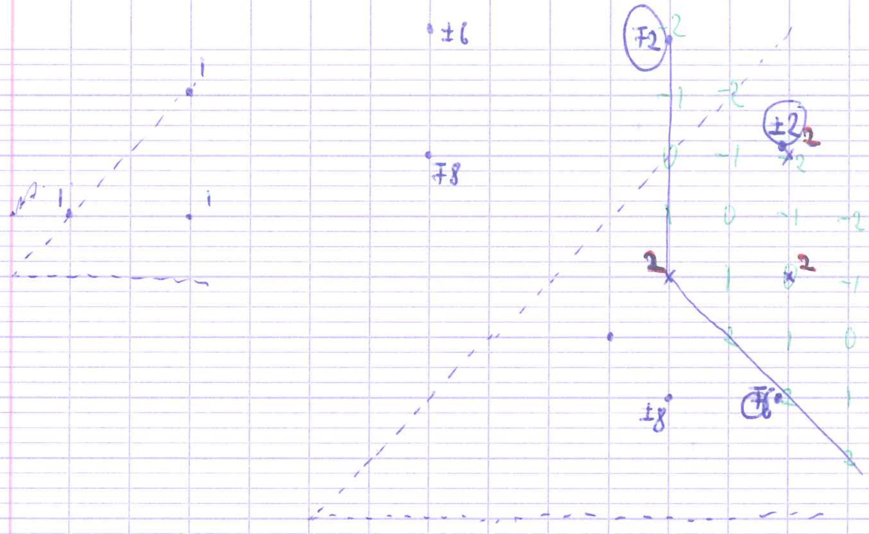
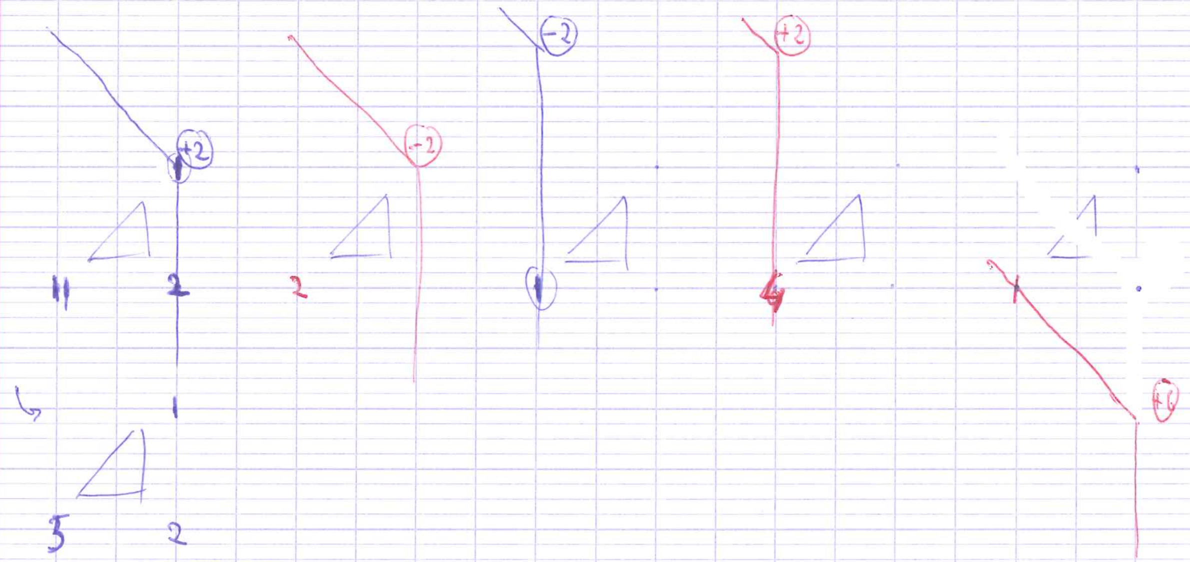
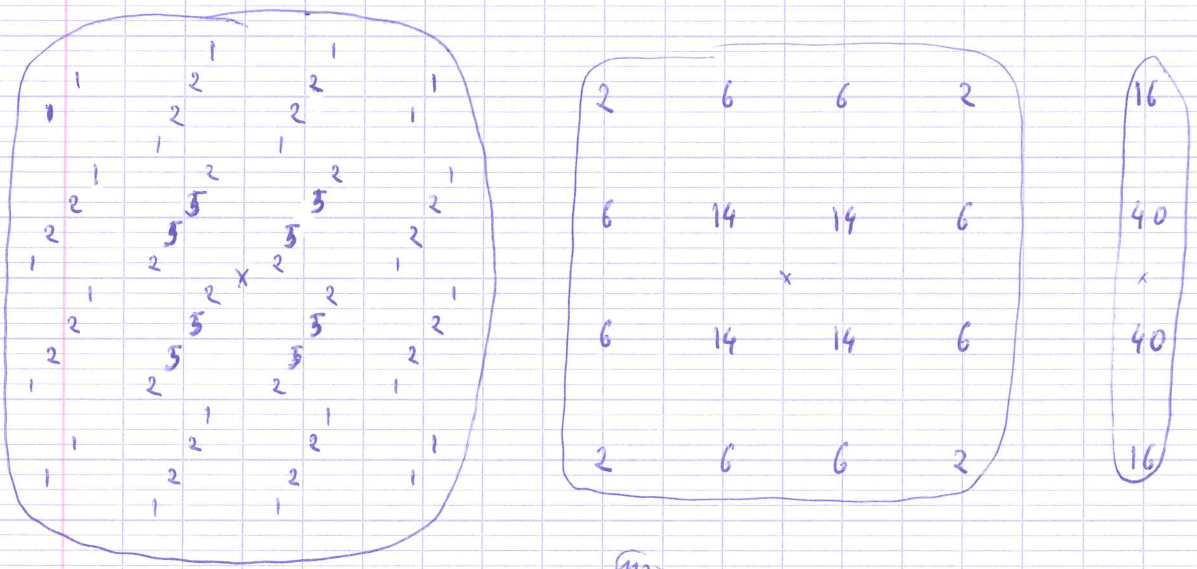


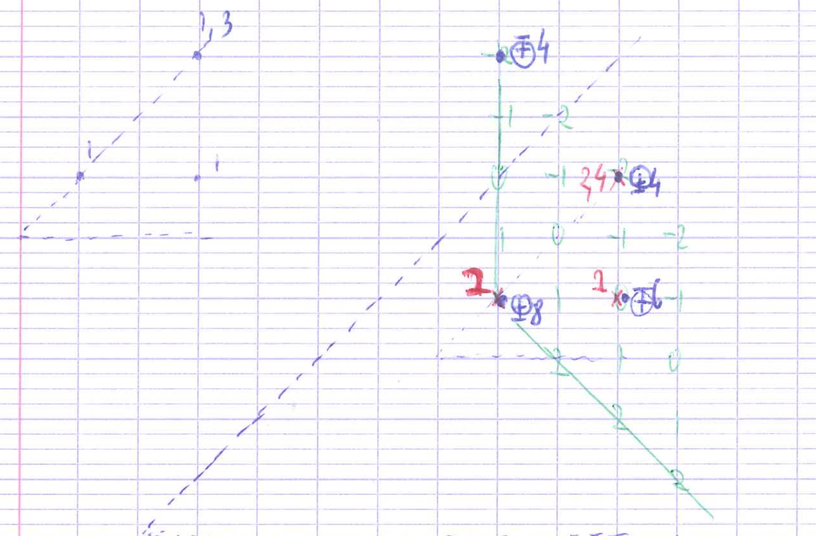




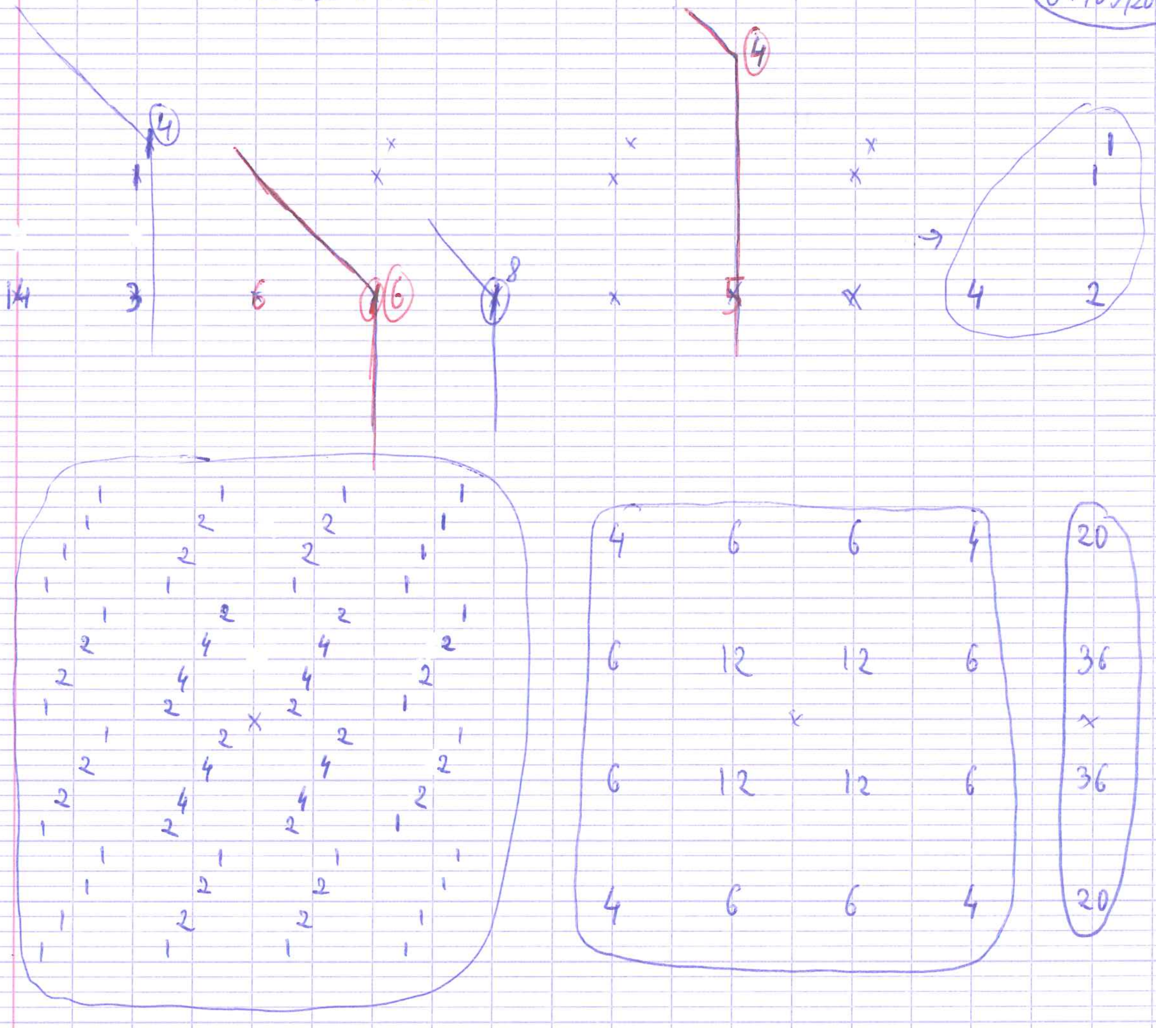
$40+72 = 112$

Dim. totale : $5\times 8 + 2\times 24 + 1\times 24 = 112$, vérif. par la formule de Weyl :

$$\prod_{\alpha\in\Delta^+} \frac{\frac{1}{8}\langle(8,6,2),\alpha\rangle}{\frac{1}{8}\langle(5,3,1),\alpha\rangle} = \frac{\cancel{2}\,\cancel{4}\,\cancel{6}\,\cancel{8}\,\cancel{2}\,\cancel{6}\,8\,\cancel{10}\,14}{1\,\cancel{2}\,\cancel{3}\,\cancel{4}\,\cancel{2}\,\cancel{4}\,\cancel{5}\,\cancel{6}\,8} \qquad 8\times 14 = 112\ldots$$



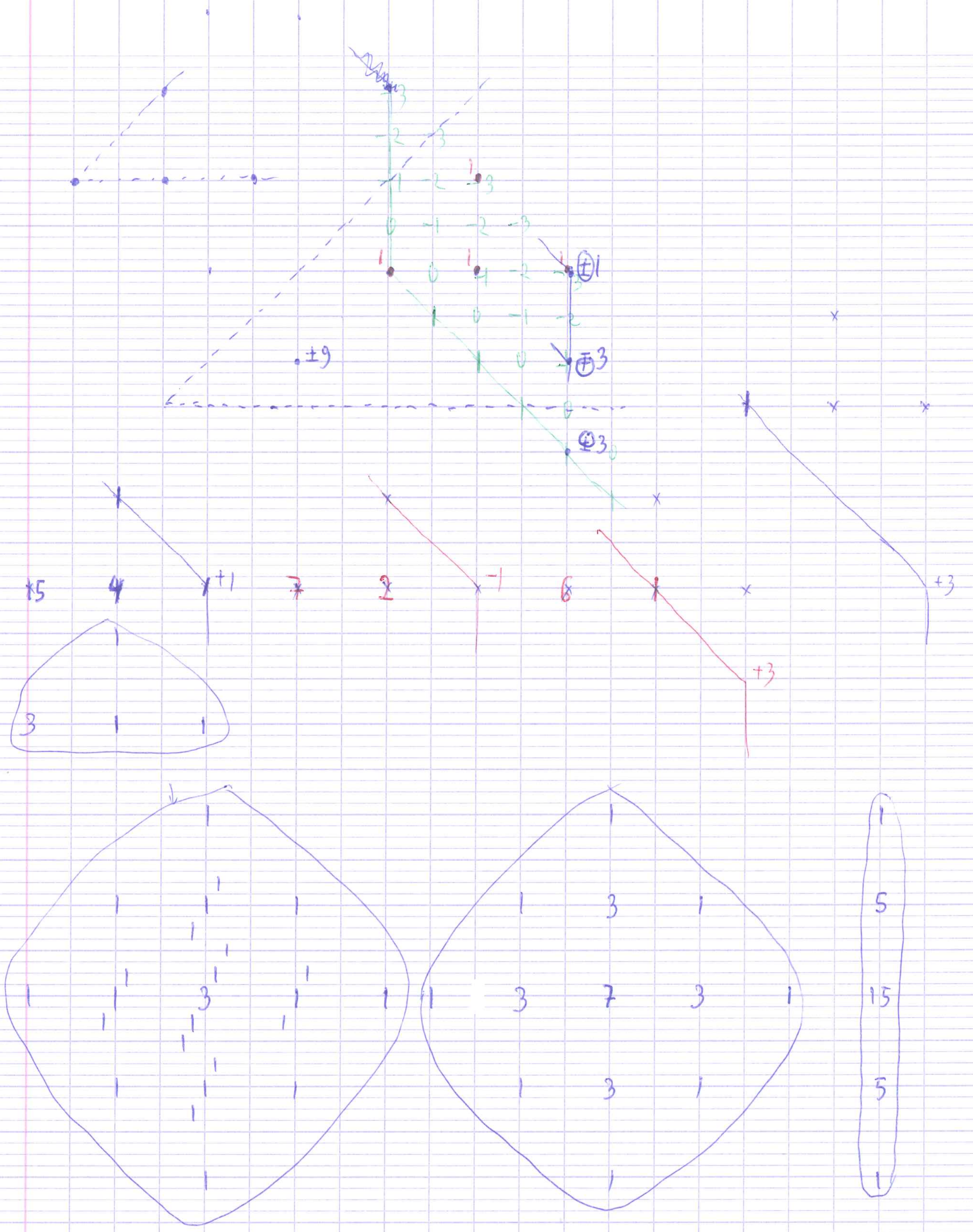
07/05/2015



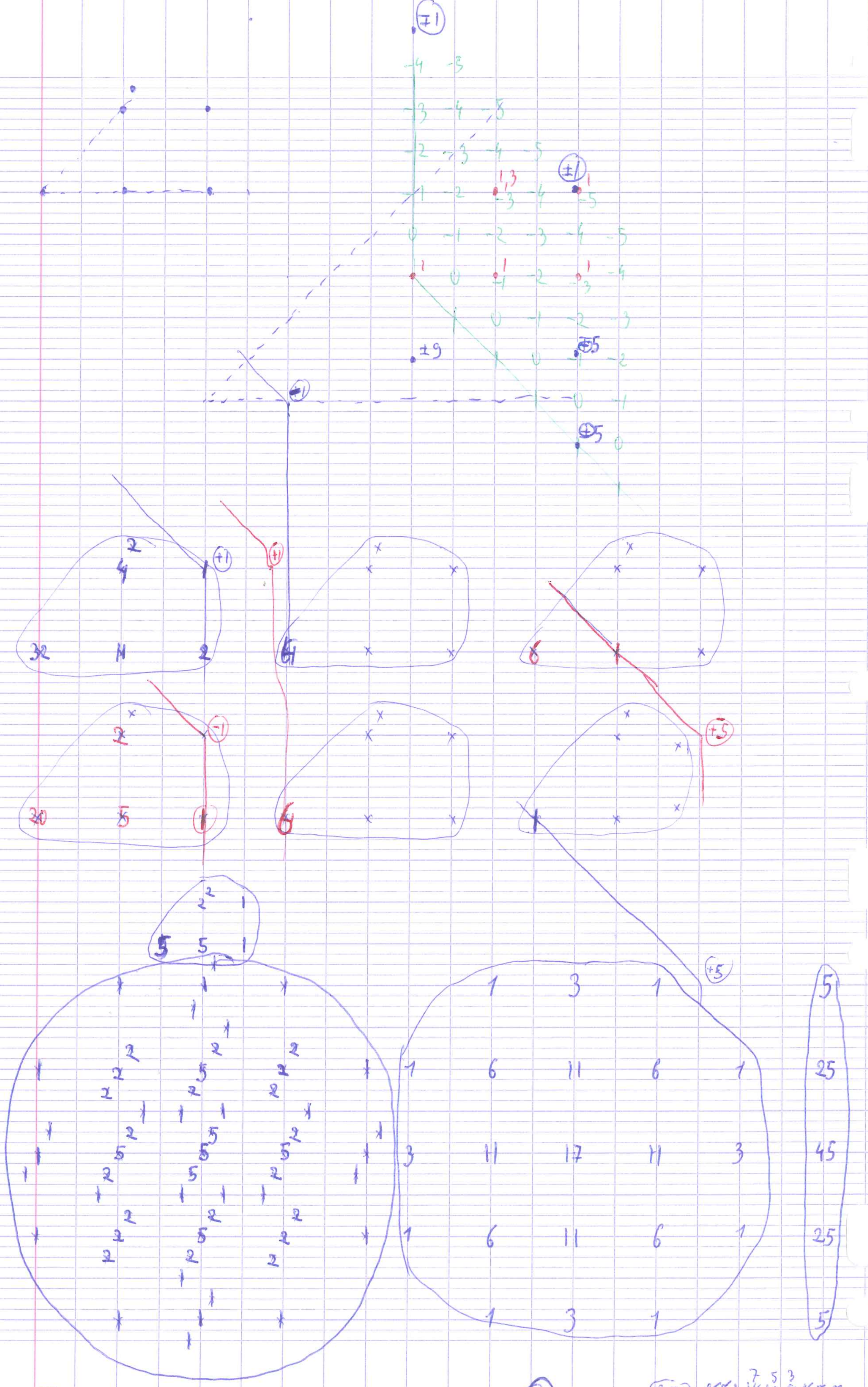
Tot. dim. $1\times 8 + 1\times 24 + 2\times 24 + 4\times 8 = (1+3+6+4)\times 8 = 14\times 8 = 112$

Formule de Weyl: $\prod_{\alpha\in\Delta^+} \frac{\frac{1}{8}\langle(8,6,4),\alpha\rangle}{\frac{1}{8}\langle(5,3,1),\alpha\rangle} = \frac{4}{1}\,\frac{\cancel{2}}{\cancel{2}}\,\frac{\cancel{6}}{\cancel{3}}\,\frac{\cancel{10}}{\cancel{4}}\,\frac{\cancel{2}}{\cancel{2}}\,\frac{\cancel{4}}{\cancel{4}}\,\frac{\cancel{8}}{\cancel{8}}\,\frac{\cancel{12}}{\cancel{6}}\,\frac{14}{\cancel{8}}$ $4\times 2\times 14$ OK.

07/05/2015



Dim. totale : 27 ; formule de Weyl : $\dfrac{9\ 3\ 1}{5\ 3\ 1} \to \cancel{\dfrac{9}{5}}\ \cancel{\dfrac{3}{3}}\ \cancel{\dfrac{1}{1}}\ \cancel{\dfrac{\overset{3}{12}}{8}}\ \cancel{\dfrac{10}{6}}\ \cancel{\dfrac{4}{4}}\ \dfrac{6}{2}\ \cancel{\dfrac{8}{4}}\ \cancel{\dfrac{2}{2}}$ OK



Total: $5 + 5\times 6 + 1\times 6 + 2\times 12 + 2\times 8 + 1\times 24 = 5 + 36 + 40 + 24 = 105$. Formule de Weyl: $\frac{(9\,5\,1)}{(5\,3\,1)} \rightarrow \frac{8}{8}\,\frac{8}{8}\,\frac{1}{1}\,\frac{14}{8}\,\frac{10}{8}\,\frac{6}{4}\,\frac{4}{2}\,\frac{8}{4}\,\frac{2}{2}$ $7\times 5\times 3 = 105$

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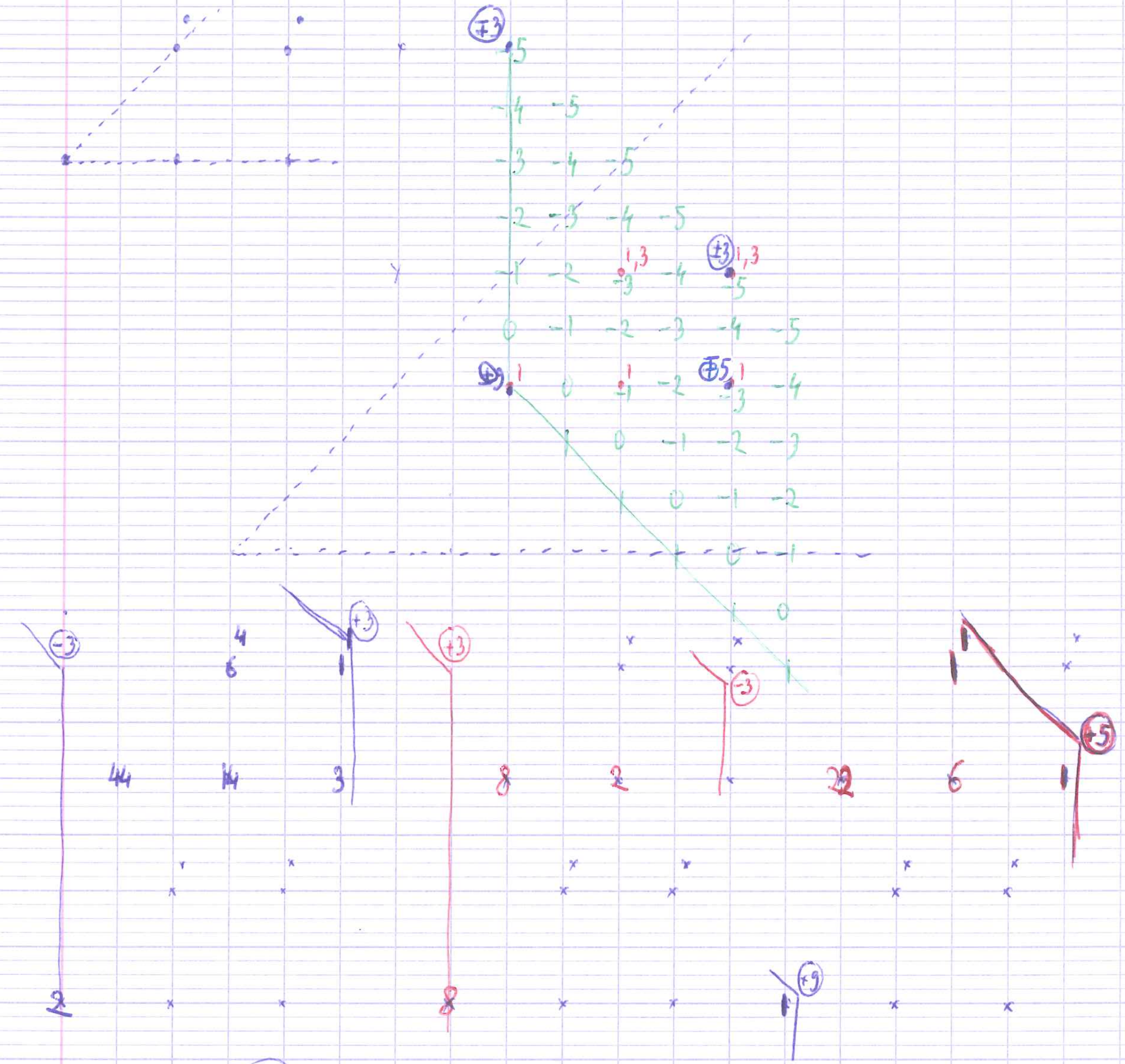
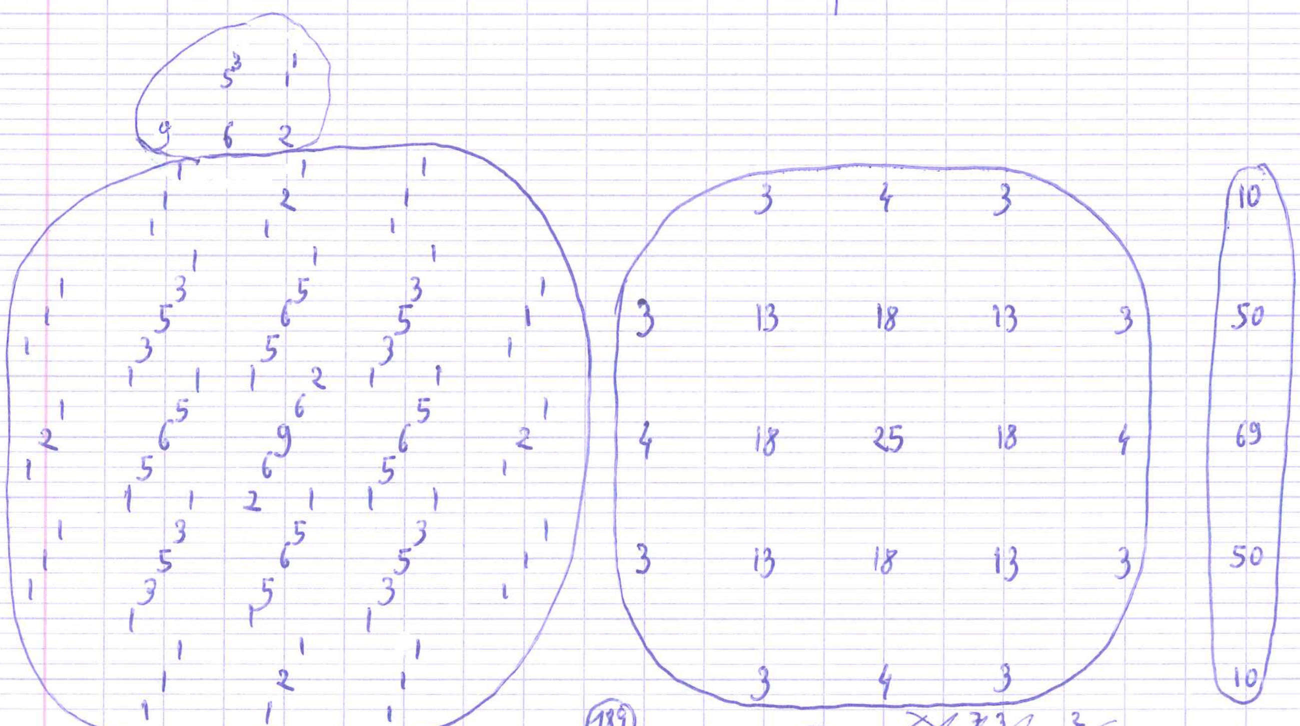
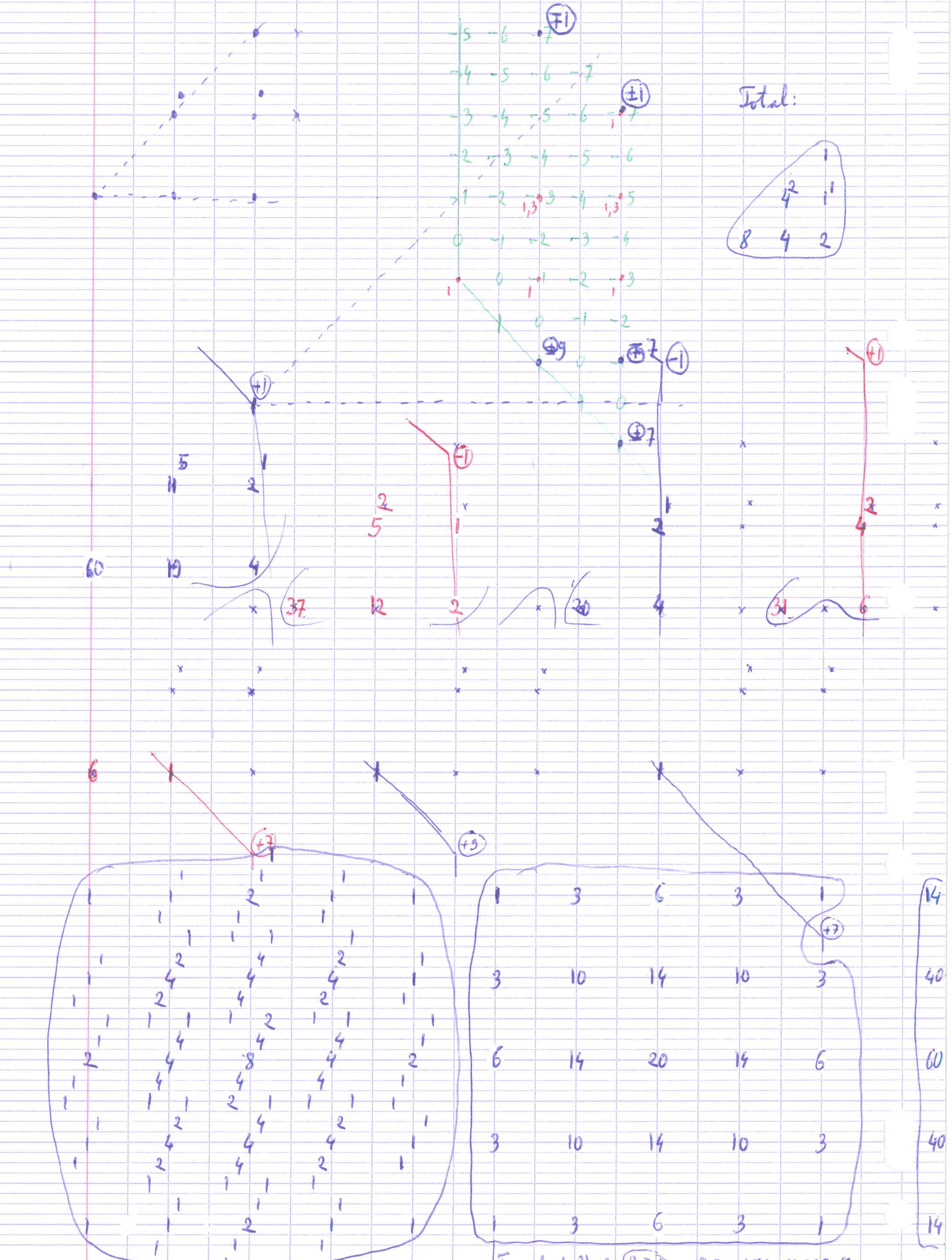




Total: $9 + 6\times 6 + 2\times 6 + 5\times 12 + 3\times 8 + 1\times 24 + 1\times 24 =$ 189

Formule de Weyl: $\dfrac{953}{531} = \dfrac{9}{5}\,\dfrac{5}{3}\,\dfrac{3}{1}\,\dfrac{14}{8}\,\dfrac{12}{6}\,\dfrac{8}{4}\,\dfrac{4}{2}\,\dfrac{6}{4}\,\dfrac{2}{2}$ 189

$\dfrac{9}{5}\,\dfrac{8}{3}\,\dfrac{3}{1}\,\dfrac{14}{7}\,\dfrac{12}{6}\,\dfrac{8}{4}\,\dfrac{4}{2}\,\dfrac{6}{4}\,\dfrac{2}{2}$ $27\times 7 = 189$ OK.



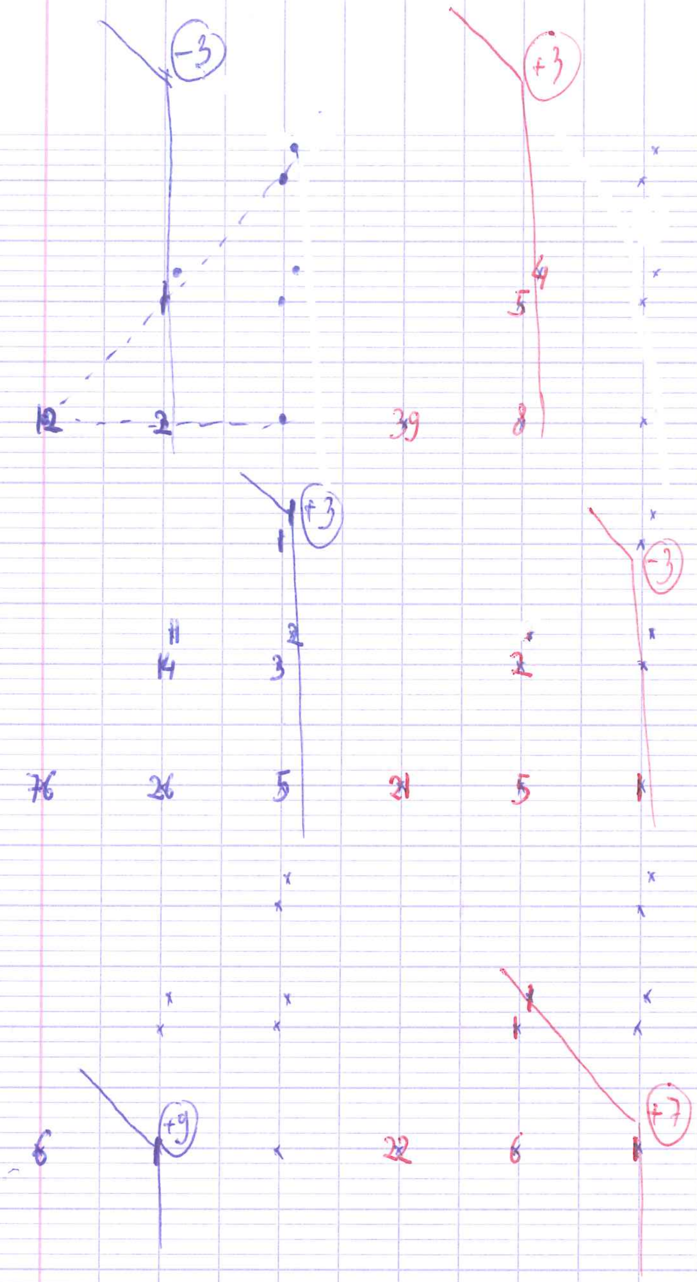
Total: $8+4\times6+2\times6+4\times12+2\times8+1\times24+1\times12+1\times24=$

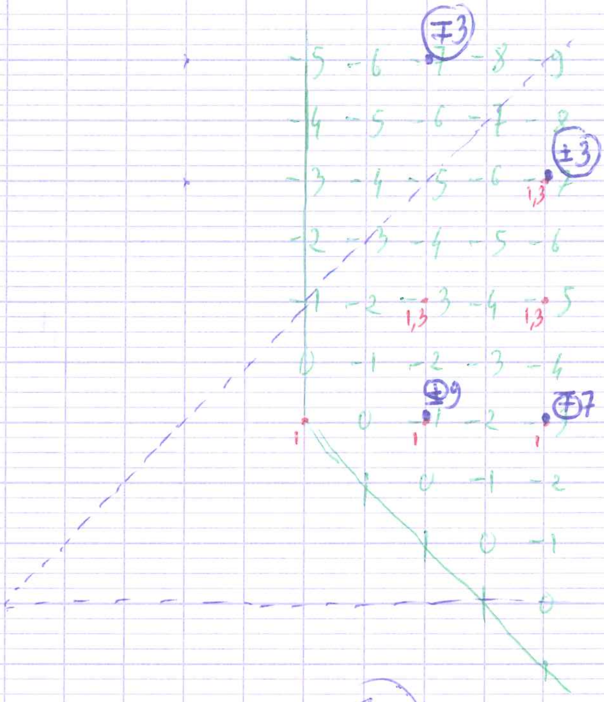
$=8+36+48+16+60$

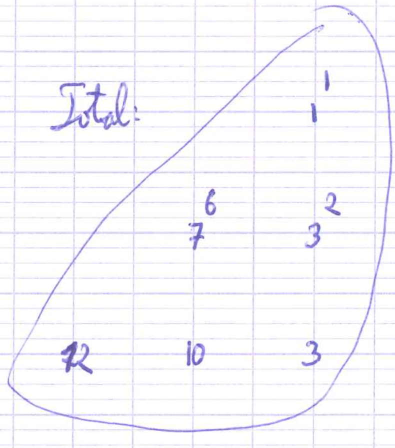
$=24+36+48+60=168$

Formule de Weyl: $\frac{9\ 7\ 1}{5\ 3\ 1} \to \frac{9}{\cancel{5}}\frac{7}{3}\frac{1}{1}\frac{\cancel{16}}{\cancel{8}}\frac{\cancel{10}}{\cancel{6}}\frac{8}{\cancel{4}}\frac{\cancel{2}}{\cancel{2}}\frac{8}{\cancel{4}}\frac{\cancel{6}}{\cancel{2}}$

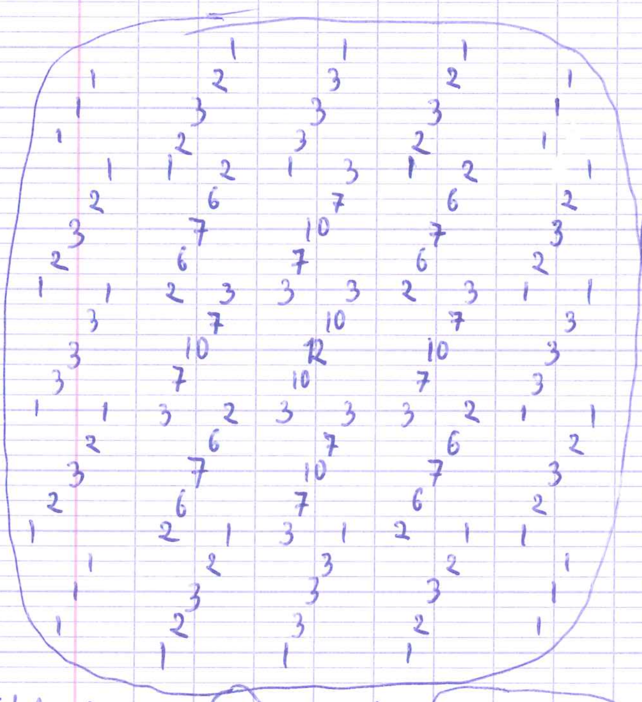
$=2^3\times3\times7=168$







94 − 82 = 12



3	9	11	9	3	35
9	23	30	23	9	94
11	30	38	30	11	120
9	23	30	23	9	94
3	9	11	9	3	35

Total: $12 + 10\times 6 + 3\times 6 + 7\times 12 + 6\times 8 + 3\times 24 + 2\times 24 + 1\times 24 + 1\times 12 =$

$= 12 + \cancel{180}\,78 + 84 + 48 + 144 + 12$

$= \cancel{180}\,90 + 60 + 144 + \cancel{106}\,84 =$ 378

Formule de Weyl: $\frac{9\,7\,3}{5\,3\,1} \to \frac{9}{5}\frac{7}{3}\frac{3}{1}\frac{16}{8}\frac{12}{6}\frac{10}{4}\frac{2}{2}\frac{8}{4}\frac{4}{2}$ 378

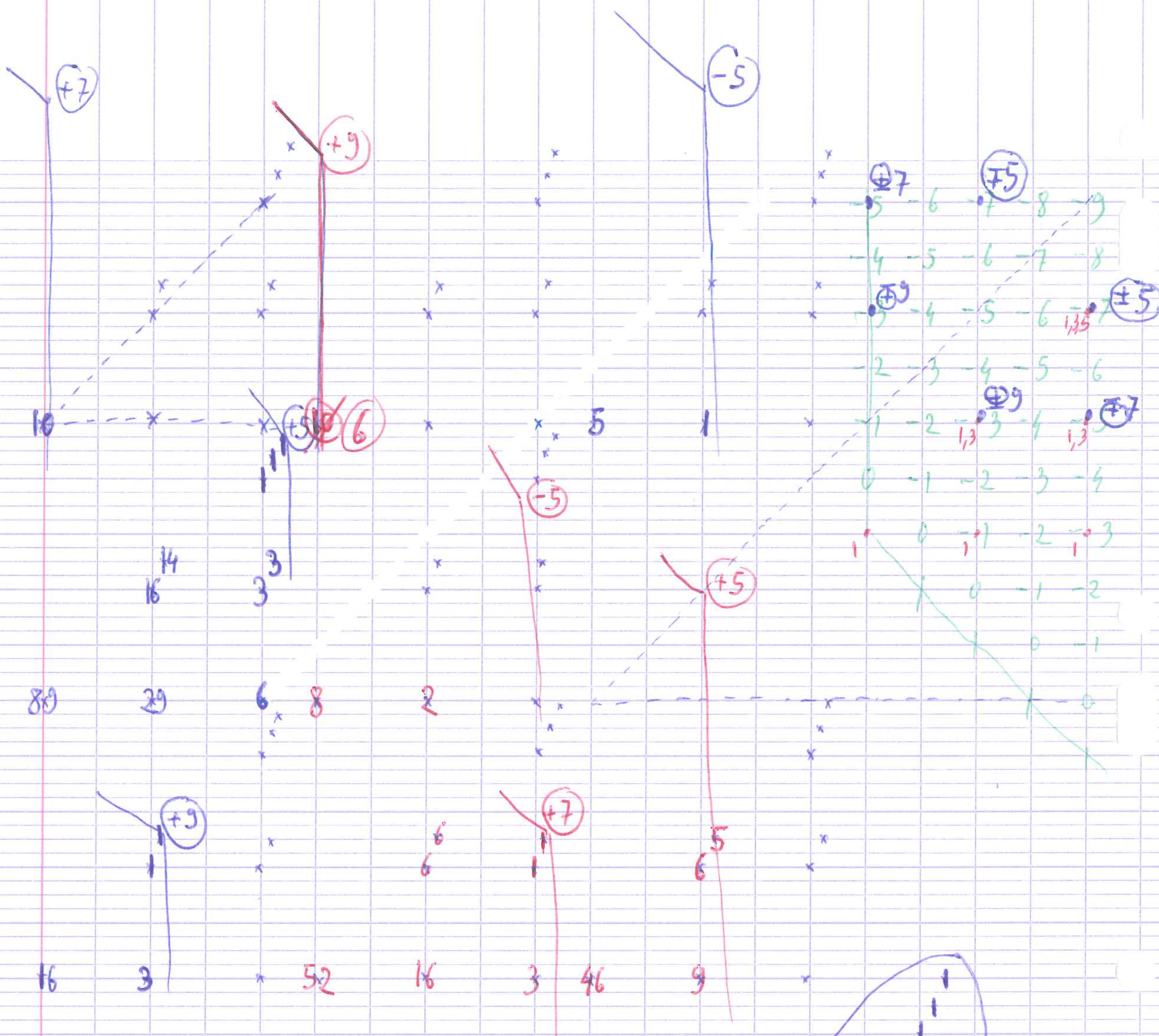
$v_1 : 1$ $v_7 : 1$

$v_3 : 3$

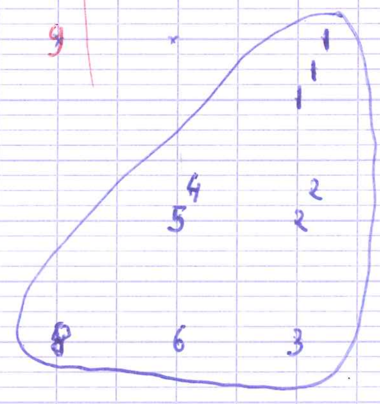
$v_5 : 0$

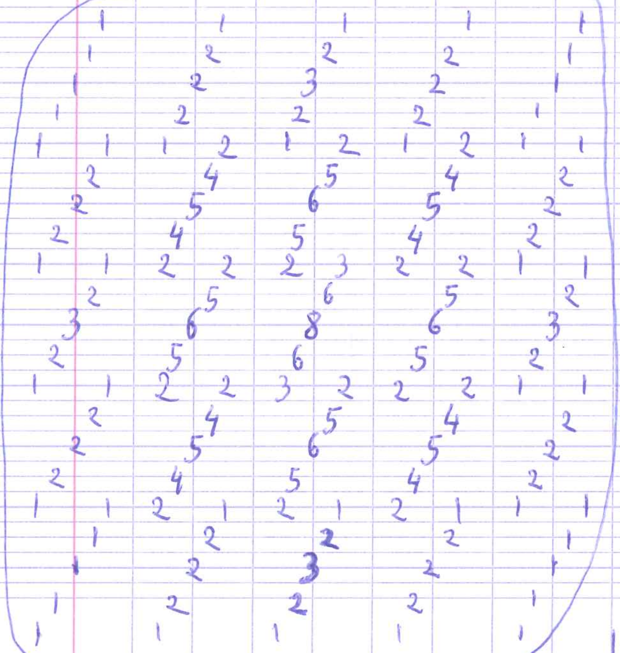
$9\times 7\times 12\times 2 = 63\times 6 = 378$

$27\times 14 = 270 + 108 = 378$



Total:





5	8	9	8	5	35
8	17	20	17	8	70
9	20	26	20	9	84
8	17	20	17	8	70
5	8	9	8	5	35

Total: $8+6\times6+5\times12+4\times8+3\times6+1\times12+1\times8+2\times24+2\times24+1\times24=$
$=8+36+60+32+18+12+8+120=100+36+30+8+120$
$=294$

Formule de Weyl: $\frac{975}{531} \to \frac{9}{5}\,\frac{7}{3}\,\frac{5}{1}\,\frac{16}{8}\,\frac{14}{6}\,\frac{12}{4}\,\frac{7}{2}\,\frac{4}{1}\,\frac{7}{2}$

$2\times3\times7^2=294$