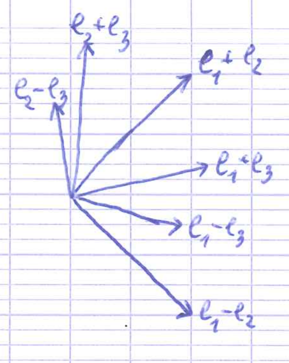
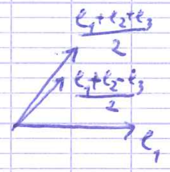
(D_3) Racines positives:



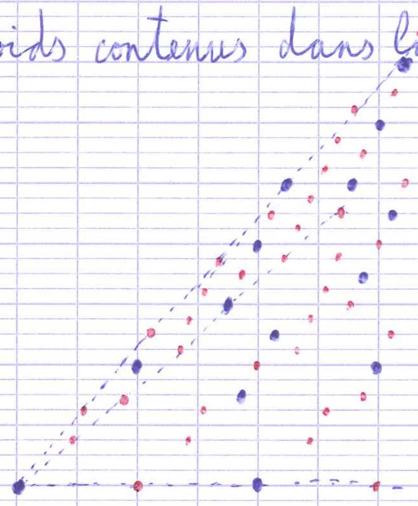
; racines simples: e_1-e_2, e_2-e_3, e_2+e_3

Chambre de Weyl: 2 fois plus grande!

$\delta = 2e_1 + e_2$; poids fondamentaux:

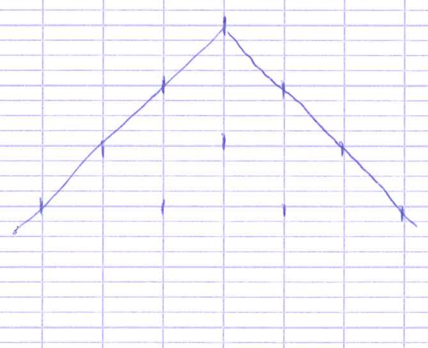


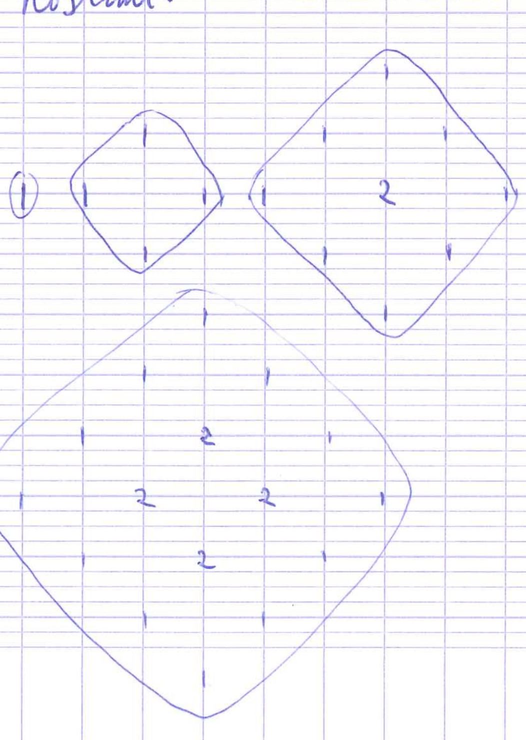
Poids contenus dans la chambre de Weyl:

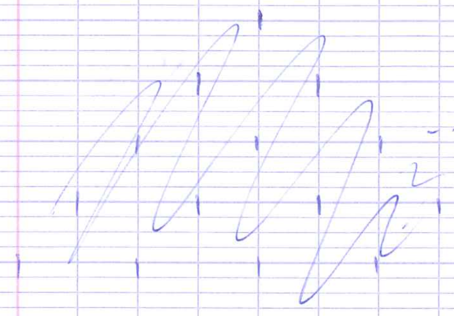


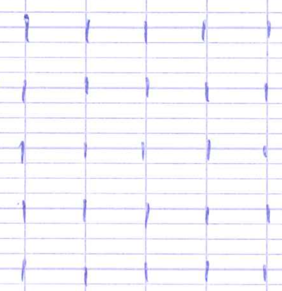
Remarque: l'automorphisme du diagramme (qui correspond à la symétrie par rapport au plan de la feuille) réduit le nombre de représentations à considérer.

Fonction de partition de Kostant:









2	2	2	2	1
4	4	4	4	2
4	4	4	4	2
4	4	4	4	2
4	4	4	4	2

⋮

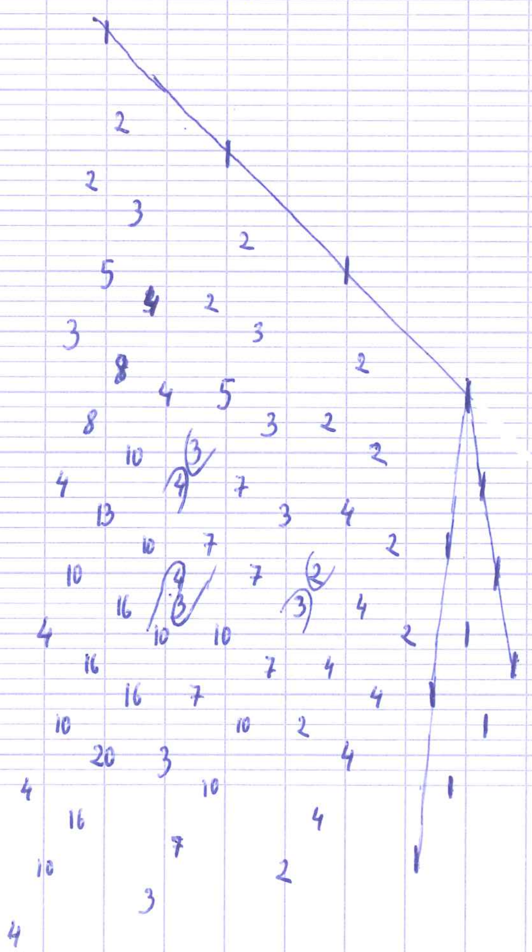
4	4	4	3	2	1
10	10	10	8	5	2
16	16	16	13	8	3
20	20	20	16	10	4
20	20	20	16	10	4
20	20	20	16	10	4

⋮

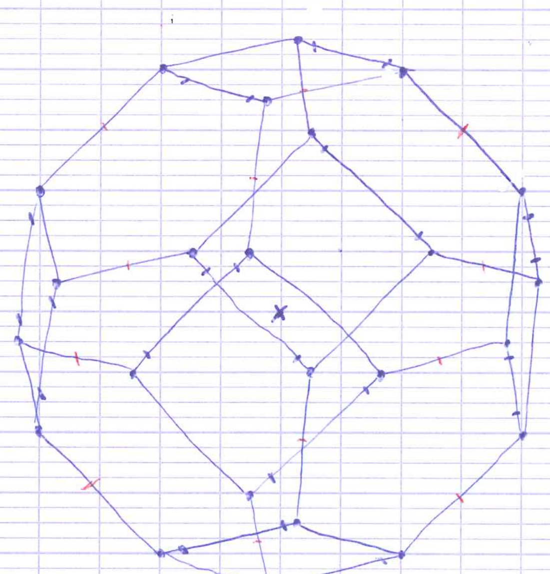
3	3	3	3	2	1
7	7	7	7	5	2
10	10	10	10	7	3
10	10	10	10	7	3
10	10	10	10	7	3
10	10	10	10	7	3

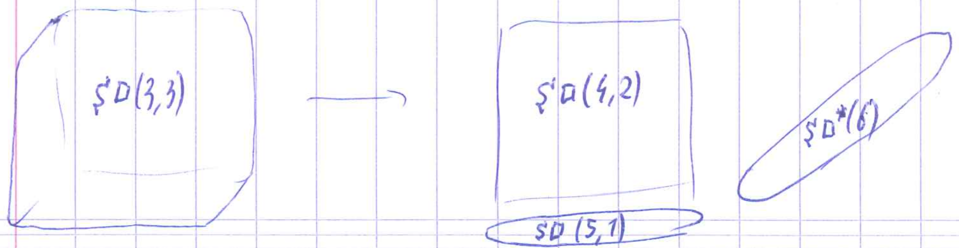
⋮

Fonction de partition de Kostant :



Orbite de δ par le groupe de Weyl :



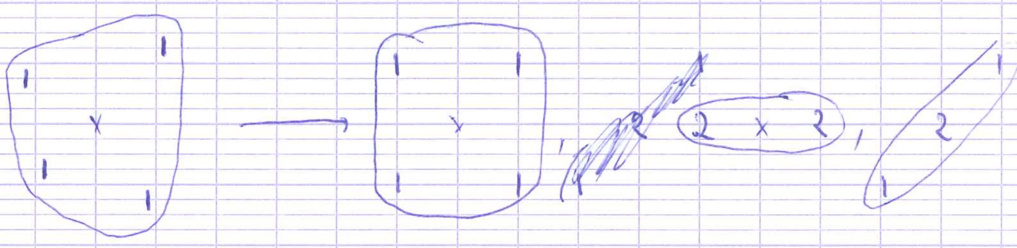


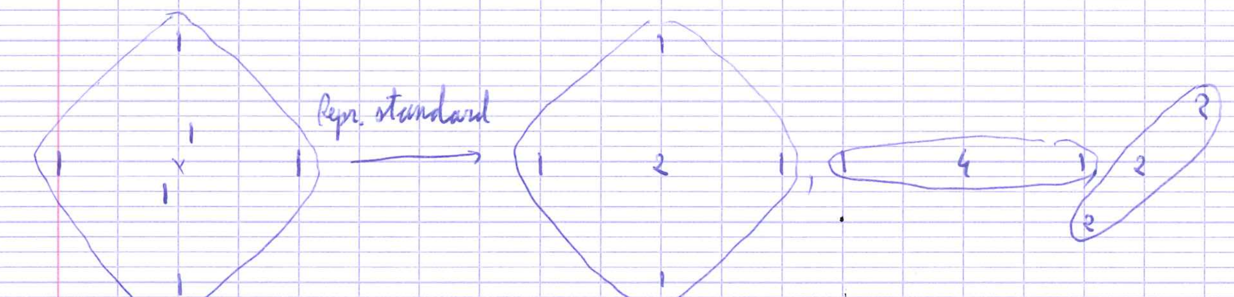
$\boxed{\mathfrak{so}(4,2)}$ Espace des racines réelles : ~~$\langle e_1+e_2 \rangle$~~ $\langle e_1+e_2, e_1-e_2 \rangle = \langle e_1, e_2 \rangle$

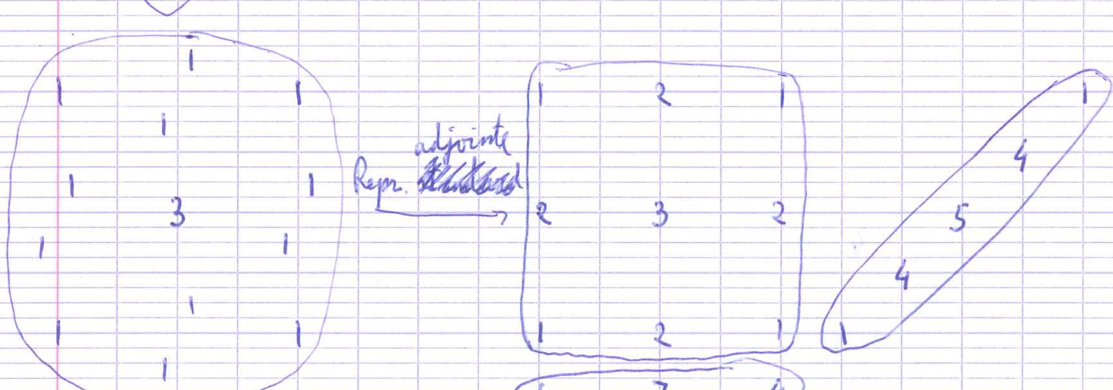
$\boxed{\mathfrak{so}(5,1)}$ Espace des racines réelles : $\langle e_1 \rangle$

$\boxed{\mathfrak{so}^*(6)}$ Espace des racines réelles : $\langle e_1 + e_2 \rangle$

① Repr. triviale → ① ① ①







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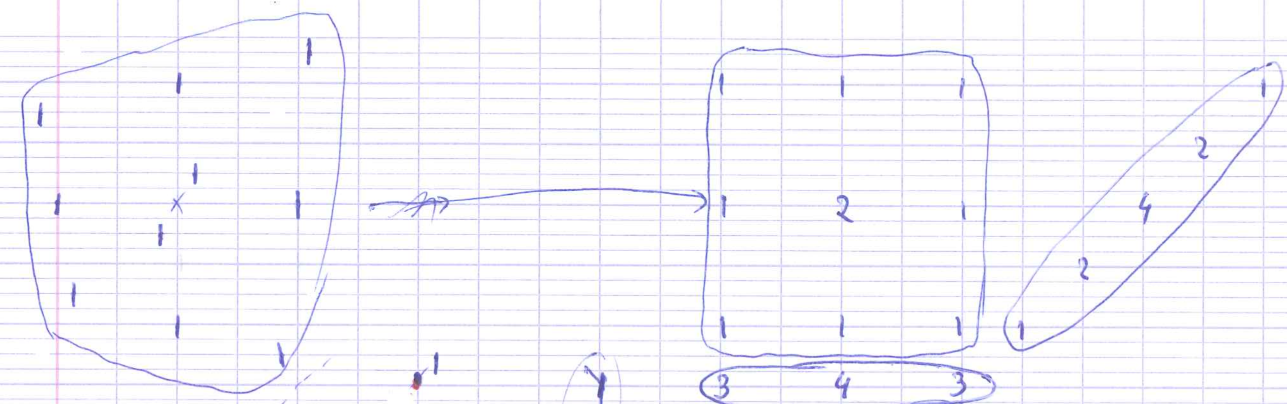
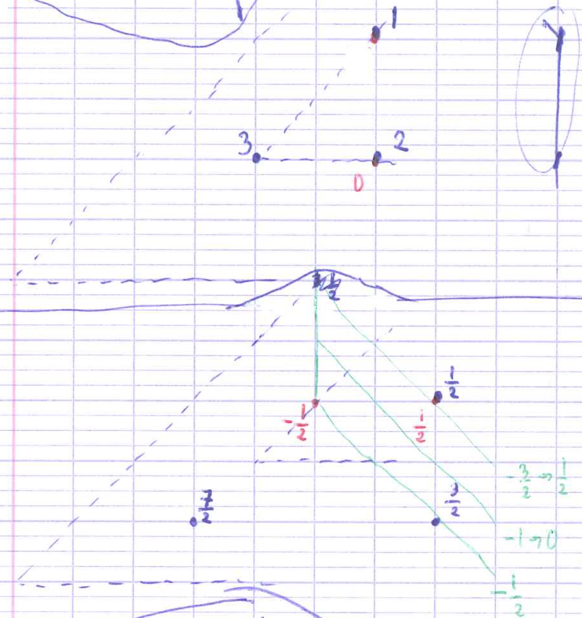
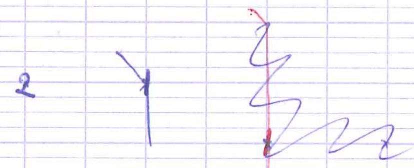
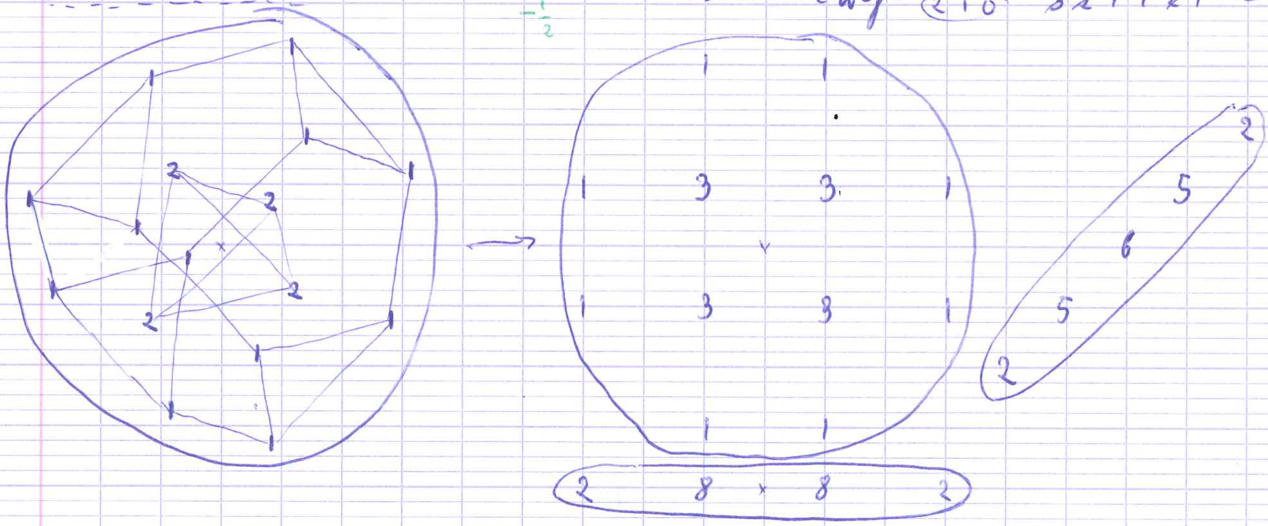


Formule de Weyl: $\dfrac{3\ 2\ 1}{2\ 1\ 0} \to \dfrac{5}{3}\,\dfrac{4}{2}\,\dfrac{3}{1}\,\dfrac{1}{1}\,\dfrac{2}{2}\,\dfrac{1}{1} \to 10$





Formule de Weyl: $\dfrac{\frac{7}{2}\ \frac{3}{2}\ \frac{1}{2}}{2\ 1\ 0} \to \dfrac{5}{3}\,\dfrac{4}{2}\,\dfrac{2}{1}\,\dfrac{2}{1}\,\dfrac{3}{2}\,\dfrac{1}{1} \to 20$



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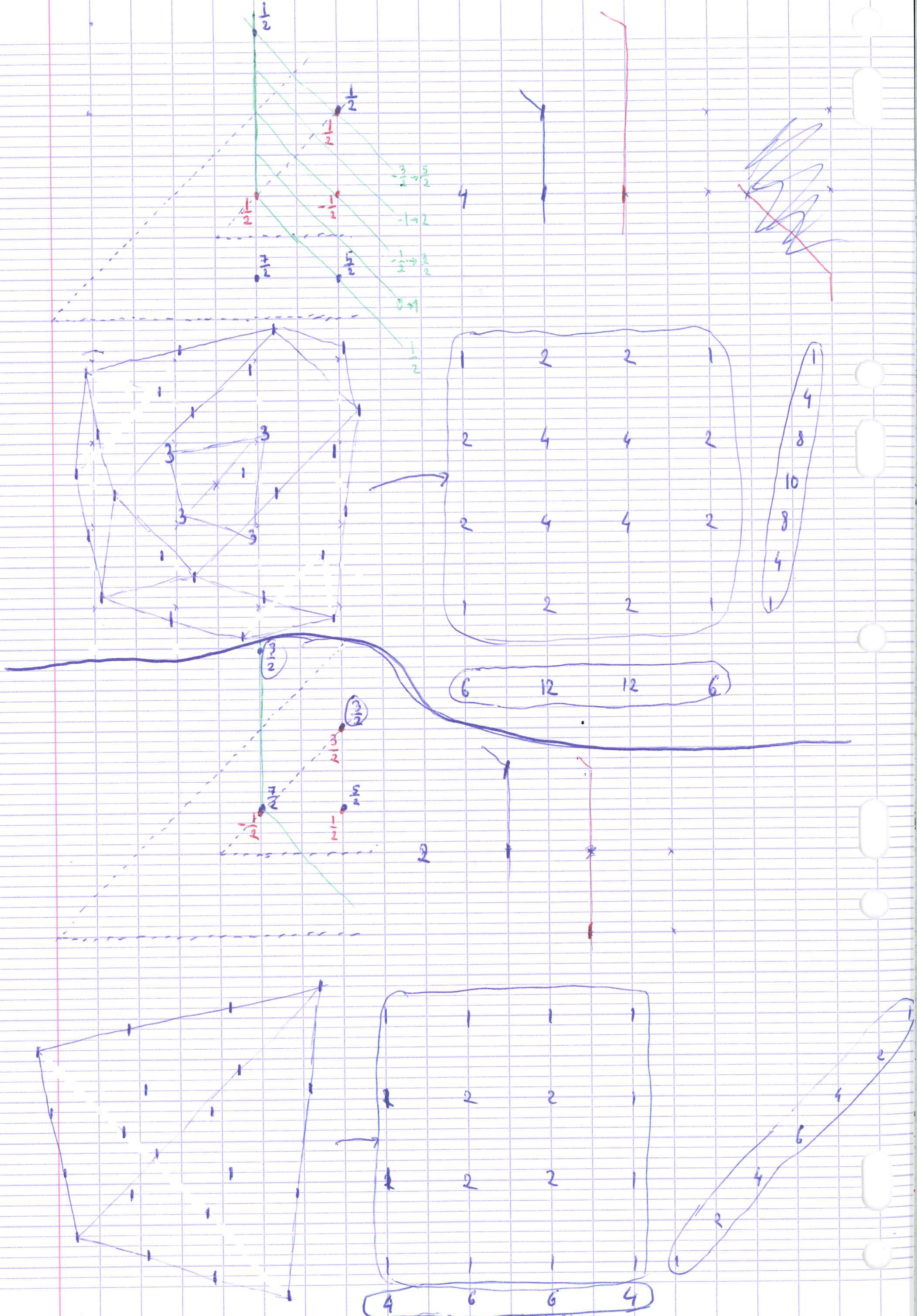
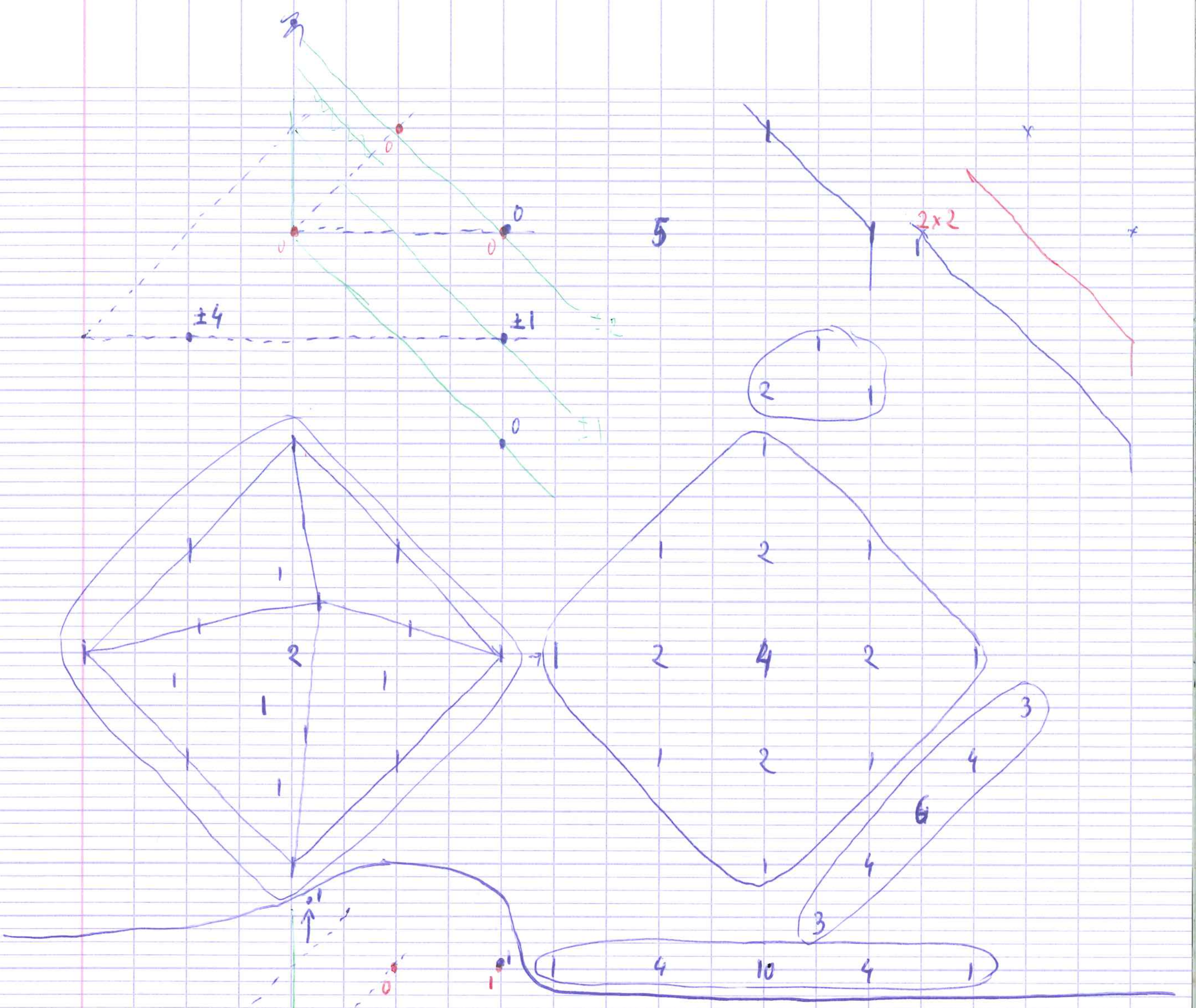
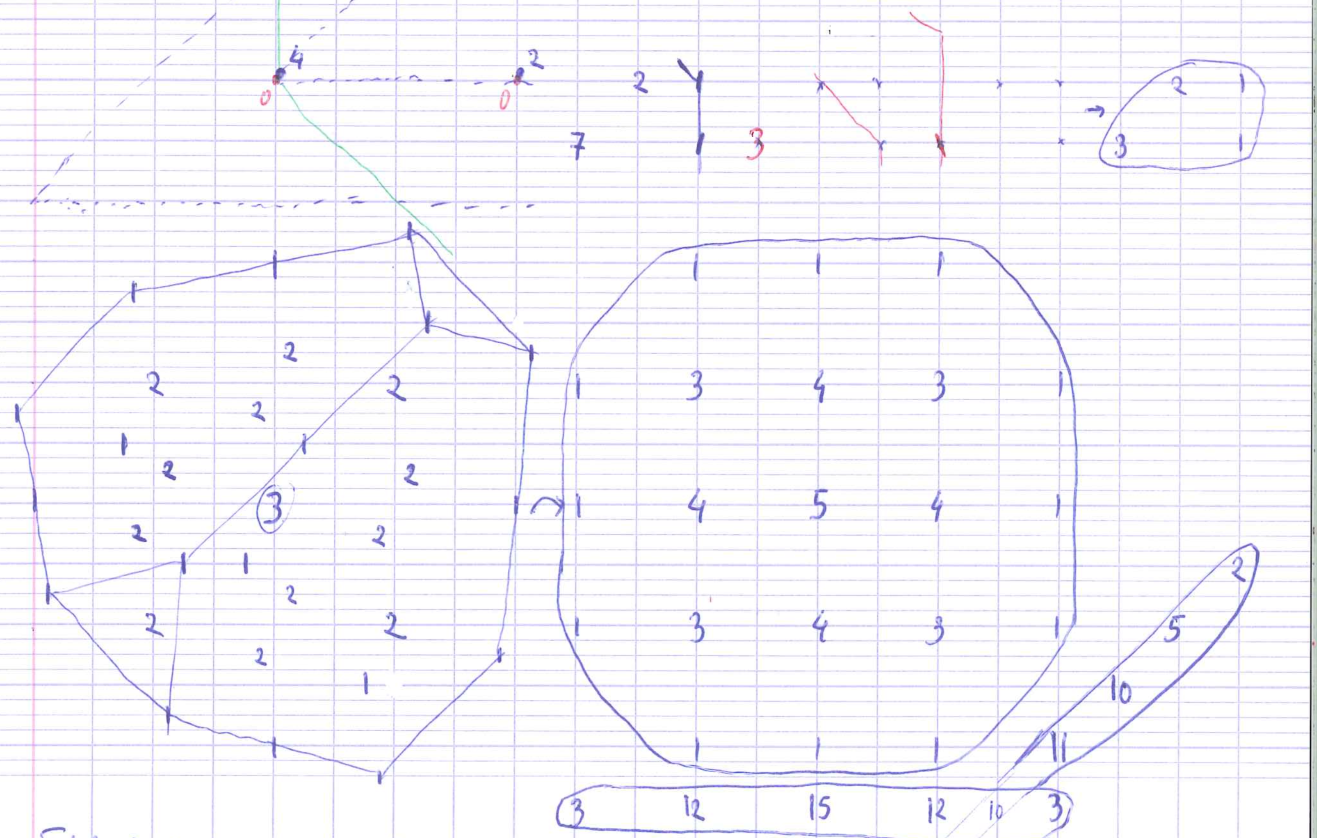
Total: $3\times 4 + 12 + 12 = 36$; Formule de Weyl: $\dfrac{\left(\frac{7}{2}\ \frac{5}{2}\ \frac{1}{2}\right)}{\left(2\ 1\ 0\right)}$ $\frac{6}{3}\ \frac{4}{2}\ \frac{3}{1}\ \frac{1}{1}\ \frac{3}{2}\ \frac{2}{1} \to 36$



Total: $1\times 4 + 1\times 12 + 1\times 4 = 20$; formule de Weyl: $\dfrac{\left(\frac{7}{2}\ \frac{5}{2}\ \frac{3}{2}\right)}{\left(2\ 1\ 0\right)}$ $\frac{6}{3}\ \frac{5}{2}\ \frac{4}{1}\ \frac{1}{1}\ \frac{2}{2}\ \frac{1}{1} \to 20$

Total : $2+1\times12+1\times6 = 20$ formule de Weyl : $\begin{pmatrix}4&1&0\\2&1&0\end{pmatrix} \to \frac{5}{3}\frac{4}{2}\frac{1}{1}\frac{3}{1}\frac{4}{2}\frac{1}{1} \to 20$

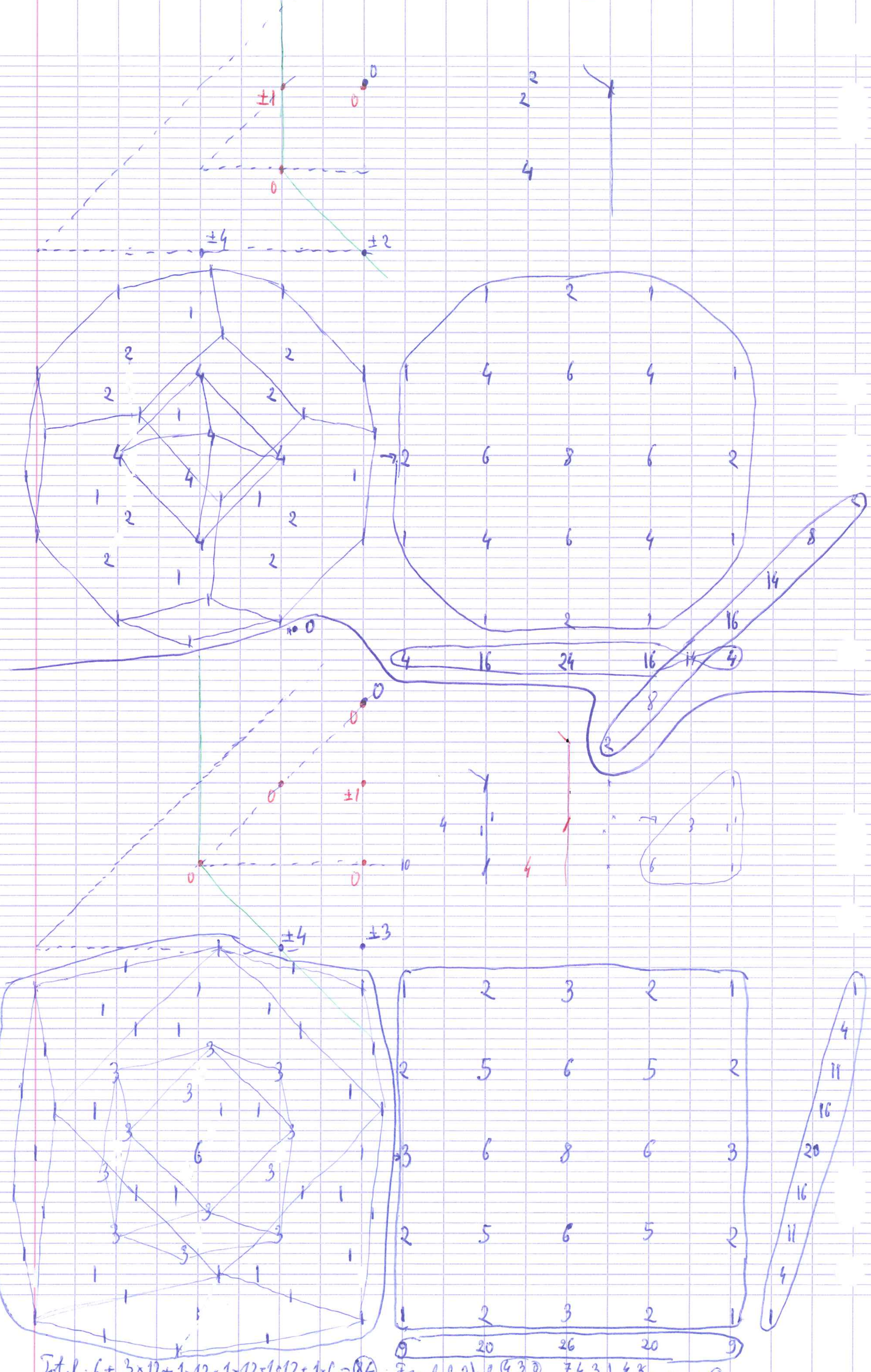




Total : $3+2\times12+1\times6+1\times12 = 45$; formule de Weyl : $\begin{pmatrix}4&2&1\\2&1&0\end{pmatrix} \to \frac{6}{3}\frac{5}{2}\frac{3}{1}\frac{2}{1}\frac{3}{2}\frac{1}{1} \to 45$

Total: $4\times 6 + 2\times 4 + 2\times 4 + 1\times 24 = 64$

Formule de Weyl: $\begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \end{pmatrix} \to \frac{6}{3}\,\frac{4}{2}\,\frac{2}{1}\,\frac{2}{1}\,\frac{4}{2}\,\frac{2}{1} \to 64$



Total: $6 + 3\times 12 + 1\times 12 + 1\times 12 + 1\times 12 + 1\times 6 = 84$; Formule de Weyl: $\begin{pmatrix} 4 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix} \to \frac{7}{3}\,\frac{4}{2}\,\frac{3}{1}\,\frac{1}{1}\,\frac{4}{2}\,\frac{3}{1} \to 7\times 12 = 84$

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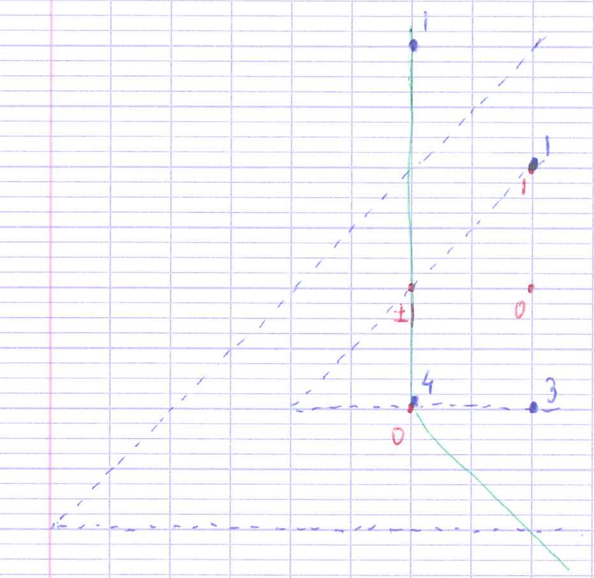
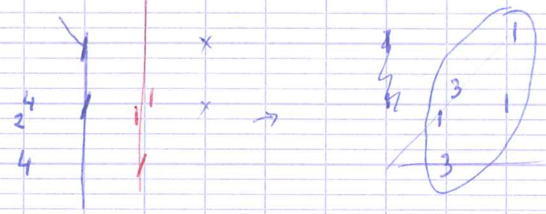
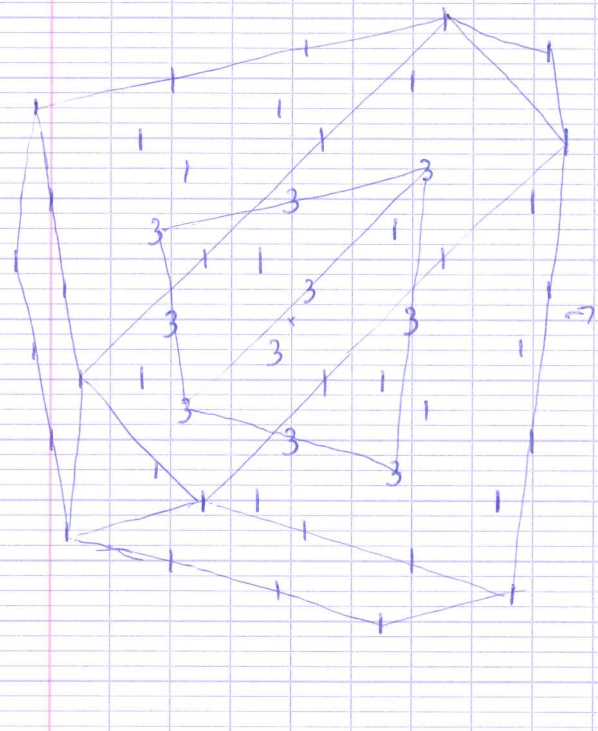
Total : $3\times 6 + 1\times 4 + 3\times 4 + 1\times 24 + 1\times 12 = 70$; formule de Weyl : $\frac{4\ 3\ 1}{2\ 1\ 0} \to \frac{7}{3}\,\frac{5}{2}\,\frac{\cancel{4}}{1}\,\frac{1}{1}\,\frac{3}{2}\,\frac{2}{1} \to 70$





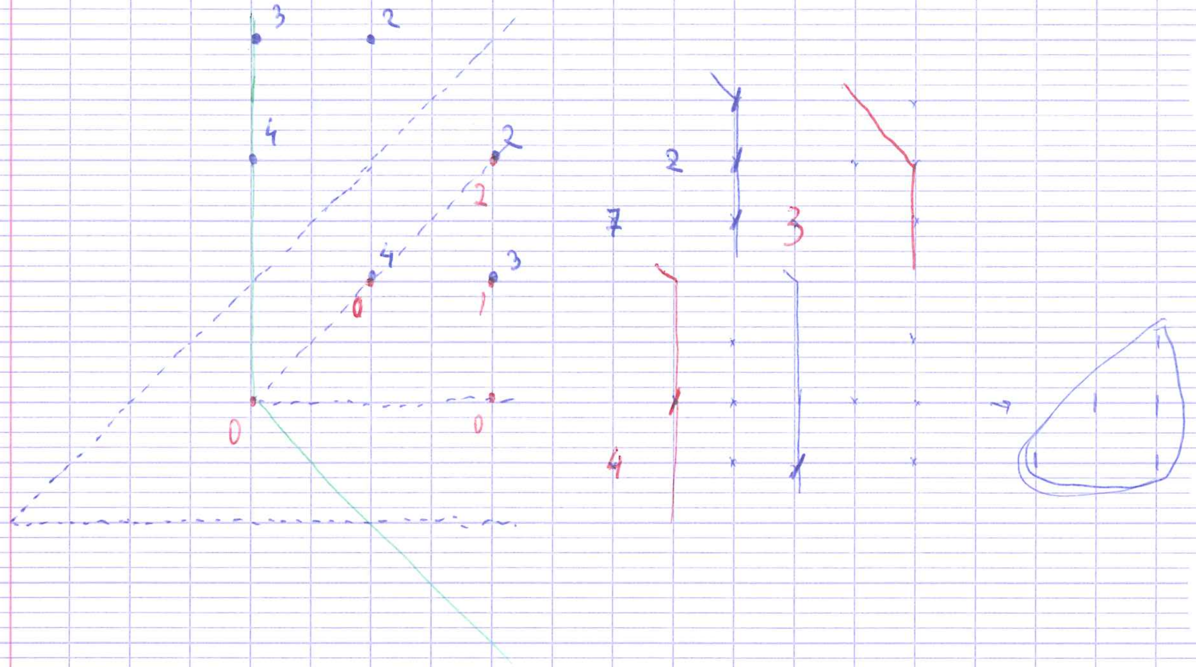


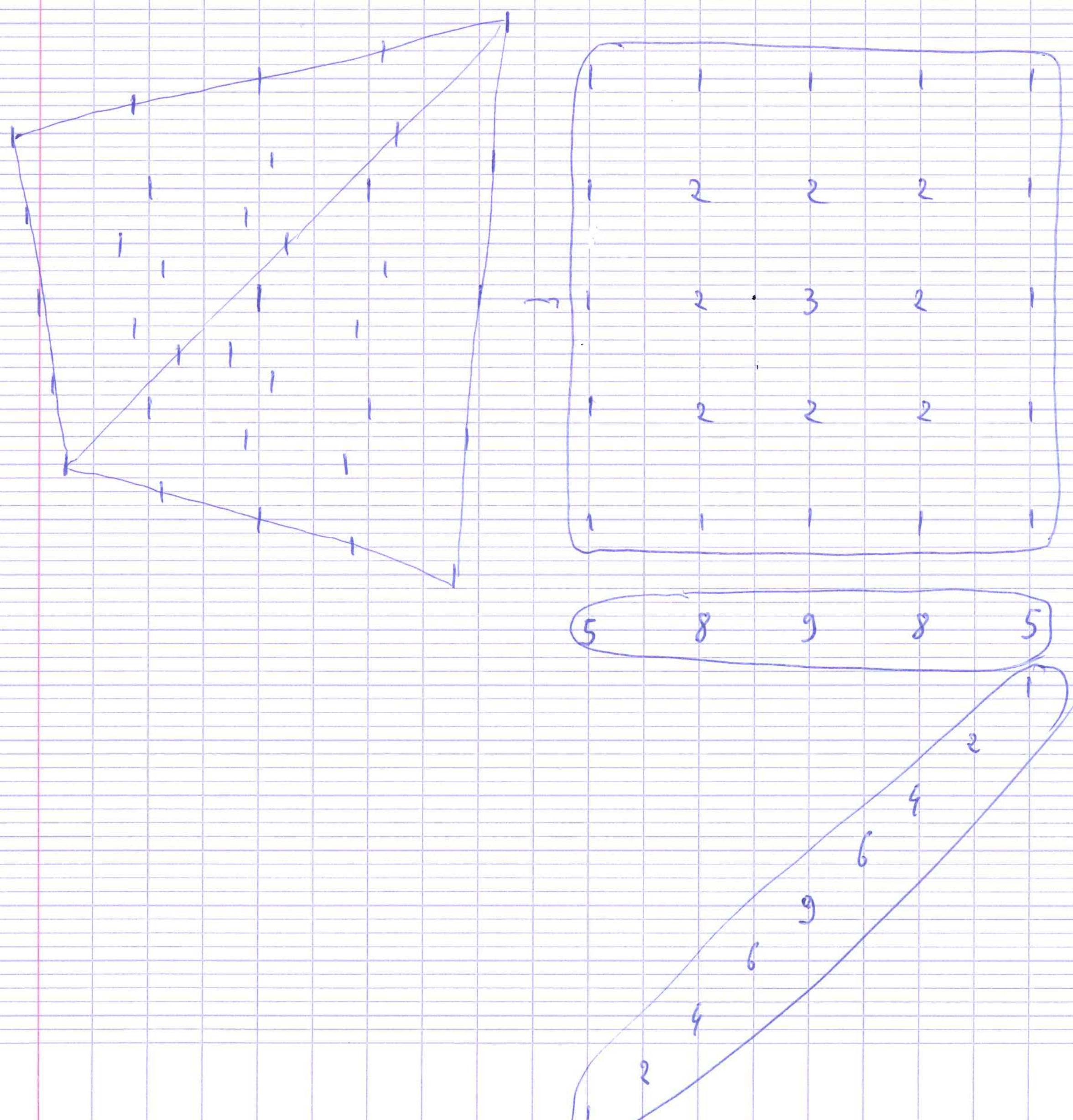
1	2	2	2	1
2	4	5	4	2
2	5	6	5	2
2	4	5	4	2
1	2	2	2	1

1, 4, 8, 14, 16, 14, 8, 4, 1

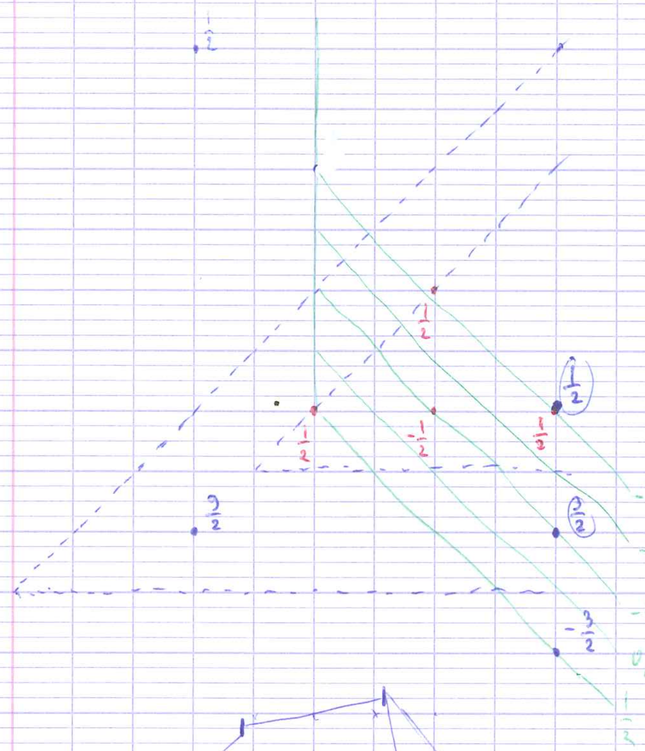
8 17 20 17 8

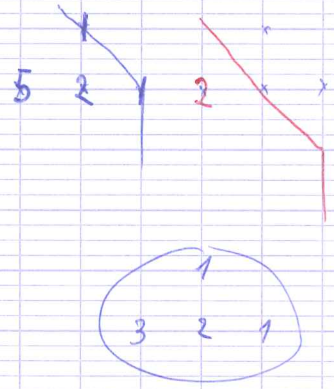
Total: $1+1\times12+1\times4+1\times12+1\times6 = 35$, formule de Weyl: $\begin{pmatrix}4&3&2\\2&1&0\end{pmatrix} \to \frac{7}{3}\,\frac{6}{2}\,\frac{5}{1}\,\frac{1}{1}\,\frac{2}{2}\,\frac{1}{1} \to 35$

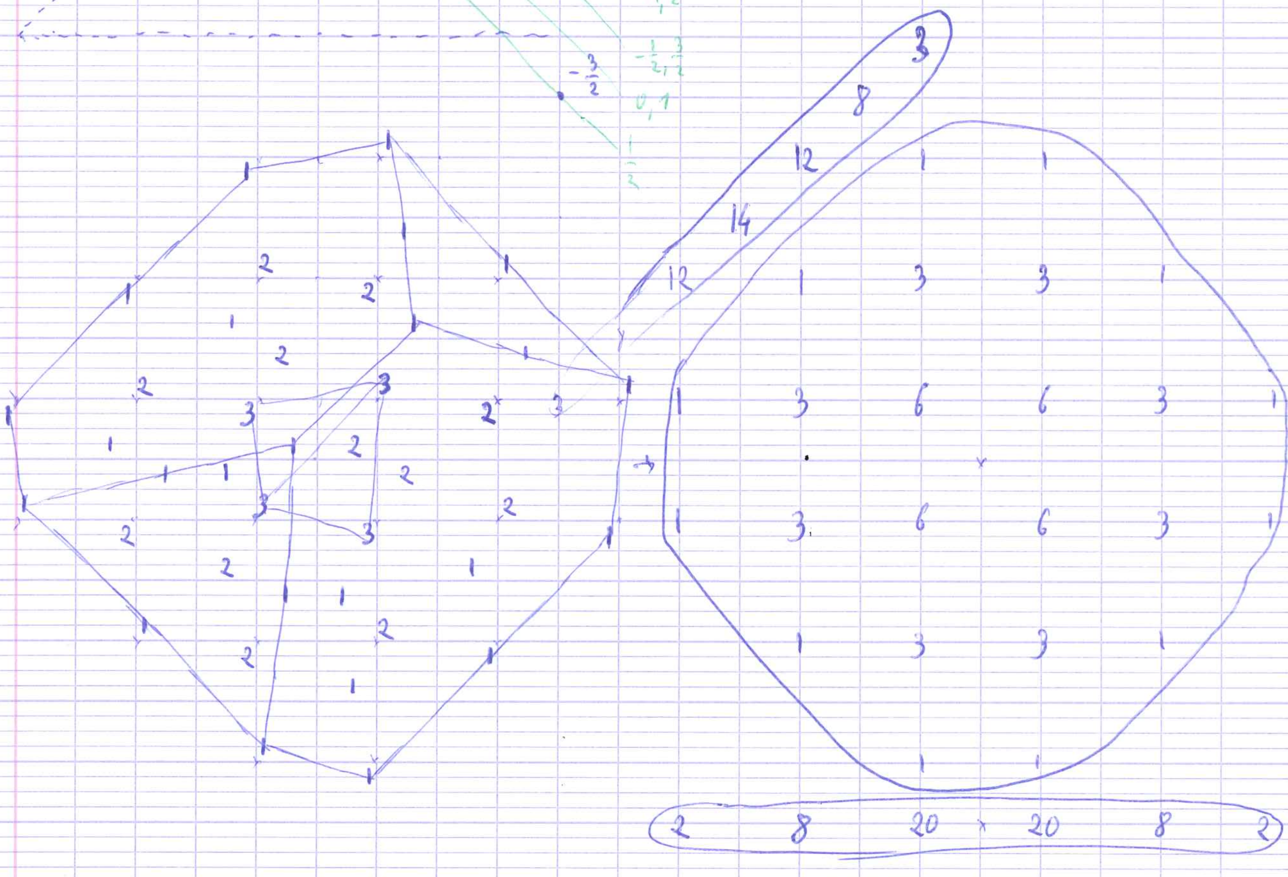




Total: $3\times 4+2\times 12+1\times 12+1\times 12=60$; formule de Weyl: $\dfrac{\frac{9}{2}\ \frac{3}{2}\ \frac{1}{2}}{2\ 1\ 0} \to \frac{6}{\cancel{3}}\ \frac{5}{\cancel{2}}\ \frac{2}{\cancel{1}}\ \frac{3}{1}\ \frac{4}{\cancel{2}}\ \frac{1}{\cancel{1}} \to 60$

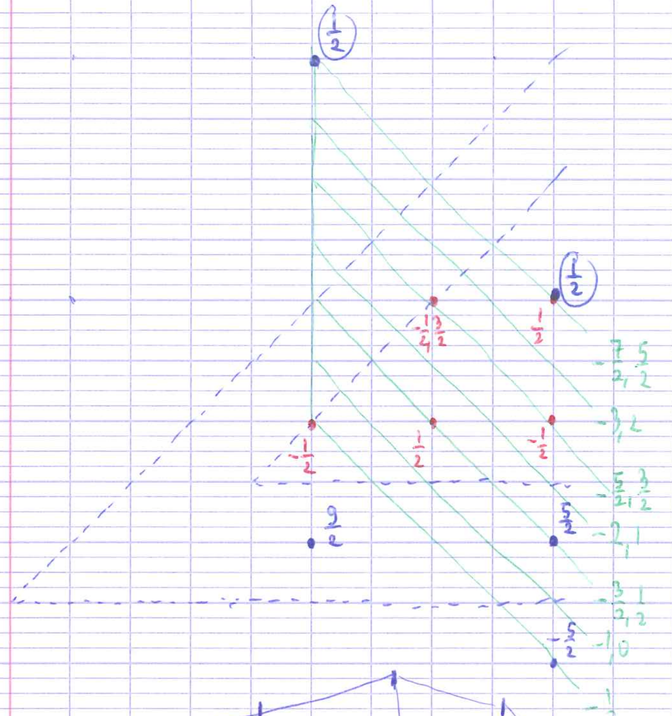


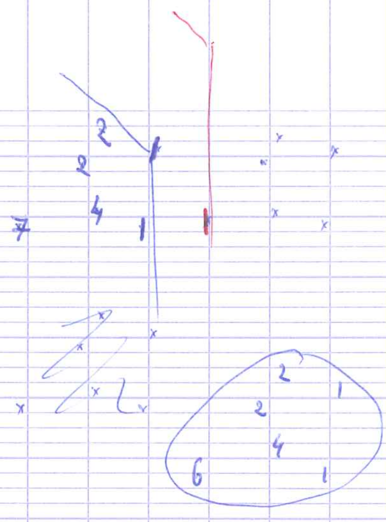


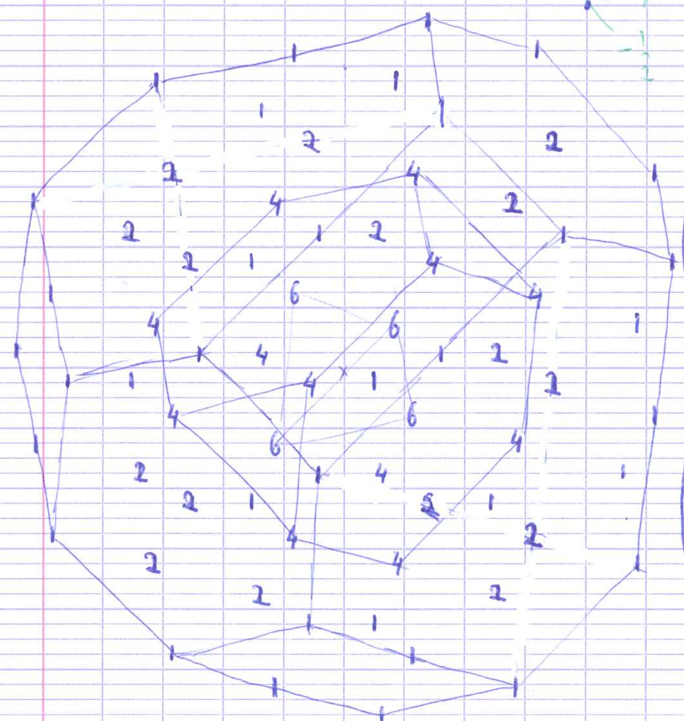


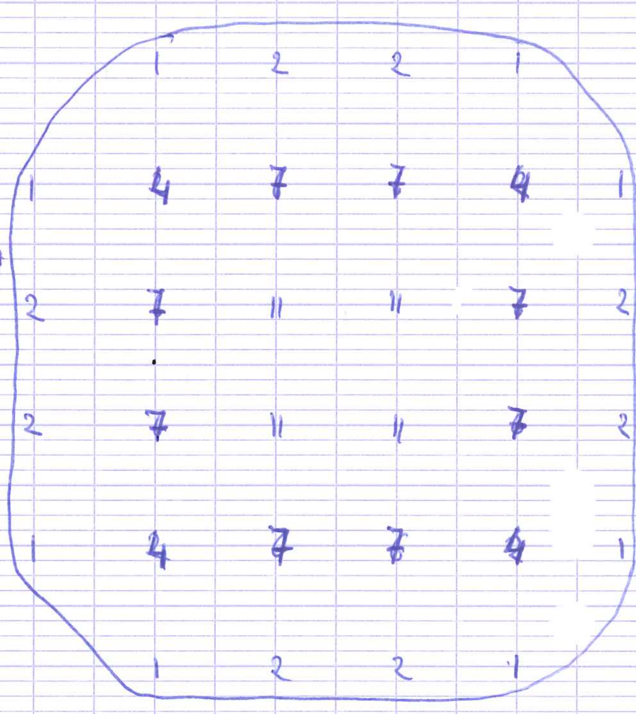
Total: $6\times4+4\times12+1\times12+2\times4+2\times12+1\times24 = 140$

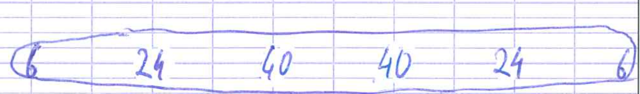
Weyl: $\begin{pmatrix} \frac{9}{2} & \frac{5}{2} & \frac{1}{2} \\ 2 & 1 & 0 \end{pmatrix} \rightarrow \frac{7}{\cancel{3}}\,\frac{5}{\cancel{2}}\,\frac{\cancel{3}}{1}\,\frac{\cancel{2}}{1}\,\frac{4}{\cancel{2}}\,\frac{\cancel{2}}{1} \rightarrow 140$

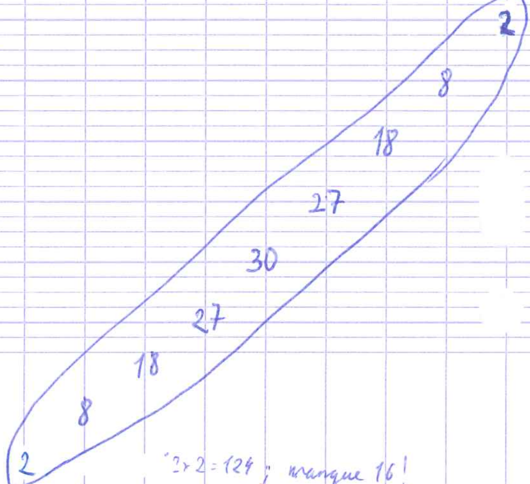












$?\times2 = 124$; manque 16 !

Total: $3\times 4 + 2\times 12 + 1\times 12 + 2\times 12 + 1\times 12 = 84$; Weyl: $\dfrac{\frac{9}{2}\ \frac{5}{2}\ \frac{3}{2}}{2\ 1\ 0} \to \frac{7}{2}\,\frac{6}{2}\,\frac{4}{1}\,\frac{2}{1}\,\frac{3}{2}\,\frac{1}{1} \to 84$

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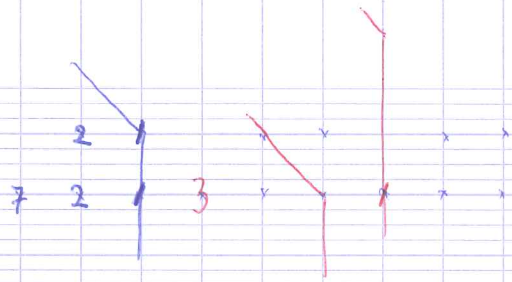
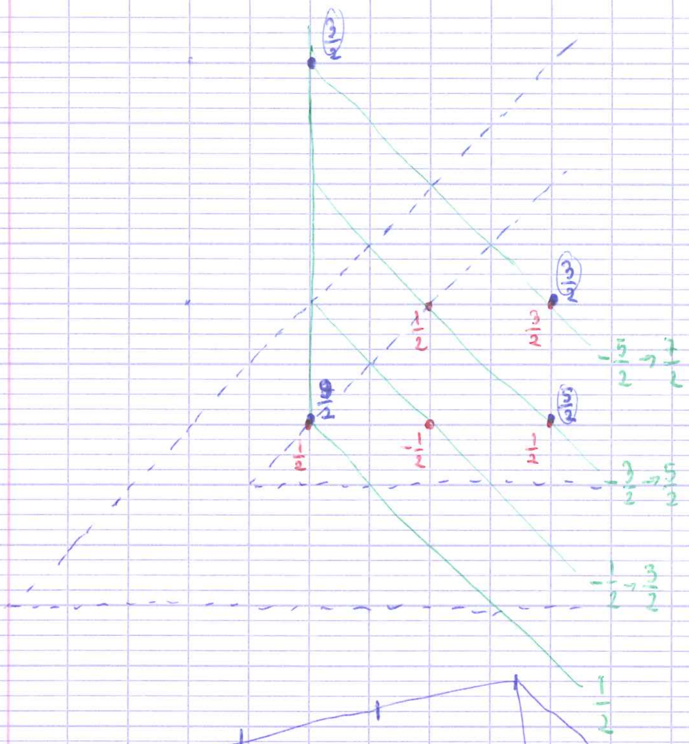
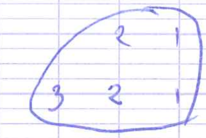
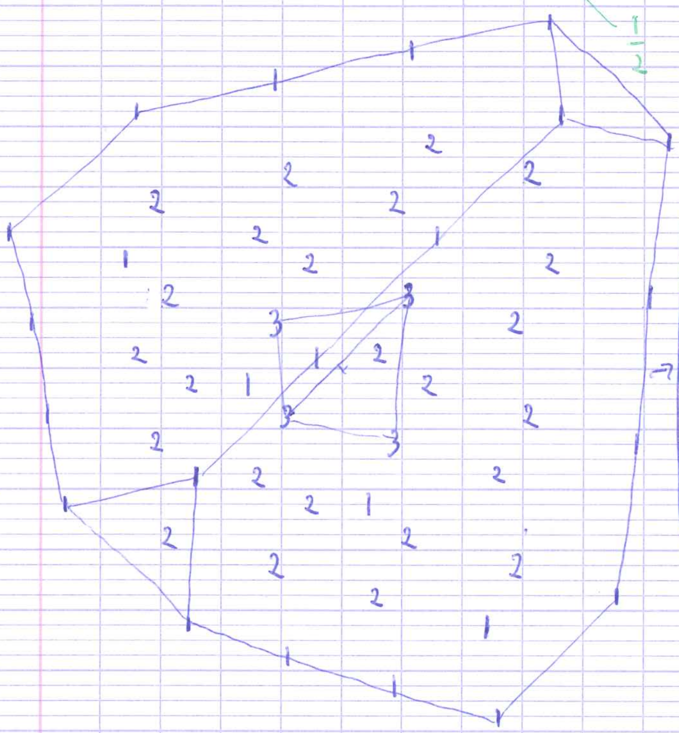








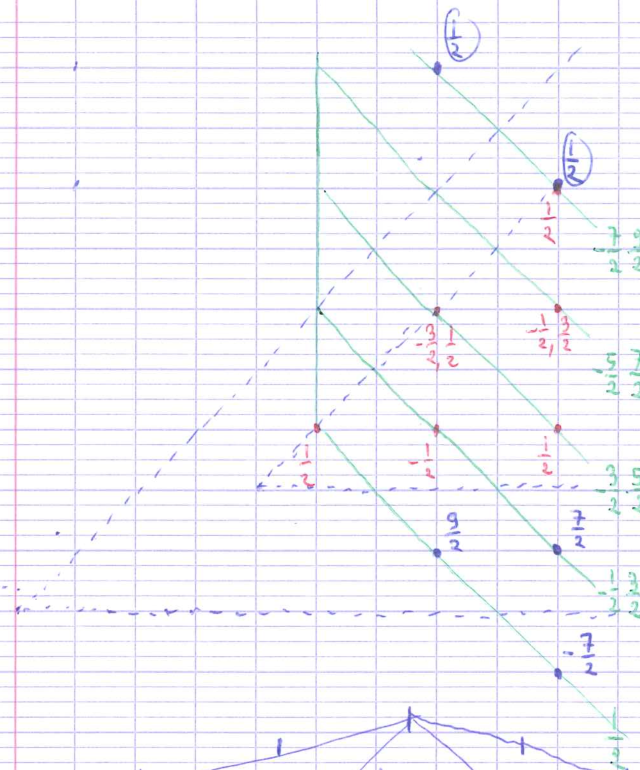
	1	1	1	1	
1	3	4	4	3	1
1	4	6	6	4	1
1	4	6	6	4	1
1	3	4	4	3	1
	1	1	1	1	

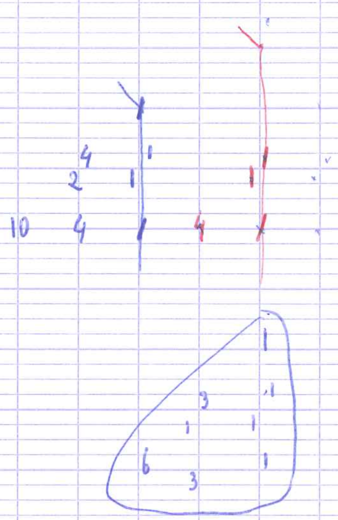
4	16	22	22	16	4

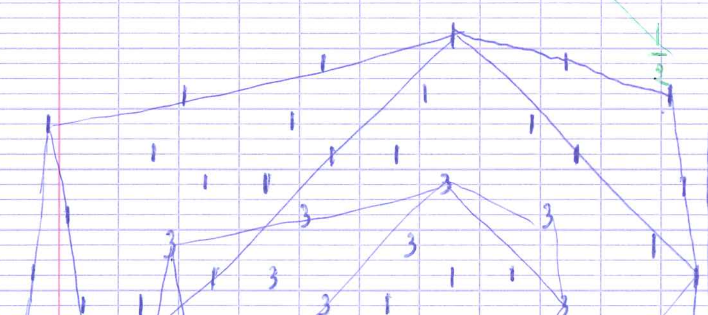
2 5 10 16 18 16 10 5 2

Total: $6\times4+3\times12+1\times12+1\times4+3\times12+1\times24+1\times12+1\times12=160$;

Weyl: $\left(\frac{3}{2}\ \frac{7}{2}\ \frac{1}{2}\right)\ (2\ 1\ 0) \rightarrow \frac{8}{3}\,\frac{5}{2}\,\frac{4}{1}\,\frac{1}{1}\,\frac{4}{2}\,\frac{3}{1} \rightarrow 160$

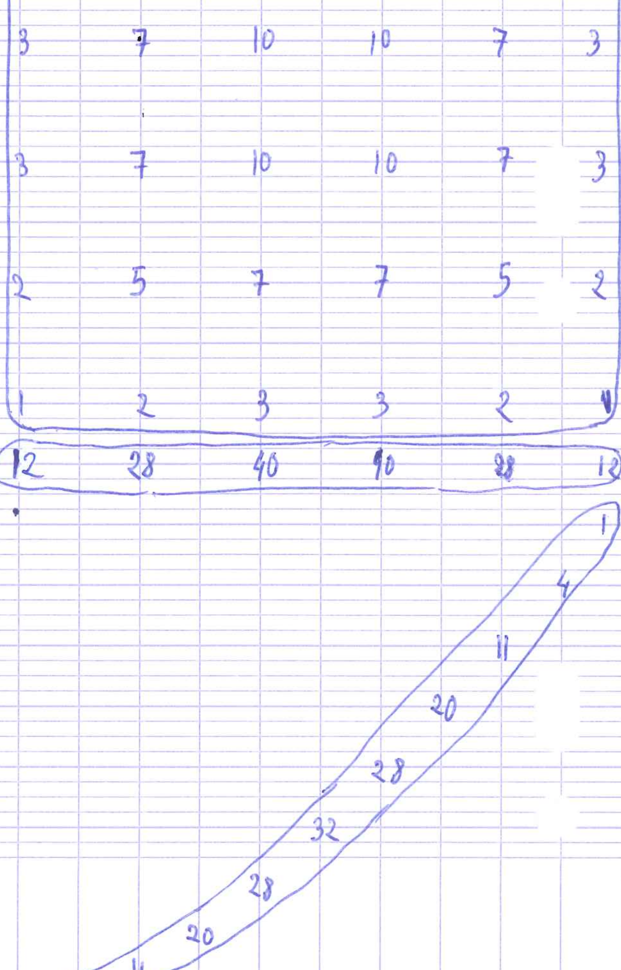






1	2	3	3	2	1
2	5	7	7	5	2
3	7	10	10	7	3
3	7	10	10	7	3
2	5	7	7	5	2
1	2	3	3	2	1

12 28 40 40 28 12



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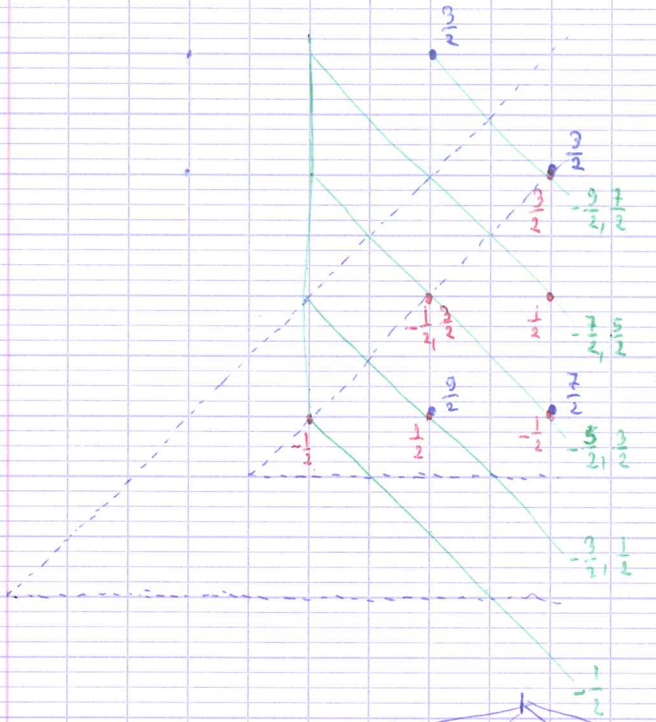
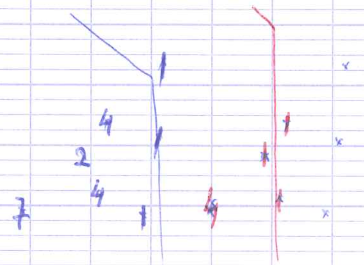
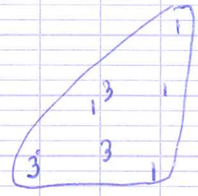
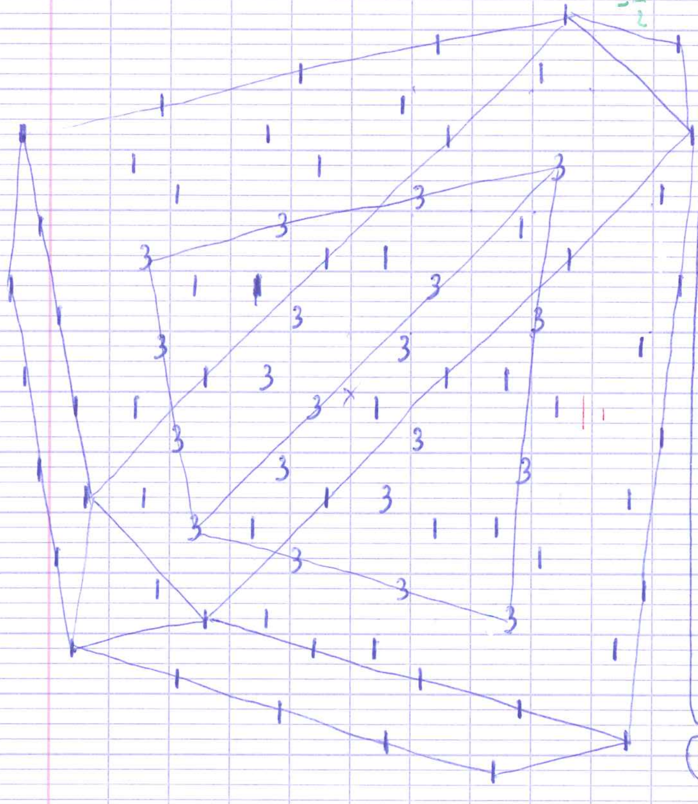
Total: $3\times4+3\times12+1\times12+1\times12+3\times4+1\times24+1\times12=120$;

Weyl: $\begin{pmatrix} \frac{3}{2} & \frac{7}{2} & \frac{3}{2} \\ 2 & 1 & 0 \end{pmatrix} \to \frac{8}{3}\,\frac{6}{2}\,\frac{5}{1}\,\frac{1}{1}\,\frac{3}{2}\,\frac{2}{1} \to 120$









1	2	2	2	2	1
2	4	5	5	4	2
2	5	7	7	5	2
2	5	7	7	5	2
2	4	5	5	4	2
1	2	2	2	2	1
10	22	28	28	22	10

1 4 8 14 21 24 21 14 8 4 1

Total: $4+12+4+12+12+12=56$; Weyl: $\dfrac{9}{2}\ \dfrac{7}{2}\ \dfrac{5}{2}$ / $2\ 1\ 0$ $\to \dfrac{8}{3}\,\dfrac{7}{2}\,\dfrac{6}{1}\,\dfrac{1}{1}\,\dfrac{2}{2}\,\dfrac{1}{1} \to 56$