On the existence of EFX allocations for more than three agents

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Partage équitable	EF MNW MMS Partage équitable de corvées
Physarum polycephalum	Plus courts chemins Réseaux efficaces

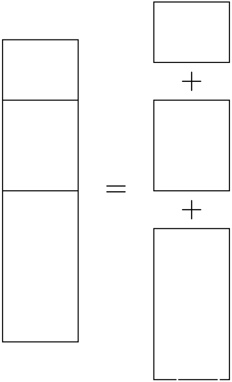
1. Partage équitable de biens
   - Le modèle
   - La famille EF
2. Existence d'allocations EFX
   - Peu de biens
   - Peu d'agents
   - Valuations identiques
   - Un espace trop grand

Le modèle

A	ensemble d'agents
G	ensemble de biens
$v : A \to \mathcal{P}(G) \to \mathbb{R}^+$	valuation, fonction d'utilité
(A, G, v)	instance
$a : G \to A$	allocation
$B_a(i)$ $(i \in A)$	Paquet alloué à i dans a

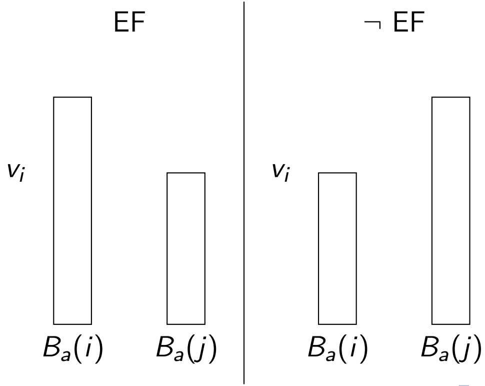
Valuations additive

$$\forall i \in A, \forall S \subset G, v_i(S) = \sum_{g \in S} v_i(\{g\})$$



EF

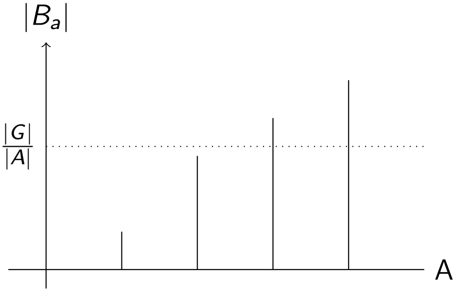
$$\forall i, j \in A, v_i(B_a(j)) \leq v_i(B_a(i))$$



EF n'existe pas toujours

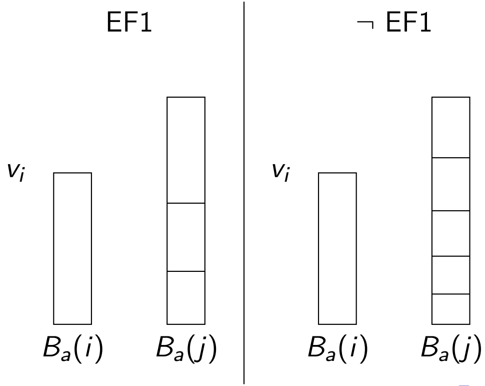
$$|A| \not| \; |G|$$

$$v : i \mapsto S \mapsto |S|$$



EF1

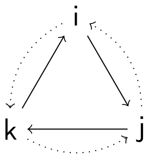
$$\forall i, j \in A, B_a(j) = \varnothing \vee \exists g \in B_a(j), v_i(B_a(j) \setminus \{g\}) \leq v_i(B_a(i))$$



Graphe des envies

$$E_a = (A, \{(i,j) \in A, v_i(B_a(i)) < v_i(B_a(j))\})$$

Lemme (Élimination des cycles)

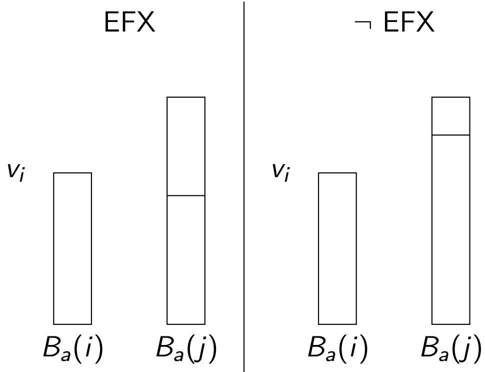


EF1 existe toujours

- Allouer aléatoirement un bien à un agent
- Éliminer les cycles de E_a

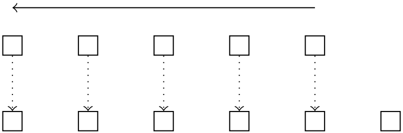
EFX

$$\forall i, j \in A, \forall g \in B_a(j), v_i(B_a(j) \setminus \{g\}) \leq v_i(B_a(i))$$

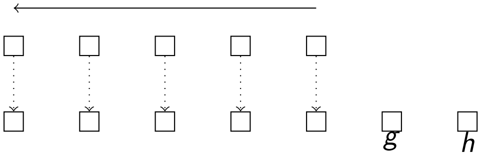


$|G| \leq |A| + 1$

- $|A|$ tours de round-robin
- Le dernier agent reçoit le dernier bien (s'il existe).



$|G| = |A| + 2$, v additive



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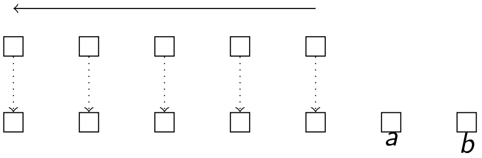

$$v_i(a) = \max(v_i(g), v_i(h))$$

$$v_i(b) = \min(v_i(g), v_i(h))$$

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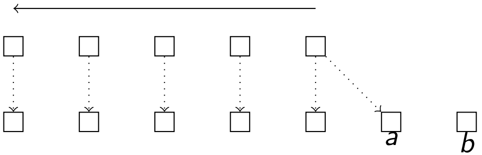

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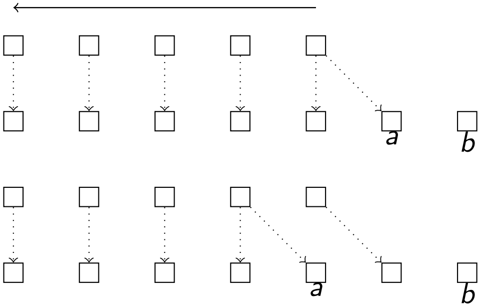

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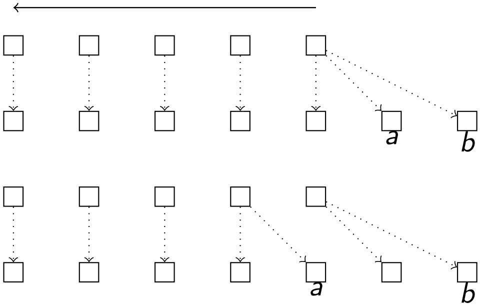

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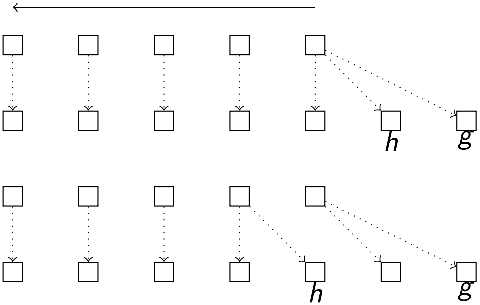


Accord sur le plus petit bien, v additive

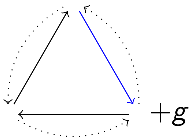
$$g \in G$$

$$\forall i \in A, \forall h \in G, v_i(g) \leq v_i(h)$$

- Trouver un allocation a EFX sans g.
- Éliminer les cycles dans E_a.
- Allouer g à une source de E_a.

Graphe des champions et $|A| = 3$

$$M_{a,g} = (A, \{(i,j) \in A^2, i \in \operatorname*{argmin}_{k \in A} \inf_{\substack{S \subset B_a(j) \cup \{g\} \\ v(a)_k < v_k(S)}} |S|\})$$



Leximin, $\forall i, j \in A, v_i = v_j, \forall i \in A, v_i$ injective

$$\Phi(a) = \text{sort}((v_i(B_a(i))))_{i \in A}$$

Allocation leximin : allocation maximale pour $\Phi(a)$ et $\leq_{\text{lex}}$

$$\text{leximin} \Rightarrow EFX$$

Taille de l'espace des instances

$$a_{|G|} = |\{v : \mathcal{P}(G) \to \mathbb{R}^+\}/\mathcal{R}|$$

$$\frac{\ln(a_n)}{2^n} \underset{n \to +\infty}{\sim} n \ln 2$$

$$a_7 = 14137791169722788711719597031620079563020547695771648 0$$

Merci de votre attention.