Trajectory correction algorithms for a 3D underwater vehicle using affine transformations

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- A transformation $\mathcal{F}$ deforms a trajectory $\mathcal{C} = (x(t), y(t))_{t \in [0,T]}$ into $\mathcal{C}'$ at a time instant τ by

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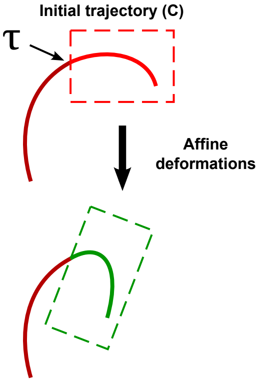

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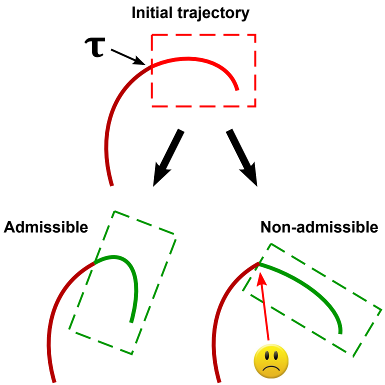

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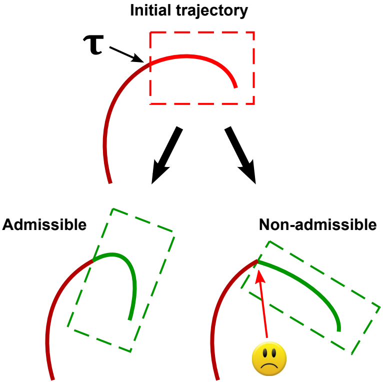

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Surprisingly, the set of admissible affine transformations can be shown to be a Lie subgroup of the General Affine group (GA_2 or GA_3) of dimension...

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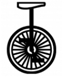
-   2 for the unicycle and omni-directional mobile robots (out of the 6 dimensions of GA_2)

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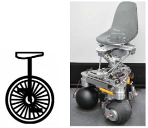
-  2 for the unicycle and omni-directional mobile robots (out of the 6 dimensions of GA_2)
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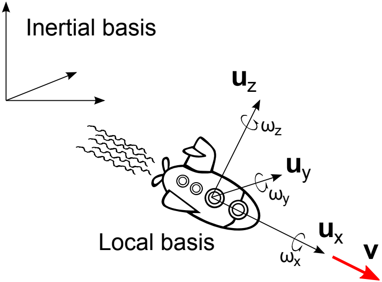
Trajectory correction for a 3D underwater vehicle

Model description:



Position of the robot: (x, y, z)
Orientation of the robot: (ϕ, θ, ψ)

Kinematic equations:

$$\left\{\begin{array}{ccc} \dot{v} & = & a \\ \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} & = & \mathbf{R}(\phi, \theta) \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \\ \dot{x} & = & v \cos\psi \cos\theta \\ \dot{y} & = & v \sin\psi \cos\theta \\ \dot{z} & = & -v \sin\theta \end{array}\right.$$

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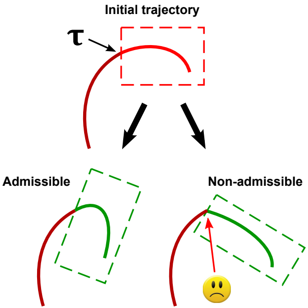

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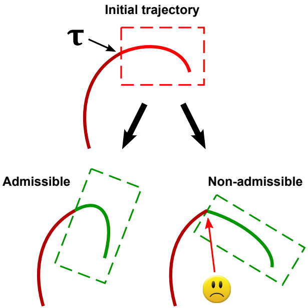

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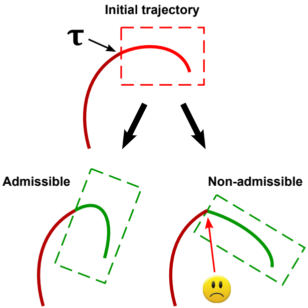

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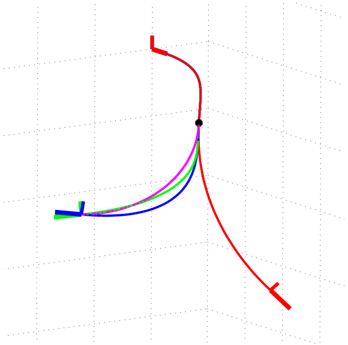
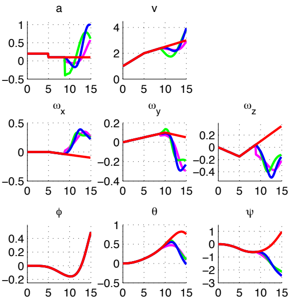
- Note: after obtaining a deformed trajectory $C'(t)$ by the above formula, one can recover the commands $(a, \omega_x, \omega_y, \omega_z)$ by some differentiations and elementary operations

Trajectory correction for a 3D underwater vehicle (IV)

Three examples of final position corrections





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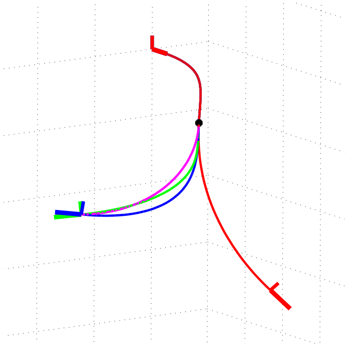

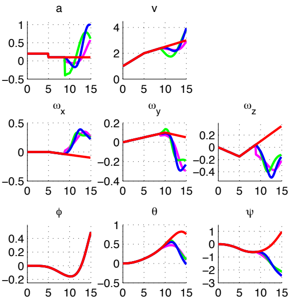

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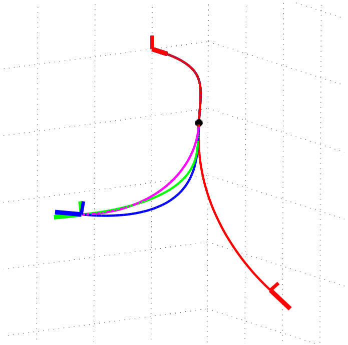

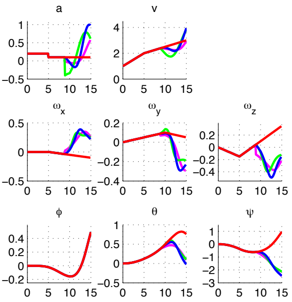

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- The green and blue corrections correct towards a same final position, but with different final orientations ("redundancy")

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  - How about using more general groups (e.g. projective)?