Maple companion for Section 5: *Explicit computations in the quartic case*.

$restart;$

Parametrization of the generating series $M_o(u)$ and $M_{o,4}(u)$ in terms of P : [Subsection 5.1].

We recall the equation which characterizes P given in **Proposition 5.1**, and the parametrizations of the generating series of the planted blossoming trees in terms of the power series P.

$$CancP := -P + u + 3\cdot x_4\cdot y_4\cdot P^3 + P\cdot\frac{(x_2 + 3\cdot x_4\cdot y_2\cdot P)\cdot(y_2 + 3\cdot y_4\cdot x_2\cdot P)}{(1 - 9\cdot x_4\cdot y_4\cdot P^2)^2} :$$

$$ParamB_{-3} := y_4\cdot u^3 :$$

$$ParamB_{-1} := u\cdot\frac{y_2 + 3\cdot x_2\cdot y_4\cdot P}{1 - 9\cdot x_4\cdot y_4\cdot P^2} :$$

$$ParamB_1 := \frac{P}{u} - 1 :$$

$$ParamW_1 := \frac{P\cdot(x_2 + 3\cdot y_2\cdot x_4\cdot P)}{u\cdot(1 - 9\cdot x_4\cdot y_4\cdot P^2)} :$$

$$ParamW_3 := \frac{x_4\cdot P^3}{u^3} :$$

We compute an parametrization of $\overline{M}_o(u)$ in terms of P using the previous equations:

$$ExprMbar_O := coeff\Big(x_2\cdot\big(\xi + (\xi^{-3}\cdot ParamB_{-3} + \xi^{-1}\cdot ParamB_{-1} + \xi^{1}\cdot ParamB_{1})\big)^2 + x_4\cdot\big(\xi + (\xi^{-3}\cdot ParamB_{-3} + \xi^{-1}\cdot ParamB_{-1} + \xi^{1}\cdot ParamB_{1})\big)^4, \xi, 0\Big)$$

$$ExprMbar_O := \frac{2\,x_2\,(3\,y_4\,x_2\,P + y_2)\,P}{-9\,x_4\,y_4\,P^2 + 1} + x_4\left(\frac{2\left(2\,y_4\,u^2\,P + \frac{u^2\,(3\,y_4\,x_2\,P + y_2)^2}{(-9\,x_4\,y_4\,P^2 + 1)^2}\right)P^2}{u^2} + \frac{4\,(3\,y_4\,x_2\,P + y_2)^2\,P^2}{(-9\,x_4\,y_4\,P^2 + 1)^2}\right) \tag{1.1}$$

It corresponds to the parametrization of $\overline{M}_o(u)$ given in **Equation (5.3)**:

$$VerificationFormula := verify\left(ExprMbar_{\mathrm{O}}, \left(2\,x_2\cdot P\frac{(y_2+3\,y_4\,x_2\,P)}{1-9\,x_4\,y_4\,P^2}+2\cdot x_4\cdot P^2\left(2\,y_4\,P+3\cdot\frac{(3\,y_4\,x_2\,P+y_2)^2}{(-9\,x_4\,y_4\,P^2+1)^2}\right)\right), equal\right)$$

$$VerificationFormula := true \tag{1.2}$$

Then, we compute a parametrization of $\mathrm{M}_o(u)$ by performing a formal integration (*with respect to u*).
To this end, we first compute a parametrization of $\partial_u P$ in terms of P, then we calculate the rational function $Rat(p)$ introduced in **Equation (5.5):**

$$ParamDerivPu := factor(subs(P(u)=P, solve(diff(subs(P=P(u), CancP), \mathrm{u}), diff(P(u), u))))$$

$$ParamDerivPu := -\left(9\,x_4\,y_4\,P^2-1\right)^3 \Big/ \left(6561\,P^8\,x_4^{\,4}\,y_4^{\,4}-2916\,P^6\,x_4^{\,3}\,y_4^{\,3}-81\,P^4\,x_2\,x_4^{\,2}\,y_2\,y_4^{\,2}+486\,P^4\,x_4^{\,2}\,y_4^{\,2}-54\,P^3\,x_2^{\,2}\,x_4\,y_4^{\,2}-54\,P^3\,x_4^{\,2}\,y_2^{\,2}\,y_4-54\,P^2\,x_2\,x_4\,y_2\,y_4-36\,x_4\,y_4\,P^2-6\,P\,x_2^{\,2}\,y_4-6\,P\,x_4\,y_2^{\,2}-x_2\,y_2+1\right) \tag{1.3}$$

$$Rat := factor\left(\frac{ExprMbar_{\mathrm{O}}}{ParamDerivPu}\right)$$

$$Rat := -\frac{1}{\left(9\,x_4\,y_4\,P^2-1\right)^5}\left(2\,P\left(162\,P^6\,x_4^{\,3}\,y_4^{\,3}-36\,P^4\,x_4^{\,2}\,y_4^{\,2}+9\,P^2\,x_2\,x_4\,y_2\,y_4+2\,x_4\,y_4\,P^2+3\,P\,x_2^{\,2}\,y_4+3\,P\,x_4\,y_2^{\,2}+x_2\,y_2\right)\left(6561\,P^8\,x_4^{\,4}\,y_4^{\,4}-2916\,P^6\,x_4^{\,3}\,y_4^{\,3}-81\,P^4\,x_2\,x_4^{\,2}\,y_2\,y_4^{\,2}+486\,P^4\,x_4^{\,2}\,y_4^{\,2}-54\,P^3\,x_2^{\,2}\,x_4\,y_4^{\,2}-54\,P^3\,x_4^{\,2}\,y_2^{\,2}\,y_4-54\,P^2\,x_2\,x_4\,y_2\,y_4-36\,x_4\,y_4\,P^2-6\,P\,x_2^{\,2}\,y_4-6\,P\,x_4\,y_2^{\,2}-x_2\,y_2+1\right)\right) \tag{1.4}$$

Note that in this form $Rat(P)$, depends on the variable u only through $P=P(u)$.
By performing the formal integration we obtain a parametrization of $M_{\mathrm{o}}(u)$ in terms of P:

$$ParamM := factor(integrate(Rat, P)-subs(P=0, factor(integrate(Rat, P))))$$

$$ParamM := -\frac{1}{\left(9\,x_4\,y_4\,P^2-1\right)^4}\left(P^2\left(39366\,P^{12}\,x_4^{\,6}\,y_4^{\,6}-24057\,P^{10}\,x_4^{\,5}\,y_4^{\,5}+5103\,P^8\,x_2\,x_4^{\,4}\,y_2\,y_4^{\,4}+5832\,P^8\,x_4^{\,4}\,y_4^{\,4}+2430\,P^7\,x_2^{\,2}\,x_4^{\,3}\,y_4^{\,4}+2430\,P^7\,x_4^{\,4}\,y_2^{\,2}\,y_4^{\,3}-324\,P^6\,x_2\,x_4^{\,3}\,y_2\,y_4^{\,3}-702\,P^6\,x_4^{\,3}\,y_4^{\,3}-702\,P^5\,x_2^{\,2}\,x_4^{\,2}\,y_4^{\,3}-702\,P^5\,x_4^{\,3}\,y_2^{\,2}\,y_4^{\,2}+81\,P^4\,x_2^{\,2}\,x_4^{\,2}\,y_2^{\,2}\,y_4^{\,2}-198\,P^4\,x_2\,x_4^{\,2}\,y_2\,y_4^{\,2}+54\,P^3\,x_2^{\,3}\,x_4\,y_2\,y_4^{\,2}+54\,P^3\,x_2\,x_4^{\,2}\,y_2^{\,3}\,y_4+42\,P^4\,x_4^{\,2}\,y_4^{\,2}+66\,P^3\,x_2^{\,2}\,x_4\,y_4^{\,2}+66\,P^3\,x_4^{\,2}\,y_2^{\,2}\,y_4+9\,P^2\,x_2^{\,4}\,y_4^{\,2}+36\,P^2\,x_2^{\,2}\,x_4\,y_2^{\,2}\,y_4+9\,P^2\,x_4^{\,2}\,y_2^{\,4}+28\,P^2\,x_2\,x_4\,y_2\,y_4+6\,P\,x_2^{\,3}\,y_2\,y_4+6\,P\,x_2\,x_4\,y_2^{\,3}-x_4\,y_4\,P^2-2\,P\,x_2^{\,2}\,y_4 \right.$$

(1.5)

$$- 2\, P\, x_4\, y_2^{\,2} + x_2^{\,2}\, y_2^{\,2} - x_2\, y_2\big)\big)$$

It can be simplified using that $CancP$ is a cancelling rational function of P. Thus, we obtain **Equation (5.2)**:

$ParamMo\_simplified := collect\big(expand\big(simplify\big(ParamM, [CancP], \{x_2\}\big)\big), P, factor\big)$

$$ParamMo\_simplified := 15\, P^6\, x_4^{\,2}\, y_4^{\,2} - 5\, P^4\, x_4\, y_4 + 4\, P^3\, u\, x_4\, y_4 + \left(\frac{x_2\, y_2}{3} - \frac{1}{3}\right) P^2 + \frac{4\, P\, u}{3} - u^2 \qquad \textbf{(1.6)}$$

We proceed similarly to compute the parametrization of $M_{o,4}(u)$ in terms of P given in **Propositon 5.2**:

$ExprMbar_{\mathrm{O},4} := simplify\Big(coeff\Big(x_4 \cdot \big(\xi + \big(\xi^{-3} \cdot ParamB_{-3} + \xi^{-1} \cdot ParamB_{-1} + \xi^{1} \cdot ParamB_{1}\big)\big)^4, \xi, 0\Big), \{u\}\Big)$

$$ExprMbar_{O,4} := \frac{324\, P^7\, x_4^{\,3}\, y_4^{\,3} - 72\, P^5\, x_4^{\,2}\, y_4^{\,2} + 54\, P^4\, x_2^{\,2}\, x_4\, y_4^{\,2} + 36\, y_4\, x_4 \left(x_2\, y_2 + \frac{1}{9}\right) P^3 + 6\, P^2\, x_4\, y_2^{\,2}}{\left(9\, x_4\, y_4\, P^2 - 1\right)^2} \qquad \textbf{(1.7)}$$

$Rat2 := factor\left(\dfrac{ExprMbar_{\mathrm{O},4}}{ParamDerivPu}\right):$

$ParamMo4 := factor(integrate(Rat2, P)) - subs(P = 0, factor(integrate(Rat2, P))):$

$ParamMo4\_simplified := factor\big(simplify\big(ParamMo4, [CancP], \{y_2\}\big)\big)$

$$\begin{aligned} ParamMo4\_simplified := {} & \frac{1}{9\left(9\, x_4\, y_4\, P^2 - 1\right) x_4}\big(1215\, P^8\, x_4^{\,4}\, y_4^{\,3} - 540\, P^6\, x_4^{\,3}\, y_4^{\,2} \\ & + 324\, P^5\, u\, x_4^{\,3}\, y_4^{\,2} - 27\, P^5\, x_2^{\,2}\, x_4^{\,2}\, y_4^{\,2} - 18\, y_2\, P^4\, x_2\, x_4^{\,2}\, y_4 + 18\, y_4\, x_4^{\,2}\, P^4 + 72\, P^3\, u\, x_4^{\,2}\, y_4 \\ & - 30\, P^3\, x_2^{\,2}\, x_4\, y_4 - 81\, P^2\, u^2\, x_4^{\,2}\, y_4 + 45\, P^2\, u\, x_2^{\,2}\, x_4\, y_4 - 3\, P^2\, x_2^{\,4}\, y_4 - 6\, P^2\, x_2\, x_4\, y_2 \\ & + 12\, P\, u\, x_2\, x_4\, y_2 - P\, x_2^{\,3}\, y_2 + 3\, x_4\, P^2 - 12\, P\, u\, x_4 + P\, x_2^{\,2} + 9\, u^2\, x_4 - u\, x_2^{\,2}\big) \end{aligned} \qquad \textbf{(1.8)}$$

We obtain the polynomial Pol from **Equation (5.7)**:

$collect(numer(ParamMo4\_simplified), P, factor)$

$$\begin{aligned} & 1215\, P^8\, x_4^{\,4}\, y_4^{\,3} - 540\, P^6\, x_4^{\,3}\, y_4^{\,2} + 27\, y_4^{\,2}\, x_4^{\,2}\left(12\, u\, x_4 - x_2^{\,2}\right) P^5 - 18\, x_4^{\,2}\, y_4\left(x_2\, y_2 - 1\right) P^4 \\ & + 6\, y_4\, x_4\left(12\, u\, x_4 - 5\, x_2^{\,2}\right) P^3 + \big(-81\, u^2\, x_4^{\,2}\, y_4 + 45\, x_4\, u\, x_2^{\,2}\, y_4 - 3\, x_2^{\,4}\, y_4 - 6\, x_4\, y_2\, x_2 \\ & + 3\, x_4\big) P^2 + \left(x_2\, y_2 - 1\right)\left(12\, u\, x_4 - x_2^{\,2}\right) P + u\left(9\, u\, x_4 - x_2^{\,2}\right) \end{aligned} \qquad \textbf{(1.9)}$$

To conclude, we verify that this parametrization of $M_{\mathrm{O},4}$ is equivalent to the parametrization given by Bousquet-Mélou and Schaeffer in **[16]**. Note that in our settings, their parametrization corresponds to the specialization: $x_2 \to \nu$, $y_2 \to \nu$, $u \to 1$.

$$ParamBMS := \frac{1}{9}\cdot\left(135 P^6 x_4^2 y_4^2 + 72 x_4 y_4^2 P^5 + 9 P^4 \nu^2 y_4^2 - 108 P^4 x_4 y_4^2 - 45 P^4 x_4 y_4 - 24 P^4 y_4^2 - 3 P^3 \nu^2 y_4 + 36 x_4 y_4 P^3 + 36 P^3 y_4^2 - 32 y_4 P^3 - 2 P^2 \nu^2 + 72 P^2 y_4 - 3 P^2 - 36 y_4 P + 12 P - 9\right) - \frac{\left(P^2 y_4 + P - 1\right)\left(3 y_4 P + 1\right)\left(3 P \nu^2 - 8 P + 12\right)}{9\left(3 P x_4 + 1\right)}:$$

$$VerificationFormula := verify\left(0, \ simplify\left(ParamBMS - subs\left(x_2 = \nu, y_2 = \nu, u = 1, ParamMo4\_simplified\right), \left[subs\left(x_2 = \nu, y_2 = \nu, \ u = 1, CancP\right)\right], \{P\}\right) \ , equal\right)$$

$$VerificationFormula := true \tag{1.10}$$

Parametrization of the generating series $I_o(u)$ in terms of Q : [Subsection 5.2].

We recall the cancelling rational function we compute for P from Equation (5.1):

$CancP$

$$-P + u + 3 x_4 y_4 P^3 + \frac{P\left(3 P x_4 y_2 + x_2\right)\left(3 P x_2 y_4 + y_2\right)}{\left(-9 x_4 y_4 P^2 + 1\right)^2} \tag{2.1}$$

We apply the following change of variables to obtain a cancelling rational function of $P\left(\nu, \nu, x_4 t^2, y_4 t^2, u\right)$: $x_4 \to t^2 \cdot x_4$, $y_4 \to t^2 \cdot y_4$, $x_2 \to \nu$, $y_2 \to \nu$

$$CancP\_tps := subs\left(x_4 = x \cdot t^2, \ y_4 = y \cdot t^2, \ x_2 = \nu, \ y_2 = \nu, \ CancP\right)$$

$$CancP\_tps := -P + u + 3 x t^4 y P^3 + \frac{P\left(3 P \nu t^2 x + \nu\right)\left(3 P \nu t^2 y + \nu\right)}{\left(-9 P^2 t^4 x y + 1\right)^2} \tag{2.2}$$

Then, we derive a cancelling rational function of the formal power series Q:

$$CancQ := subs\left(tP = \frac{Q}{t}, subs\left(t = t \cdot \left(1 - \nu^2\right), subs\left(P = \frac{tP}{t}, CancP\_tps\right)\right)\right)$$

$$CancQ := -\frac{Q}{t^2\left(-\nu^2 + 1\right)} + u + \frac{3 x\left(-\nu^2 + 1\right) y Q^3}{t^2} + \frac{Q\left(3 \nu\left(-\nu^2 + 1\right) Q x + \nu\right)\left(3 \nu\left(-\nu^2 + 1\right) Q y + \nu\right)}{t^2\left(-\nu^2 + 1\right)\left(-9\left(-\nu^2 + 1\right)^2 Q^2 x y + 1\right)^2} \tag{2.3}$$

Finally, we obtain Equation (5.9):

$$EquationQ := t^2 = factor\left(solve\left(\mathrm{subs}\left(\mathrm{t} = \mathrm{sqrt}(\mathrm{T}), CancQ\right), T\right)\right)$$

$$EquationQ := t^2 = \frac{1}{\left(9 Q^2 \nu^4 x y - 18 Q^2 \nu^2 x y + 9 Q^2 x y - 1\right)^2 u}\left(\left(243 Q^6 \nu^{10} x^3 y^3\right.\right. \tag{2.4}$$

$$- 1215\, Q^6 \nu^8 x^3 y^3 + 2430\, Q^6 \nu^6 x^3 y^3 - 2430\, Q^6 \nu^4 x^3 y^3 + 1215\, Q^6 \nu^2 x^3 y^3$$
$$- 135\, Q^4 \nu^6 x^2 y^2 - 243\, Q^6 x^3 y^3 + 405\, Q^4 \nu^4 x^2 y^2 - 405\, Q^4 \nu^2 x^2 y^2 + 135\, Q^4 x^2 y^2$$
$$+ 9\, Q^2 \nu^4 x y + 12\, Q^2 \nu^2 x y - 21\, Q^2 x y - 3\, x \nu^2 Q - 3\, Q \nu^2 y + 1)\, Q)$$

The numerator of the second term in this equation factorizes as follows:

$$Q \cdot \left(collect\left(numer\left(\frac{op(2, EquationQ)}{Q} \right), Q, factor \right) \right)$$

$$Q \left(1 + 243\, x^3 y^3 (\nu - 1)^5 (\nu + 1)^5 Q^6 - 135\, x^2 y^2 (\nu - 1)^3 (\nu + 1)^3 Q^4 + 3\, x y\, (\nu - 1)\, (\nu + 1) \left(3\, \nu^2 + 7\right) Q^2 - 3\, \nu^2 (x + y)\, Q\right) \quad \textbf{(2.5)}$$

To conclude we apply the same change of variable -- denoted by Θ -- to the parametrization of $\boldsymbol{M_{o,4}(u)}$ in terms of $\boldsymbol{P}$ to obtain a parametrization of $\boldsymbol{I_o(u)}$ **in** terms of $\boldsymbol{Q}$.

We recall the parametrization of $\boldsymbol{M_{o,4}(u)}$ from **Equation (5.7)**, and perform the change of variables: $x_4 \to t^2 \cdot x_4$, $y_4 \to t^2 \cdot y_4$, $x_2 \to \nu$, $y_2 \to \nu$

$$Param1M_{O,4} := subs\Big(x_4 = t^2 \cdot x, y_4 = t^2 \cdot y, x_2 = \nu, y_2 = \nu, \frac{1}{9\left(9\, x_4 y_4 P^2 - 1\right) x_4} \cdot \big(1215\, P^8 x_4^4 y_4^3$$
$$- 540\, P^6 x_4^3 y_4^2 + 324\, P^5 u\, x_4^3 y_4^2 - 27\, P^5 x_2^2 x_4^2 y_4^2 - 18\, P^4 x_2 x_4^2 y_2 y_4 + 18\, x_4^2 y_4 P^4$$
$$+ 72\, P^3 u\, x_4^2 y_4 - 30\, P^3 x_2^2 x_4 y_4 - 81\, P^2 u^2 x_4^2 y_4 + 45\, P^2 u\, x_2^2 x_4 y_4 - 3\, P^2 x_2^4 y_4$$
$$- 6\, P^2 x_2 x_4 y_2 + 12\, P u\, x_2 x_4 y_2 - P x_2^3 y_2 + 3\, x_4 P^2 - 12\, P u\, x_4 + P x_2^2 + 9\, u^2 x_4 - u\, x_2^2\big)\Big):$$

The change of variables Θ gives the following:

$$ParamI_O := factor\left(subs\left(tP = \frac{Q}{t}, subs\left(t = t \cdot \left(1 - \nu^2\right), subs\left(P = \frac{tP}{t}, subs\left(P = \frac{tP}{t}, Param1M_{O,4} \right)\right)\right)\right)\right)$$

$$ParamI_O := \big(1215\, Q^8 \nu^{12} x^4 y^3 - 7290\, Q^8 \nu^{10} x^4 y^3 + 18225\, Q^8 \nu^8 x^4 y^3 - 324\, Q^5 \nu^{10} t^2 u\, x^3 y^2 \quad \textbf{(2.6)}$$
$$- 24300\, Q^8 \nu^6 x^4 y^3 + 1620\, Q^5 \nu^8 t^2 u\, x^3 y^2 + 18225\, Q^8 \nu^4 x^4 y^3 - 540\, Q^6 \nu^8 x^3 y^2$$
$$- 3240\, Q^5 \nu^6 t^2 u\, x^3 y^2 - 81\, Q^2 \nu^8 t^4 u^2 x^2 y - 7290\, Q^8 \nu^2 x^4 y^3 + 2160\, Q^6 \nu^6 x^3 y^2$$
$$+ 27\, Q^5 \nu^8 x^2 y^2 + 3240\, Q^5 \nu^4 t^2 u\, x^3 y^2 + 324\, Q^2 \nu^6 t^4 u^2 x^2 y + 1215\, Q^8 x^4 y^3$$
$$- 3240\, Q^6 \nu^4 x^3 y^2 - 81\, Q^5 \nu^6 x^2 y^2 - 1620\, Q^5 \nu^2 t^2 u\, x^3 y^2 - 72\, Q^3 \nu^6 t^2 u\, x^2 y$$
$$- 486\, Q^2 \nu^4 t^4 u^2 x^2 y + 2160\, Q^6 \nu^2 x^3 y^2 + 81\, Q^5 \nu^4 x^2 y^2 + 324\, Q^5 u\, x^3 y^2 t^2$$
$$- 18\, Q^4 \nu^6 x^2 y + 216\, Q^3 \nu^4 t^2 u\, x^2 y + 45\, Q^2 \nu^6 t^2 u\, x y + 324\, Q^2 \nu^2 t^4 u^2 x^2 y$$
$$- 540\, Q^6 x^3 y^2 - 27\, Q^5 \nu^2 x^2 y^2 + 54\, Q^4 \nu^4 x^2 y - 216\, Q^3 \nu^2 t^2 u\, x^2 y - 90\, Q^2 \nu^4 t^2 u\, x y$$
$$- 81\, Q^2 t^4 u^2 x^2 y + 9\, \nu^4 t^4 u^2 x - 54\, Q^4 \nu^2 x^2 y + 30\, Q^3 \nu^4 x y + 72\, Q^3 u\, x^2 y\, t^2$$
$$+ 45\, Q^2 \nu^2 u\, x y\, t^2 - 12\, Q \nu^4 t^2 u\, x - 18\, \nu^2 t^4 u^2 x + 18\, Q^4 x^2 y - 30\, Q^3 \nu^2 x y - 3\, Q^2 \nu^4 y$$

$$+ 24 Q \nu^2 u x t^2 + 9 t^4 u^2 x - 6 Q^2 \nu^2 x - 12 Q u x t^2 - \nu^2 u t^2 + 3 Q^2 x + Q \nu^2 \Big) \Big/ \left(9 x \left(9 Q^2 \nu^4 x y - 18 Q^2 \nu^2 x y + 9 Q^2 x y - 1\right) (\nu - 1)^2 (\nu + 1)^2 t^4\right)$$

We verify that it is equivalent to the parametrization given in **Equation (5.11)**. To do this, we use the cancelling rational function of Q.

$$ParamPaper := \frac{1}{9\left(1-\nu^2\right) t^4 \left(1 + 3x\left(1-\nu^2\right) Q\right)} \cdot \Big(405 x^3 y^2 \cdot \left(1-\nu^2\right)^4 Q^7 + 351 x^2 y^2 \left(1 - \nu^2\right)^3 Q^6 + 27 x y \left(1-\nu^2\right)^2 \cdot \left(\nu^2 y - \left(5 + 12 t^2 u \cdot \left(1-\nu^2\right)^2 y\right) x\right) Q^5 + 3 x y \left(1 - \nu^2\right) \left(36 t^2 u \cdot \left(1-\nu^2\right)^2 x - 3\nu^2 - 47\right) Q^4 + \left(\left(252 t^2 u \cdot \left(1-\nu^2\right)^2 y - 6\nu^2 - 9\right) x - 15 \nu^2 y\right) Q^3 + \left(\left(36 t^2 u \cdot \left(1-\nu^2\right) - 108 t^4 u^2 y \cdot \left(1-\nu^2\right)^3\right) x + 5 + 9 \nu^2 t^2 u \cdot \left(1 - \nu^2\right) y\right) Q^2 - t^2 u \left(27 t^2 u \cdot \left(1-\nu^2\right)^2 x - 3\nu^2 + 8\right) Q + 3 t^4 u^2 \left(1-\nu^2\right)\Big):$$

$VerificationFormula := verify\left(0,\ simplify\left(ParamPaper - ParamI_{O}, [\mathrm{CancQ}], \{Q\}\right),\ equal\right)$

$$VerificationFormula := true \tag{2.7}$$

To conclude, we compute the first terms of the expansion in t of the series Q and I_{O}:

$with(gfun) : Qser := op(2, algeqtoseries(numer(CancQ), t, Q, 8, pos\_slopes))$

$$Qser := u t^2 + 3 u^2 \nu^2 (x + y) t^4 + 3\left(6\nu^4 x^2 + 3\nu^4 x y + 6\nu^4 y^2 + 8 x y \nu^2 + x y\right) u^3 t^6 + 9\left(15\nu^4 x^3 + 6\nu^4 x^2 y + 6\nu^4 x y^2 + 15 \nu^4 y^3 + 28\nu^2 x^2 y + 28\nu^2 x y^2 + 11 x^2 y + 11 x y^2\right) \nu^2 u^4 t^8 + O\left(t^{10}\right) \tag{2.8}$$

$factor(series(subs(Q = Qser, ParamPaper), t, 15))$

$$2 u^3 \nu^2 x t^2 + \left(9\nu^4 x + 8\nu^2 y + y\right) u^4 x t^4 + 18\left(3\nu^4 x^2 + 4 x y \nu^2 + 2\nu^2 y^2 + 2 x y + y^2\right) \nu^2 u^5 x t^6 + O\left(t^8\right) \tag{2.9}$$