# String topology seminar

## Plan of the talks

**Talk 1**: **String topology seminar introduction**. (Thibaut Mazuir)

General introduction to the topic of string topology and presentation of the plan of the seminar.

**Talk 2**: **The BV algebra structure on loop homology**. (Gerard Bargallo)

(i) Definition of a Gerstenhaber algebra. Definition of a Batalin-Vilkovisky algebra. A BV algebra induces a Gerstenhaber algebra. [Maz, Section 1.3.4]
(ii) The BV algebra structure on loop homology $\mathbb{H}_*(LM)$ for a manifold $M$. [CS99, Sections 1-5]
(iii) The intersection product and the Thom collapse map. Outline of the equivalent construction of the BV algebra structure on $\mathbb{H}_*(LM)$ in [CHV06, Section 1].
(iv) Homotopy-invariance of the loop product and comments. [CKS08]
(v) (If time permits) Outline of the definition of the Goresky–Hingston coproduct. The Goresky–Hingston coproduct is NOT homotopy-invariant. [NRW22, Section 1] and [Nae21]

**Talk 3**: **Hochschild homology and Jones isomorphism**. (Léo Mousseau)

(i) Definition of Hochschild homology. [Lod98, Chapter 1]
(ii) The Gerstenhaber algebra structure on Hochschild homology. [Ger63]
(iii) The Jones isomorphism $H^*(LM) \simeq HH_*(C^*(M))$. Outline of its proof. Statement of the Burghelea-Fiedorowicz-Goodwillie isomorphism $H_*(LM) \simeq HH_*(C_*(\Omega M))$ without proof. [LO15, Introduction, Chapter 4]
(iv) Comparison of the BV algebra structure on $\mathbb{H}_*(LM)$ and the Gerstenhaber algebra structure on $HH_*(C^*(M))$. [FT08]

**Talk 4**: **The cacti operad and its action on the free loop space**. (Matthias Görg)

(i) Definition of an operad. Definition of an algebra over an operad. [LV12, Maz, Tam]
(ii) Example of the little disks operad/the framed little disks operad. Outline of the proof that the framed little disks operad is the operad $BV$. [Maz, Section 3.4.2]
(iii) Definition of the cacti operad. Action of the cacti operad on the free loop space. This action lifts the BV algebra structure on loop homology. [CHV06, Section 1.2]

**Talk 5**: **Combinatorial models of moduli spaces of surfaces**. (Gaëtan Borot)

(i) Definition of mapping class groups [FM12, Chapters 1 and 2], Teichmüller spaces [FM12, Chapter 10] and moduli spaces of Riemann surfaces [FM12, Section 12.1]. The moduli space of Riemann surfaces as a classifying space [FM12, Section 12.6].

(ii) Category of admissible fat graphs and space of metric admissible fat graphs [DK22, Section 3].
(iii) Visual sketch of the homotopy equivalence between the moduli space of Riemann surfaces and the space of metric fat graphs using the radial slit configurations model of Bödigheimer [DK22, Sections 1, 2 and 4].

**Talk 6**: **The cyclic Deligne conjecture**. (Wennan Zhang)

(i) Motivation and statement of the Deligne conjecture without proof. [Vor00, Introduction]
(ii) Statement of the cyclic Deligne conjecture. Statement of Theorem 3.3 and Corollary 3.4 in [TZ06, Section 1] and comments.
(iii) Definition of a PROP and examples. [CHV06, Sections 2.1.1 and 2.1.2]
(iv) Definition of the PROP of cyclic Sullivan chord diagrams. Comments on the links with Talks 3 and 4. [TZ06, Section 2]
(v) Definition of the normalized Hochschild cochain complex. Outline of the proof of Theorem 3.3. [TZ06, Section 3]
(vi) Comments about the commutative case. [TZ06, Section 4]

**Talk 7**: **Generalized string topology operations**.

(i) Definition of topological quantum field theories and examples. [CHV06, Section 3.1] and [Ati88]
(ii) Definition of topological conformal field theories. The case of string topology. [CHV06, Section 3.1]
(iii) Generalized string topology operations. [CHV06, Section 3.2], see also [CG04] and [God08]
(iv) Some comments about open-closed string topology. [Ram06]

**Talk 8**: **Introduction to symplectic field theory** [CL09]. The talk will be in two parts of 1h each.

(i) Principles of symplectic field theory.
(ii) The Weyl algebra of differential operators.
(iii) The hamiltonian of symplectic field theory as solution of quantum master equation.

Original source on symplectic field theory [EGH00].

(i) String homology. Linearised contact homology.
(ii) Isomorphism between the two in the case of the zero-section of $T^*M$.

**Talk 9**: **Homotopy IBL-algebras**. (Pedro Tamaroff)

Section 2–9 of [CFL20], in particular covering Proposition 1.2 and 1.3.

(i) $\mathrm{IBL}_\infty$-algebras.
(ii) Deformation theory and homotopy transfer.

(iii) Maurer-Cartan elements in the Weyl algebra.

(Avoid discussion of the filtered case)

**Talk 10**: **Homotopy IBL-algebra structure on cyclic bar complex of a dga**. (Dingyu Yang)

Section 10–12 of [CFL20], in particular covering Proposition 1.4, Theorem 1.5, Proposition 1.6-1.7.

(i) Dual cyclic bar complex of a cyclic dga.
(ii) dIBL-algebra structure via sums over fatgraphs, canonical Maurer–Cartan element.

**Talk 11**: **Applications to string topology and Floer homology**. (Naageswaran Manikandan)

Section 12–13 of [CFL20] in particular Theorem 1.10, Conjecture 1.12-1.13 and recent progress [CHV22].

(i) The case of the de Rham complex.
(ii) The case of the Floer complex.

## References


[Ati88] M. Atiyah. Topological quantum field theories. *Inst. Hautes Études Sci. Publ. Math.*, (68):175–186 (1989), 1988.

[CFL20] K. Cieliebak, K. Fukaya, and J. Latschev. Homological algebra related to surfaces with boundary. *Quantum Topol.*, 11(4):691–837, 2020. math.QA/1508.02741.

[CG04] R.L. Cohen and V. Godin. A polarized view of string topology. In *Topology, geometry and quantum field theory*, volume 308 of *London Math. Soc. Lecture Note Ser.*, pages 127–154. Cambridge Univ. Press, Cambridge, 2004. math.AT/0303003.

[CHV06] R.L. Cohen, K. Hess, and A.A. Voronov. *String topology and cyclic homology.* Adv. Courses in Math. – CRM Barc. Basel: Birkhäuser, 2006.

[CHV22] K. Cieliebak, P. Hajek, and E. Volkov. Chain-level equivariant string topology for simply connected manifolds. 2022. math.AT/2202.06837.

[CKS08] R.L. Cohen, J.R. Klein, and D. Sullivan. The homotopy invariance of the string topology loop product and string bracket. *J. Topol.*, 1(2):391–408, 2008.

[CL09] K. Cieliebak and J. Latschev. The role of string topology in symplectic field theory. In *New perspectives and challenges in symplectic field theory: dedicated to Yasha Eliashberg on the occasion of his 60th birthday*, pages 113–146. Amer. Math. Soc., 2009. math.SG/0706.3284.

[CS99] M. Chas and D. Sullivan. String topology, 1999. math.GT/9911159.

[DK22] Egas Santander D and A. Kupers. Comparing combinatorial models of moduli space and their compactifications, 2022. math.GT/1506.02725.

[EGH00] Y. Eliashberg, A. Givental, and H. Hofer. Introduction to symplectic field theory. Number Special Volume, Part II, GAFA 2000 (Tel Aviv, 1999), pages 560–673. 2000. math.SG/0010059.

[FM12] B. Farb and D. Margalit. *A primer on mapping class groups*, volume 49 of *Princeton Mathematical Series.* Princeton University Press, Princeton, NJ, 2012.

[FT08] Y. Félix and J.-C. Thomas. Rational BV-algebra in string topology. *Bull. Soc. Math. France*, 136(2):311–327, 2008. math.AT/0705.4194.

[Ger63] M. Gerstenhaber. The cohomology structure of an associative ring. *Ann. of Math. (2)*, 78:267–288, 1963.

[God08] V. Godin. Higher string topology operations, 2008. math.AT/0711.4859.

[LO15] J. Latschev and A. Oancea, editors. *Free loop spaces in geometry and topology. Including the monograph Symplectic cohomology and Viterbo's theorem by Mohammed Abouzaid*, volume 24 of *IRMA Lect. Math. Theor. Phys.* Zürich: European Mathematical Society (EMS), 2015.

[Lod98] J.-L. Loday. *Cyclic homology*, volume 301 of *Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences].* Springer-Verlag, Berlin, second edition, 1998. Appendix E by María O. Ronco, Chapter 13 by the author in collaboration with Teimuraz Pirashvili.

[LV12] J.-L. Loday and B. Vallette. *Algebraic operads*, volume 346 of *Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences].* Springer, Heidelberg, 2012.

[Maz] T. Mazuir. Introduction to algebraic operads. Lectures notes available at `http://www.normalesup.org/~mazuir/files/lecturenotes-6.pdf`.

[Nae21] F. Naef. The string coproduct "knows" reidemeister/whitehead torsion, 2021. math.AT/2106.11307.

[NRW22] Florian Naef, Manuel Rivera, and Nathalie Wahl. String topology in three flavours, 2022. math.AT/2203.02429.

[Ram06] A. Ramirez. Open-closed string topology via fat graphs, 2006. math.AT/0606512.

[Tam] P. Tamaroff. Algebraic operads, koszul duality and gröbner bases: an introduction. Lectures notes available at `https://ptamarov.github.io/lecture-notes.pdf`.

[TZ06] Thomas Tradler and Mahmoud Zeinalian. On the cyclic Deligne conjecture. *J. Pure Appl. Algebra*, 204(2):280–299, 2006. math.QA/0404218.

[Vor00] A.A. Voronov. Homotopy Gerstenhaber algebras. In *Conférence Moshé Flato 1999, Vol. II (Dijon)*, volume 22 of *Math. Phys. Stud.*, pages 307–331. Kluwer Acad. Publ., Dordrecht, 2000.