

# FLOER HOMOTOPY THEORY SEMINAR

## PLAN OF THE TALKS

**DISCUSSION (15/04).** – Thibaut Mazuir & Chris Wendl

**TALK 1: Introduction to spectra (29/04).** – Ruochong Huang

**TALK 2: Morse theory and classifying spaces (06/05).** – Apratim Choudhury

(i) Quick recollections on Morse theory and the Morse-Smale assumption

*Section 1 of [CJS95b]*

(ii) Gluing for moduli spaces of broken Morse trajectories

*Sections 2 and 5 of [CJS95b]*

(iii) Definition of the flow category  $\mathcal{C}_f$  and of the classifying space of a topological category

*Sections 1 and 3 of [CJS95b], see also [Seg68] if necessary*

(iv) Idea of proof of the two parts of the main theorem of [CJS95b]

*Sections 3 and 6 of [CJS95b], see also [Cal] if necessary*

(v) Relationship with the Morse complex

*Section 4 of [CJS95b]*

(vi) Compare these results to the main theorem of [Fra79]

**TALK 3: Realizing chain complexes by spectra (13/05).** – Thibaut Mazuir

(i) Rough sketch of the Floer homotopy theory program by Cohen, Jones and Segal

(ii) The Puppe construction following [Hat02]. The category  $\mathcal{F}$  and the stable homotopy type of a filtered space.

(iii) Realizing chain complexes with spectra.

*Section 5 in [CJS95a] and Section 1.1 in [Coh20]*

**TALK 4: Framed manifolds with corners and framed Morse-Smale categories (27/05).**  
– Thibaut Mazuir

(i) Recollections of Talk 3

(ii) Manifolds with corners and  $\langle k \rangle$ -manifolds

(iii) Framed embeddings of  $\langle k \rangle$ -manifolds and the Pontrjagin-Thom construction.

(iv) Framed Morse-Smale categories, their associated "Floer homotopy type" and chain complexes

*Selected pieces of* [CJS95a], *Section 1 in* [Coh10] *and Section 1 in* [Coh20]

**TALK 5: Floer homotopy types and Floer homology with coefficients (10/06).** – Thibaut Mazuir

(i) Recollections of Talk 4

(ii) The Floer homotopy type of a Morse function

(iii) Symplectic homology, the Floer homotopy type of the cotangent bundle and the generalization of Viterbo's theorem

(iv) Floer homology with coefficients: commutative ring spectra and orientations of Morse-Smale categories

(v) Sketch of the proof of Arnold's conjecture by Abouzaid and Blumberg using the Morava K-theory spectra

*Selected pieces of* [CJS95a], [Coh10], [Coh09], [Coh20] *and* [AB21]

**TALK ? : A Khovanov stable homotopy type (?)** – Naageswaran Manikandan

Selected pieces of [LS14]

**TALK ? : Classical bordism theory (?)** – ?

Chapter 1 of [CF64]

**TALK ? : Bordism theory of flow categories (?)** – ?

[Hir] - and maybe selected pieces of [PS24] in this talk or in a later talk

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