



# Precursory knowledge of commutative multiplication relies on geometric representations

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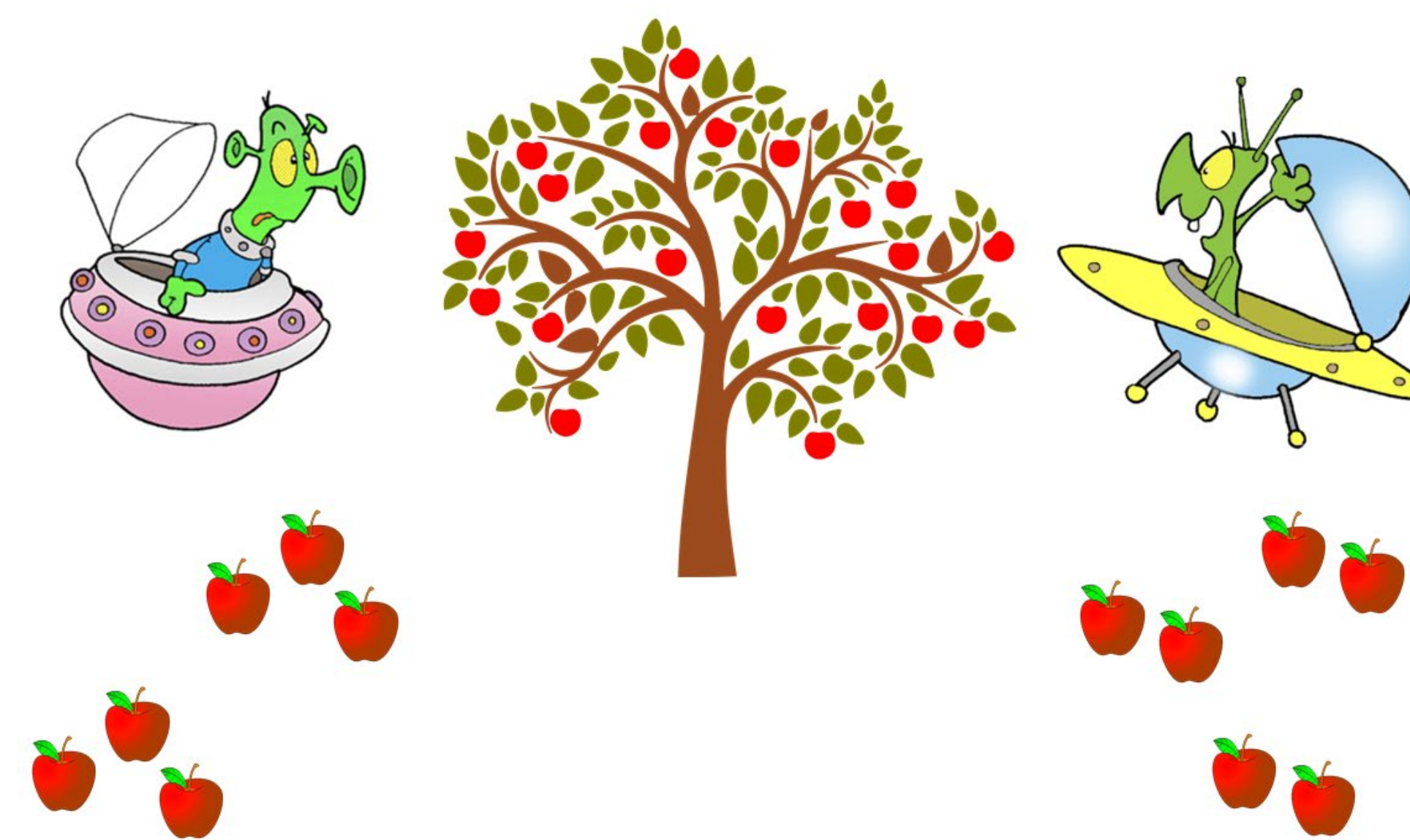
## Introduction

- ❖ An efficient way for children to learn large numbers is by understanding that they are composed of smaller numbers.
- ❖ Realizing that each number is unique even if it can be composed in different ways is essential to develop comprehensive numerical skills.
- ❖ Adults know that both buying 2 bags of 4 apples and 4 bags of 2 apples will result in buying 8 apples.
- ❖ Before mastering multiplication, most school-age children do not spontaneously perceive this result.
- ❖ Does that mean that, contrary to addition, children do not possess precursory knowledge of the commutative principle of multiplication?

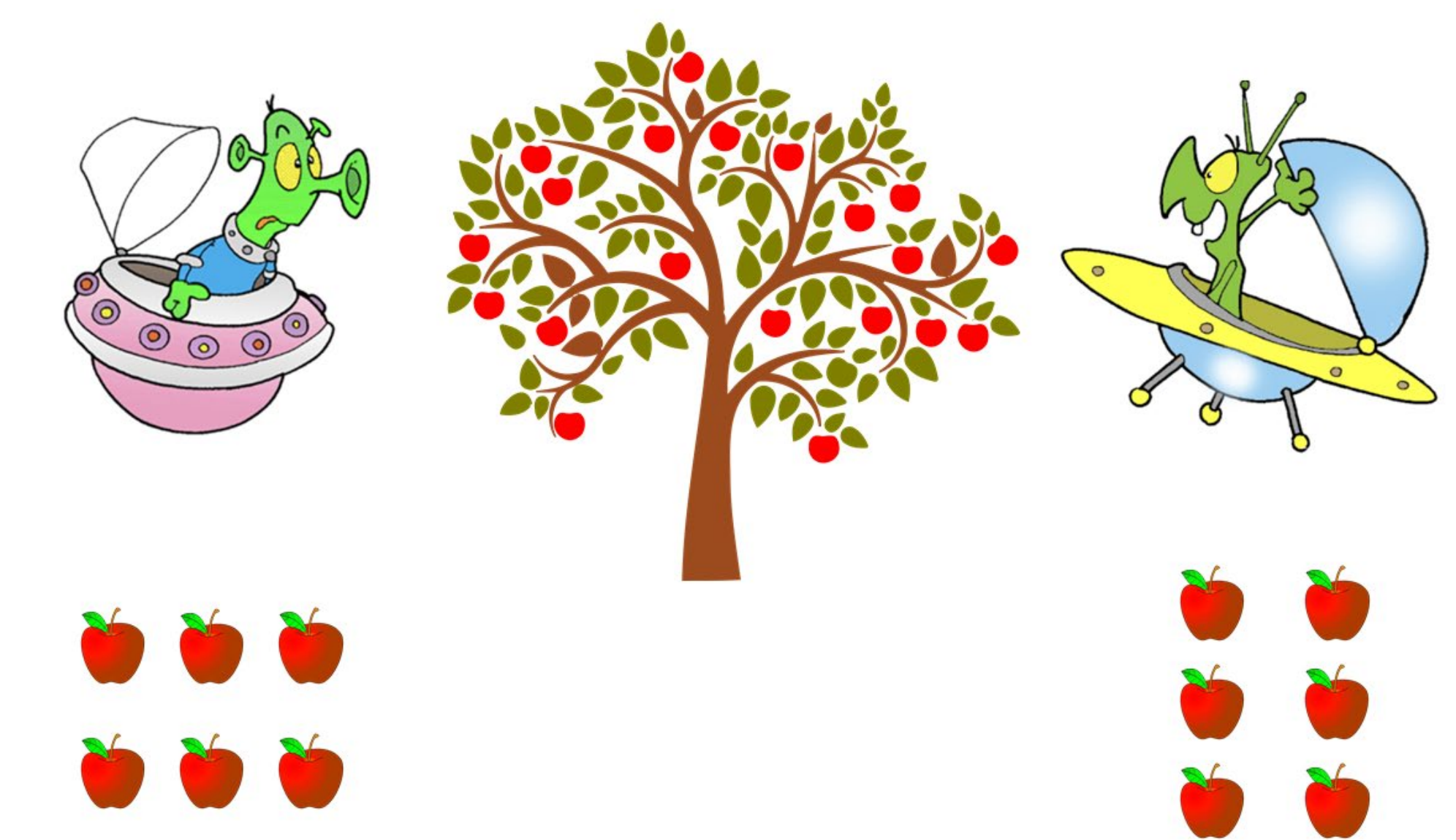
## Methods

- ❖ Participants: 18 5-year-old children
- ❖ Two-alternative forced-choice task: do two characters have a fair or an unfair share of apples?
- ❖ Test of addition, multiplication, and identity

- ❖ Two types of grouping: simple or geometric
- ❖ One session without verbal description, the other with
- ❖ Unfair trials = fair trials with one missing or more element
- ❖ If answer unfair, children are asked who has more apples.



Commutativity: multiplication with simple grouping

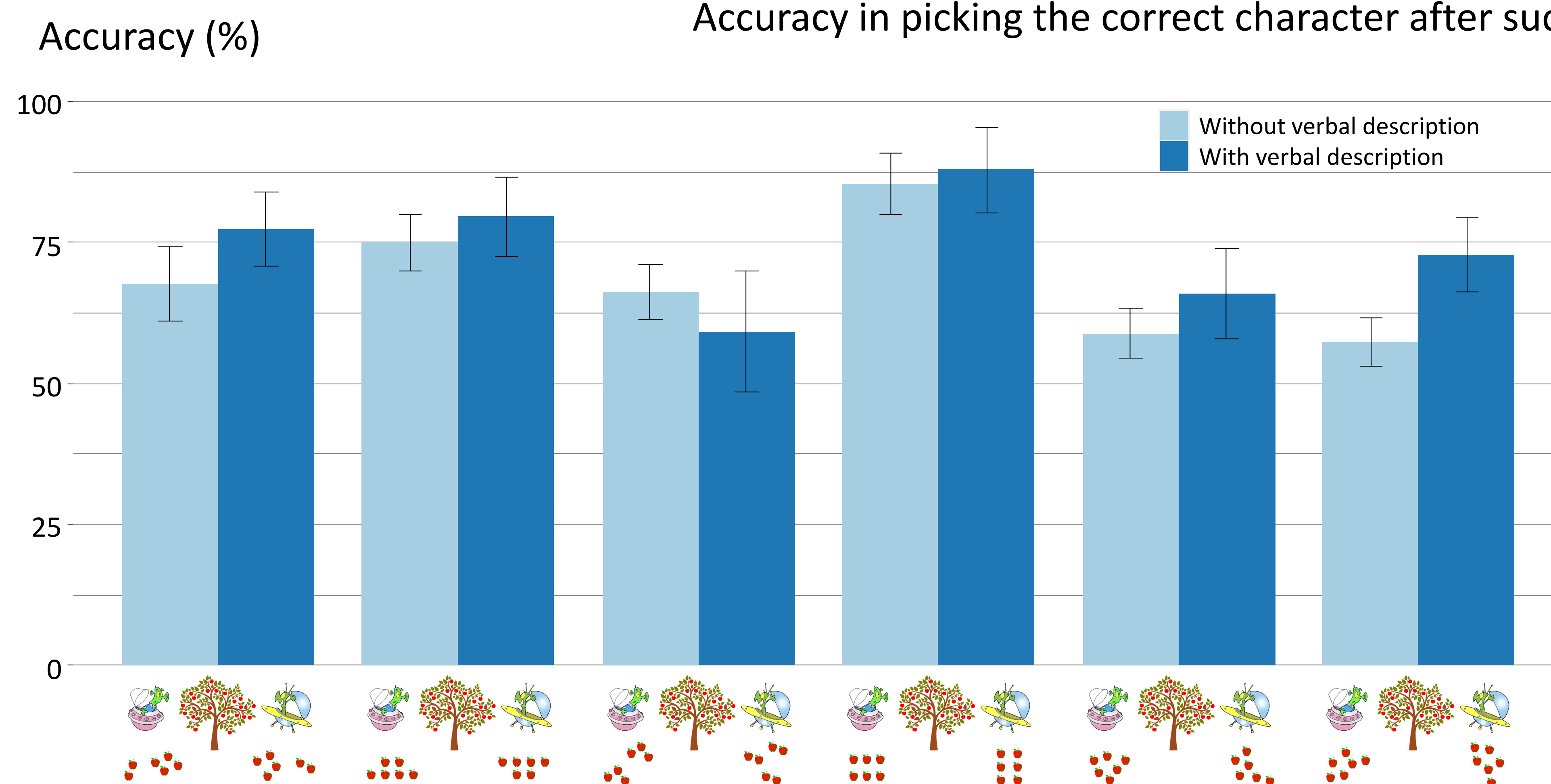


Commutativity: multiplication with geometric grouping

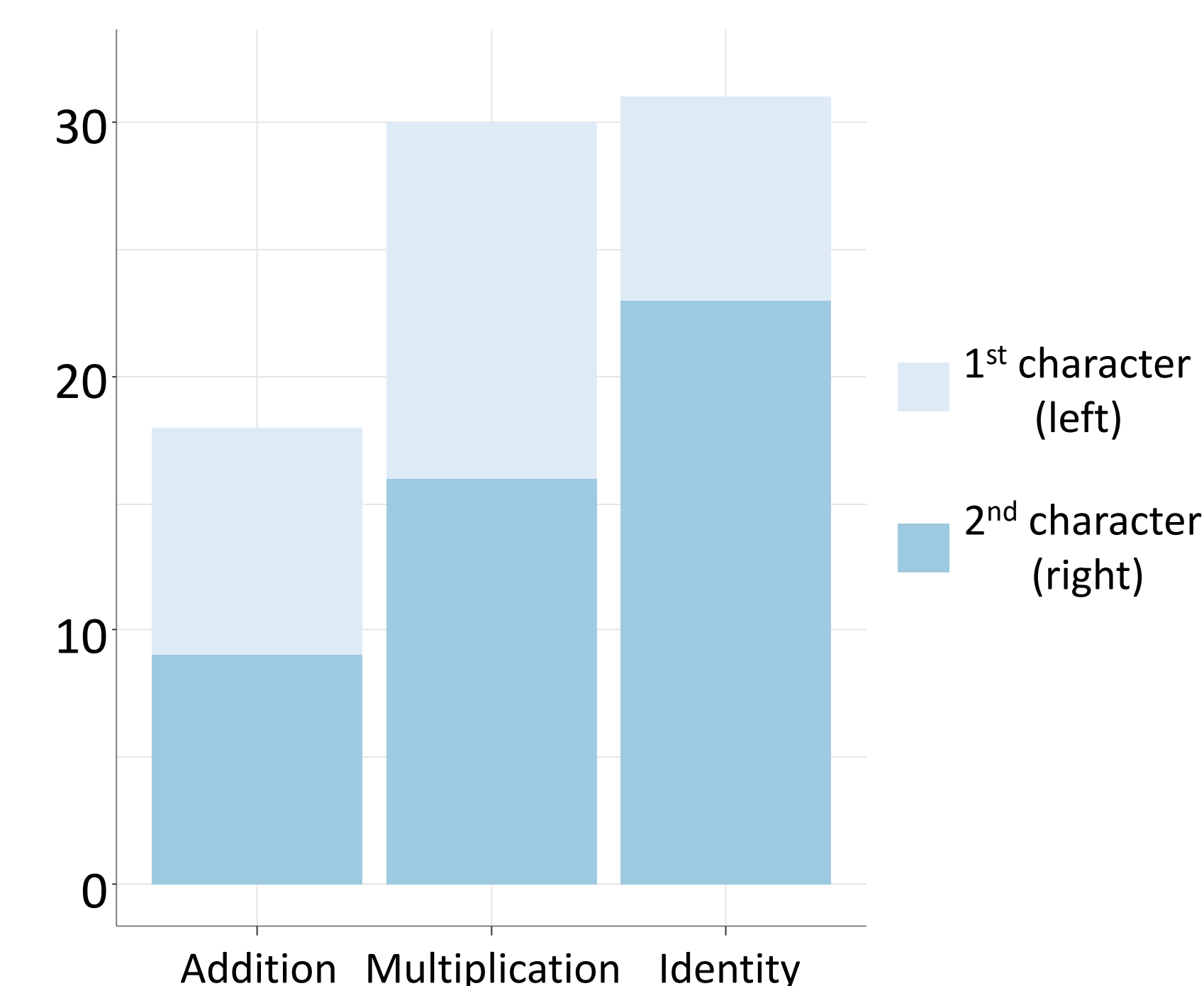
## Results

Global accuracy: 70.0 ± 2.29 %

Accuracy in picking the correct character after successfully answering "unfair": 80.2 ± 4.04 %



Character children picked when they wrongly answered "unfair"



## Conclusions

- ❖ Children possess early intuitions of commutativity before learning formal arithmetic at school.
- ❖ Additive commutativity is slightly more accurately perceived than multiplicative commutativity.
- ❖ Verbal descriptions of the groups are not helpful to understand commutativity.
- ❖ Multiplicative commutativity is better perceived when supported by representations that make the symmetry clearly apparent.
- ❖ Our results may provide useful insights on how to best introduce commutativity at school.

## References and funding

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