Defining contextual refinement for capability machines

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Logic and Semantics Seminar

Outline

1. Introduction
2. Capability machines
3. Components and contexts
4. Defining contextual refinement
5. Validity relation
6. Conclusion

Introduction

Contextual refinement

- Binary relation between two open programs
- Any observable behavior from p is also observable in p'

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- Binary relation between two open programs
- Any observable behavior from p is also observable in p'

General definition

$$p \preccurlyeq_c p' \quad := \quad \forall\, C, C[p] \text{ terminates } \Rightarrow \; C[p'] \text{ terminates}$$

Applications of contextual refinement

- Reasoning on open programs using the concrete semantics

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BUT: often hard to prove

Example: specification as a program

Formal specification:

$$
\begin{array}{l}
\forall\, P, I, f, xs, \ell, \\
\quad \left\{ \begin{array}{l} \texttt{isList}\ \ell\ xs * \texttt{all}\ P\ xs * I\ []\ a\ * \\ (\forall x,\ x,\ a',\ ys, \{P\ x * I\ ys\ a'\}\ \ f\ x\ a'\ \ \{r.\ I\ (x :: ys)\ r\}) \end{array} \right\} \\
\quad \texttt{foo}\ f\ a\ \ell \\
\quad \{r.\ \texttt{isList}\ \ell\ xs * I\ xs\ r\}
\end{array}
$$

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$$\texttt{foo}\ f\ a\ \ell$$
$$\{r.\ \texttt{isList}\ \ell\ xs * I\ xs\ r\}$$

Specification as a program:

```
let rec foo_spec f a l = match l with
  | [] -> a
  | x::xs -> f x (foo_spec f a xs)
```

Example: representation independence

```
let counter () =
  let x = ref 0 in
  let incr () =
    x := !x + 1
  in
  let read () = !x
  in incr, read
```

```
let counter_neg () =
  let x = ref 0 in
  let incr () =
    x := !x - 1
  in
  let read () = - !x
  in incr, read
```

Capability machines

What is a capability machine

- Security oriented CPU
- Check memory access via special machine words:

$$\mathtt{Word} = \mathbb{Z} \sqcup \mathtt{Cap}$$

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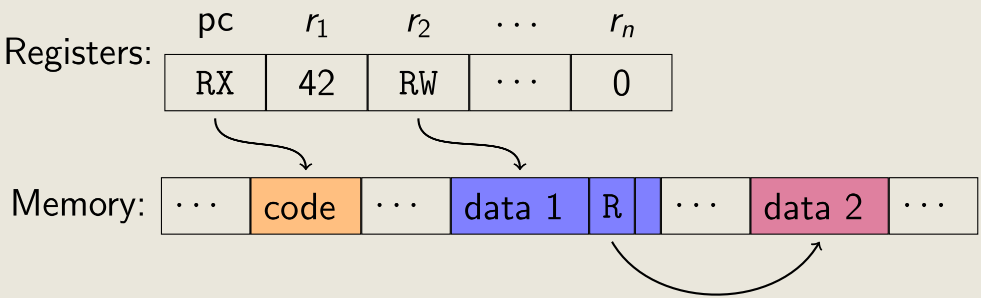
Capability

$$c \in \texttt{Cap} \;:=\; (p, b, e, a)$$

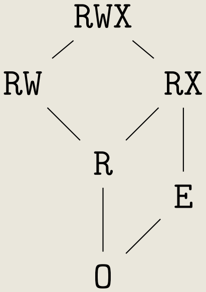
where $p \in \{\texttt{O},\ \texttt{E},\ \texttt{R},\ \texttt{RW},\ \texttt{RX},\ \texttt{RWX}\}$

$\Rightarrow$ gives access to $[b; e)$ with permission p

Memory access via capabilities



Permission order



Memory access via capabilities

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Memory access via capabilities

Very few instructions modify capabilities :

- $\texttt{lea}\ r\ z$ changes a capability's address to $a + z$
- $\texttt{subseg}\ r\ b'\ e'$ modifies the range to $[b'; e') \subseteq [b; e)$
- $\texttt{restrict}\ r\ p'$ modifies the permission to $p' \preccurlyeq p$
- $\texttt{jmp}\ r$ and $\texttt{jnz}\ r\ \rho$ change $\texttt{E}$ to $\texttt{RX}$.

Cerise capability machine model

Simple model:

- Single core
- No interruptions
- No privilege levels
- No virtual memory
- Limited instruction set

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But captures:

- Finite memory
- Fixed set of registers
- Instructions encoded as integers

Cerise instruction set

$$\rho \in \mathbb{Z} \sqcup \mathtt{RegName}$$

$$\begin{aligned} i \in \mathtt{Instr} \quad := \quad & \mathtt{fail} \mid \mathtt{halt} \mid \mathtt{jmp}\ r \mid \mathtt{jnz}\ r\ r \mid \\ & \mathtt{move}\ r\ \rho \mid \mathtt{load}\ r\ r \mid \mathtt{store}\ r\ \rho \mid \\ & \mathtt{add}\ r\ \rho\ \rho \mid \mathtt{sub}\ r\ \rho\ \rho \mid \mathtt{lt}\ r\ \rho\ \rho \mid \\ & \mathtt{restrict}\ r\ \rho \mid \mathtt{subseg}\ r\ \rho\ \rho \mid \mathtt{lea}\ r\ \rho \mid \mathtt{isptr}\ r\ r \mid \\ & \mathtt{getp}\ r\ r \mid \mathtt{getb}\ r\ r \mid \mathtt{gete}\ r\ r \mid \mathtt{geta}\ r\ r \end{aligned}$$

Machine state

$$
\begin{aligned}
(\mathtt{mem}, \mathtt{regs}) \in \mathtt{ExecConf} \quad &:= \quad (\mathtt{Addr} \to \mathtt{Word}) \times (\mathtt{RegName} \to \mathtt{Word}) \\
\delta \in \mathtt{ExecMode} \quad &:= \quad \mathtt{Halted} \mid \mathtt{Failed} \mid \mathtt{Running}
\end{aligned}
$$

Machine state: $\mathtt{ExecMode} \times \mathtt{ExecConf}$

Small step semantics

$\textsc{ExecStep}$

$$(\mathtt{Running}, (\mathtt{mem}, \mathtt{regs})) \rightarrow \begin{cases} \mathtt{execInstr}\ \mathtt{mem}\ \mathtt{regs}\ i & \text{if } \mathtt{regs}(\mathtt{pc}) = (p, b, e, a) \wedge \\ & \mathtt{RX} \preccurlyeq p \wedge a \in [b; e) \wedge \\ & \mathtt{decodeInstr}(\mathtt{mem}(a)) = \mathtt{Some}\ i \\ \mathtt{Failed}, (\mathtt{mem}, \mathtt{regs}) & \text{otherwise} \end{cases}$$

Components and contexts

Defining open and closed program

What is a program?

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- A region of memory containing encoded instructions
- A register state $\texttt{RegName} \to \texttt{Word}$

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An open program?

A closed program?

A context?

Defining open programs: components

Open program:

- segment of memory
- interface to access it

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Component

$$\texttt{component} \;:=\; \left\{ \begin{array}{lcl} \texttt{segment} & : & \texttt{Addr} \rightharpoonup \texttt{Word} \\ \texttt{imports} & : & \texttt{Addr} \rightharpoonup \texttt{Symbols} \\ \texttt{exports} & : & \texttt{Symbols} \rightharpoonup \texttt{Word} \end{array} \right\}$$

Well-formed components

- imports and exports symbols are disjoint:
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- imports and exports symbols are disjoint:
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- import addresses are part of the component's memory:
  $\mathtt{dom}\,(\mathtt{imports}) \subseteq \mathtt{dom}\,(\mathtt{segment})$
- contained capabilities only point to its memory:

$$\forall\,(\_, b, e, \_) \in \mathtt{img\ segment} \cup \mathtt{img\ exports},$$
$$[b; e) \subseteq \mathtt{dom\ segment}$$

Closed program

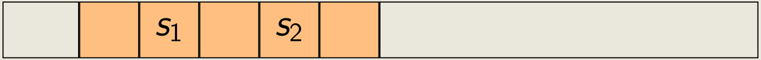
Program

A program is a pair $(p, \texttt{regs})$:

- p is a well-formed component with no imports
- $\texttt{regs} \in \texttt{RegName} \to \texttt{Word}$ is a register state
- capabilities in $\texttt{regs}$ point to p

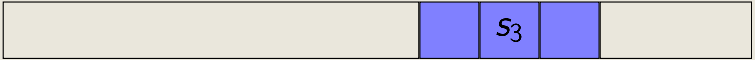
Linking

x:



$$\texttt{exports} = \{s_3 \mapsto w_3,\ s_4 \mapsto w_4\}$$

y:



$$\texttt{exports} = \{s_1 \mapsto w_1\}$$

Linking

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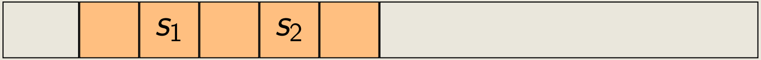

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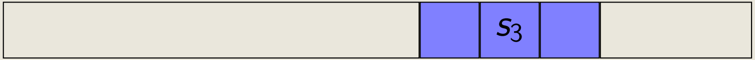

$\texttt{exports} = \{s_1 \mapsto w_1\}$

$x \bowtie y$:

$\texttt{exports} = \{s_3 \mapsto w_3,\ s_4 \mapsto w_4,\ s_1 \mapsto w_1\}$

Linking

Requires components to be **disjoint** and well-formed:

$$x \bowtie y :=$$

$$\left\{ \begin{array}{lcl} \texttt{exports} & := & x.\texttt{exports} \uplus y.\texttt{exports} \\ \texttt{imports} & := & \left\{ a \mapsto s \;\middle|\; \begin{array}{l} a \mapsto s \in x.\texttt{imports} \uplus y.\texttt{imports} \wedge \\ s \mapsto \_ \notin x.\texttt{exports} \uplus y.\texttt{exports} \end{array} \right\} \\ \texttt{segment} & := & x.\texttt{segment}[y.\texttt{exports} \circ x.\texttt{imports}] \uplus \\ & & y.\texttt{segment}[x.\texttt{exports} \circ y.\texttt{imports}] \end{array} \right\}$$

Properties of linking

- $x \#_\ell y \Rightarrow x \bowtie y$ well-formed
- commutative: $x \#_\ell y \Rightarrow x \bowtie y = y \bowtie x$
- associative:
  $x \#_\ell y \wedge y \#_\ell z \wedge x \#_\ell z \Rightarrow x \bowtie (y \bowtie z) = (x \bowtie y) \bowtie z$

Context

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Context

A context for a component x is a pair $(z, \mathtt{regs})$ where:

- $x \#_\ell z$
- $\mathtt{img}\ x.\mathtt{imports} \subseteq \mathtt{dom}\ z.\mathtt{exports}$
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  capabilities in $\texttt{regs}$ point to z
- if $y.\texttt{exports} = \emptyset$ and $(z, \texttt{regs})$ is a context of $x \bowtie y$ then
  $(z, \texttt{regs})$ is a context of y

Defining contextual refinement

Contextual refinement

General idea: $x \preccurlyeq_{\text{ctx}} y$ when:

- for all context $(z, \texttt{regs})$
- for all values $v \in \{\texttt{Halted}, \texttt{Failed}\}$

if $\exists n,\ \texttt{machine\_run}\ n\ (z \bowtie x)\ \texttt{regs} = v$
then $\exists n,\ \texttt{machine\_run}\ n\ (z \bowtie y)\ \texttt{regs} = v$

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What quantification on context?

Multiple options:

1. quantify on context of both x and y

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Contextual refinement

Improved definition: $x \preccurlyeq_{\text{ctx}} y$ when:

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$$\begin{cases} (z, \texttt{regs}) \text{ is a context of } x \\ \exists n,\ \texttt{machine\_run}\ n\ (z \bowtie x)\ \texttt{regs} = v \end{cases} \Rightarrow$$

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Components can be too different:

$\Rightarrow$ require that $\texttt{dom}\ y\texttt{.exports} \subseteq \texttt{dom}\ x\texttt{.exports}$

Final definition

- $\text{dom } x.\mathit{segment} \cap [0; \text{ctxt\_size}) = \emptyset$
- $\text{dom } y.\texttt{exports} \subseteq \text{dom } x.\texttt{exports}$
- for all $(z, \texttt{regs})$, for all $v \in \{\texttt{Halted}, \texttt{Failed}\}$

$$\left\{\begin{array}{l} (z, \texttt{regs}) \text{ is a context of } x \\ \exists n,\ \texttt{machine\_run}\ n\ (z \bowtie x)\ \texttt{regs} = v \end{array}\right. \Rightarrow$$

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Good properties of contextual refinement

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reflexive: x well-formed $\ \Rightarrow\ x \ \preccurlyeq_{\text{ctx}}\ x$

transitive: $x \ \preccurlyeq_{\text{ctx}}\ y \wedge y \ \preccurlyeq_{\text{ctx}}\ z \ \Rightarrow\ x \ \preccurlyeq_{\text{ctx}}\ z$

compositional: if x and y disjoint

$$x \ \preccurlyeq_{\text{ctx}}\ x' \wedge y \ \preccurlyeq_{\text{ctx}}\ y' \ \Rightarrow\ (x \ \bowtie\ y) \ \preccurlyeq_{\text{ctx}}\ (x' \ \bowtie\ y')$$

Other properties of contextual refinement

Other consequences: if $x \preccurlyeq_{\text{ctx}} y$ then

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Other consequences: if $x \preccurlyeq_{\mathrm{ctx}} y$ then

- All public memory of x and y is the same
- Depends on absolute memory position
- Non-terminating programs refine pretty-much anything
- E capabilities behave in the same way
- $\mathrm{dom}(\texttt{segment y}) \subseteq \mathrm{dom}(\texttt{segment x})$

Growing and shrinking components

if z has no exports then:

- if $x \preccurlyeq_{\text{ctx}} y$ then $x \bowtie z \preccurlyeq_{\text{ctx}} y$
- if $x \preccurlyeq_{\text{ctx}} y \bowtie z$ then $x \preccurlyeq_{\text{ctx}} y$

Validity relation

Unary validity relation

Goal: capture values safe to share with unknown code

$$
\begin{aligned}
\mathcal{V}(z) &:= \texttt{True} \\
\mathcal{V}(\texttt{0}, b, e, a) &:= \texttt{True}
\end{aligned}
$$

Unary validity relation

Goal: capture values safe to share with unknown code

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\mathcal{V}(z) &:= \texttt{True} \\
\mathcal{V}(\texttt{O}, b, e, a) &:= \texttt{True} \\
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\mathcal{V}(\texttt{R}/\texttt{RX}, b, e, a) &:= \mathop{\Huge *}_{a \in [b;e)} \exists P, \begin{cases} \boxed{\exists w,\ a \mapsto_a w \ * \ P(w)} \ * \\ \triangleright \Box\, \forall w,\ P(w) \ -\!\!* \ \mathcal{V}(w) \end{cases}
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\end{array}
$$

Recursive definition possible thanks to Iris' later modality ($\triangleright$)

Unary expression relation

Goal: capture values safe to execute with unknown code

$$
\begin{aligned}
\mathcal{E}(w) \;:=\; & \forall\, \mathtt{regs} \in \mathtt{RegName} \to \mathtt{Addr},\; \mathtt{regs}_{\ell}(\mathrm{pc}) = w \;\Rightarrow \\
& \left(\mathop{\scalebox{2}{$\ast$}}_{r \in \mathtt{RegName}} r \mapsto_r \mathtt{regs}(r) \,\ast\, \mathcal{V}(\mathtt{regs}(r)) \right) \mathrel{-\!\!\ast} \\
& \mathtt{WP}\ \mathtt{Running}\ \{v,\ v = \mathtt{Halted}\}
\end{aligned}
$$

Binary validity relation

Defined on equal values:

$$
\begin{array}{lll}
\mathcal{V}(z,\ z) & := & \texttt{True} \\
\mathcal{V}((\texttt{O}, b, e, a),\ \_) & := & \texttt{True} \\
\mathcal{V}((\texttt{E}, b, e, a),\ \_) & := & \triangleright \Box\, \mathcal{E}((\texttt{RX}, b, e, a),\ (\texttt{RX}, b, e, a)) \\
\mathcal{V}((\texttt{R}/\texttt{RX}, b, e, a),\ \_) & := & \displaystyle\mathop{\Huge *}_{a \in [b;e)} \exists P, \begin{cases} \boxed{\exists\, w\, w',\ a \mapsto_a w * a \rightarrowtail_a w' * P(w, w')} \ * \\ \triangleright \Box\, \forall\, w\, w',\ P(w, w') \ \twoheadrightarrow \mathcal{V}(w,\ w') \end{cases} \\
\mathcal{V}((\texttt{RW}/\texttt{RWX}, b, e, a),\ \_) & := & \displaystyle\mathop{\Huge *}_{a \in [b;e)} \boxed{\exists\, w\, w',\ a \mapsto_a w \ * \ a \rightarrowtail_a w' \ * \ \mathcal{V}(w,\ w')}
\end{array}
$$

Binary expression relation

$$\begin{aligned}\mathcal{E}\,(w_\ell,\, w_r) \;:=\;& \forall\, \mathtt{regs}_\ell,\, \mathtt{regs}_r,\; \mathtt{regs}_\ell(\mathrm{pc}) = w_\ell \;\wedge\; \mathtt{regs}_r(\mathrm{pc}) = w_r \;\Rightarrow \\ & \left(\mathop{\Huge\ast}_{r \in \mathtt{RegName}} r \mapsto_r \mathtt{regs}_\ell(r) \ast r \rightarrowtail_r \mathtt{regs}_r(r) \ast \mathcal{V}\,(\mathtt{regs}_\ell(r),\, \mathtt{regs}_r(r)) \right) \mathrel{-\!\!\ast} \\ & \mathtt{WP}\;(\mathtt{Running}, \mathtt{Running})\;\{(v_\ell, v_r),\; v_\ell = \mathtt{Halted} \Rightarrow v_r = \mathtt{Halted}\}\end{aligned}$$

Binary expression relation

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$$\left(\mathop{\Huge\ast}_{r \in \mathtt{RegName}} r \mapsto_r \mathtt{regs}_\ell(r) * r \rightarrowtail_r \mathtt{regs}_r(r) * \mathcal{V}\,(\mathtt{regs}_\ell(r),\, \mathtt{regs}_r(r)) \right) \mathrel{-\!\!*}$$
$$\mathtt{WP}\;(\mathtt{Running}, \mathtt{Running})\;\{(v_\ell, v_r),\; v_\ell = \mathtt{Halted} \Rightarrow v_r = \mathtt{Halted}\}$$

$\Rightarrow$ similar implication to the one in contextual refinement

Fundamental theorem on logical relations

If a capability is safe to share, it is safe to execute

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FTLR

$$\begin{array}{l} \texttt{spec\_ctx} \Rightarrow \\ \mathcal{V}\left((p,b,e,a),\ (p,b,e,a)\right) \Rightarrow \\ \mathcal{E}\left((p,b,e,a),\ (p,b,e,a)\right) \end{array}$$

Exports relation

Goal: link validity (words) to CR (components)

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$$\mathcal{V}_{\text{exp}}(x, y) := \mathop{\Huge\ast}_{s \mapsto w_r \in y.\texttt{exports}} \exists\, w_\ell,\ s \mapsto w_\ell \in x.\texttt{exports} * \mathcal{V}(w_\ell, w_r)$$

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Implies $\texttt{dom}\ y.\texttt{exports} \subseteq \texttt{dom}\ x.\texttt{exports}$

Compatibility with link

Let x, y, z be components such that:

- x and z are disjoint; y and z are disjoint;
- $\mathtt{img}\,(z.\mathtt{segment}) \subseteq \mathbb{Z}$;
- $\mathtt{dom}\ x.\mathtt{exports} \subseteq \mathtt{dom}\ y.\mathtt{exports}$;

Then:

$$\mathtt{spec\_ctx} * \mathcal{V}_{\mathsf{exp}}\,(x,\ y) * \mathsf{mem\_map}_{\ell}(x, z) * \mathsf{mem\_map}_{r}(y, z) \\ \Rrightarrow \quad \mathcal{V}_{\mathsf{exp}}\,(x \ \bowtie\ z,\ y \ \bowtie\ z)$$

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Remaining work:

- Show link between $\mathcal{V}_{\text{exp}}$ and CR
- Strenghten theorem on $\mathcal{V}_{\text{exp}}$ of links

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Reflexions on CR:

- Too strong relation for many practical cases
- Maybe try to restrict observable behaviors

Thank you for your attention

Questions?