ERRATUM TO: FINITELY PRESENTABLE, NON-HOPFIAN GROUPS WITH KAZHDAN'S PROPERTY (T) AND INFINITE OUTER AUTOMORPHISM GROUP

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Abstract. In the article mentioned in the title, the second main result is the construction of a non-Hopfian, finitely presented Kazhdan group. The proof of its finite presentability has a little flaw, which is fixed here.

In my article [C], a certain group Γ is constructed (Definition 2.4) and is claimed (Theorem 1.3) to be finitely presented. This claim is true, but the proof relies on a misquotation of Abels' results. Namely, Theorem 3.1, attributed to Abels, is not true as I stated it (especially [C, "Theorem" 3.1(iii)] is irrelevant). Fortunately, the computation in the subsequent proof is not only correct, but allows to easily fix the issue. We now describe in detail the necessary corrections.

Theorem 3.1 of [C] has to be replaced by

Theorem 3.1 (corrected) [A]**.** *Let G be a connected linear algebraic group over $\mathbf{Q}_p$. Suppose that the following assumptions are fulfilled:*

(i) *G is $\mathbf{Q}_p$-split.*
(ii) *G is unipotent-by-semisimple, i.e. $G = US$, where U is the unipotent radical and $S \neq \{1\}$ a semisimple Levi factor.*
(iii) *No root of S is colinear to any weight of the adjoint representation of S on $H_1(\mathfrak{u})$.*
(iv) *0 is not a weight of the adjoint representation of S on $H_2(\mathfrak{u})$.*

Then $G(\mathbf{Q}_p)$ is compactly presented.

This relies on [A, Theorem 6.4.3 and Remark 6.4.5]. In order not to misquote it a second time, a few comments are necessary.

- Because of (ii), Condition (1a) of [A, Remark 6.4.5], which involves the orthogonal of the subspace generated by roots, reduces to say that $H_1(\mathfrak{u})$ does not contain the trivial representation (this is a necessary and sufficient condition for $G(\mathbf{Q}_p)$ to be compactly generated, see [A, Theorem 6.4.4]), and since $S \neq \{1\}$ this is ensured by (iii).
- Condition (1b) of [A, Remark 6.4.5] is weaker than (iii) since it involves dominant weights and positive roots. In order not to introduce these notions, we assume the much stronger (iii) which is satisfied in our example.
- In [A, Theorem 6.4.3], Condition (iv) is replaced by the much weaker condition that the representation of S on $H_2(\mathfrak{u})$ does not contain the trivial representation.

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Turning back to [C], the group Γ is written as $G(\mathbf{Z}[1/p])$ and it is explained that finite presentability of Γ follows from compact presentability of $G(\mathbf{Q}_p)$. We have to check that G satisfies the conditions of the corrected version of Theorem 3.2. The group G is written $G = US$, with U the unipotent radical and $S = \mathrm{SL}_{n_2} \times \mathrm{SL}_{n_3}$ ($n_2, n_3 \geq 3$), so (i) and (ii) are clear. Condition (iv), which is the most technical point, has been checked [C, Lemma 3.6]. It remains to check (iii), which follows from

Lemma 3.2 (corrected). *For any root φ of S, no weight ω of the action of D on $H_1(\mathfrak{u})$ lies on the line $\mathbf{R}\varphi$.*

To prove the lemma, most of the job has been done, since the weights of the adjoint representation of S on $H_1(\mathfrak{u})$ were computed in the proof of [C, Lemma 3.2]. This proof is correct and the reader can refer to it, although the former statement of the lemma itself is no longer of interest.

Recall from the context that we have two finite sets I_2, I_3 of cardinality $n_2, n_3 \geq 3$. We use the convenient convention that a letter such as i_k implicitly means $i_k \in I_k$. The weights live in M/P, where $M = \mathbf{Z}^{n_2+n_3} = \mathbf{Z}^{I_2 \sqcup I_3}$ with basis (u_i), and P is the plane generated by $\sum_{j_2} u_{j_2}$ and $\sum_{k_3} u_{k_3}$. The weights are, modulo P

$$-u_{j_2}, \quad u_{j_2} - v_{k_3}, \quad v_{k_3} \quad (j_2 \in I_2,\ k_3 \in I_3)$$

So far all this is contained in the proof of [C, Lemma 3.2]. Now, to apply the corrected version of Theorem 3.1, we need to write down the roots; since $S = \mathrm{SL}_{n_2} \times \mathrm{SL}_{n_3}$ these are, in M (modulo P)

$$u_{j_2} - u_{k_2}, \quad u_{j_3} - u_{k_3} \quad (j_2 \neq k_2 \in I_2,\ j_3 \neq k_3 \in I_3).$$

Since $n_2, n_3 \geq 3$ it is clear that any non-trivial linear combination of a root and a weight never lies in P. This proves the lemma.

References

A. H. Abels. *Finite presentability of S-arithmetic groups. Compact presentability of solvable groups.* Lecture Notes in Math. **1261**, Springer, 1987.

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