Origin of large scale magnetic fields

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9th November, 2016

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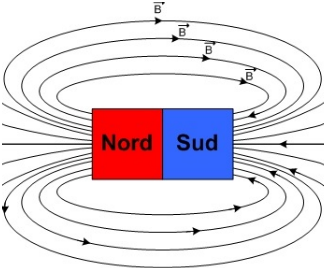
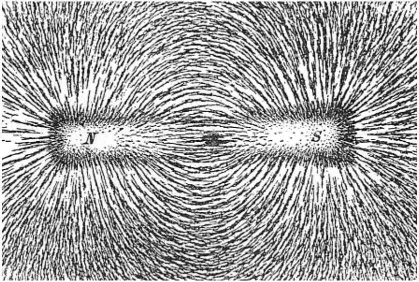
Compass

Use of the magnetic property of the Earth for traveling.

Magnet and metal dust



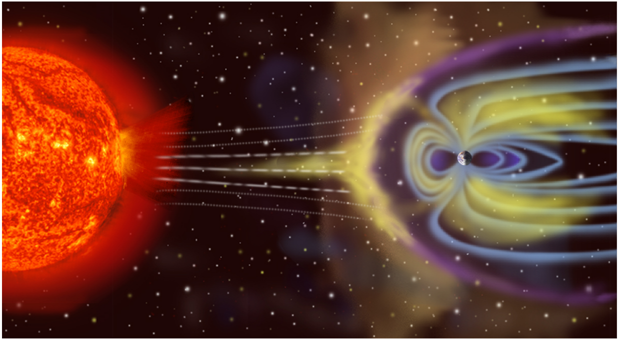


Similarities with magnets.

Aurora

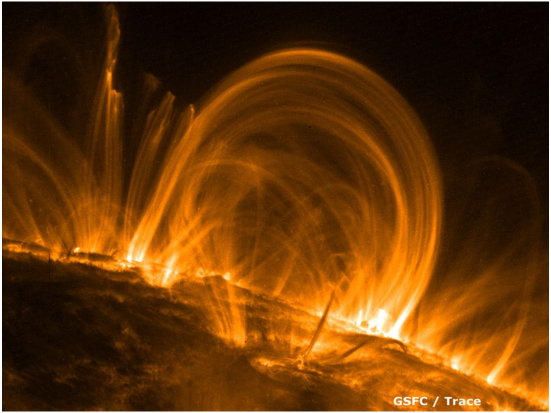
Appreciating the visual beauty of the magnetic activity.

Magnetosphere



Interaction of solar particles with the magnetosphere.

Solar activity



Magnetic activity on the Sun.

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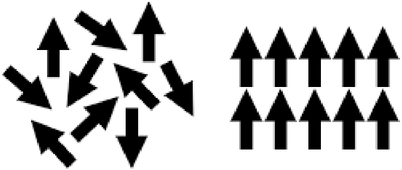
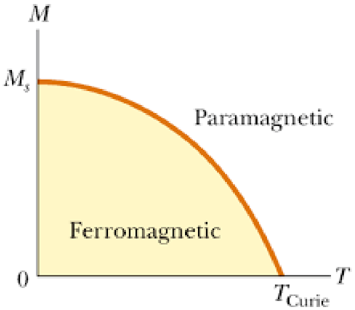
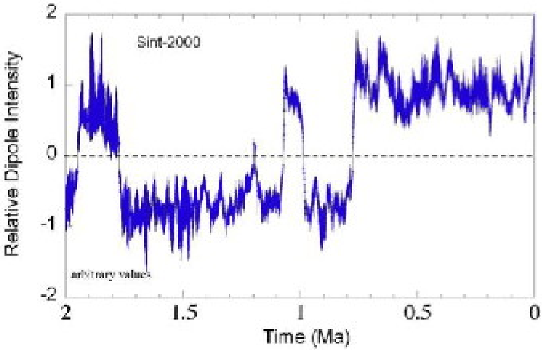
Curie temperature





No magnetization at high temperature.
How can you make a magnet at high temperature ?

Reversal



Reversal of the magnetic field.
How can the magnet flip ?

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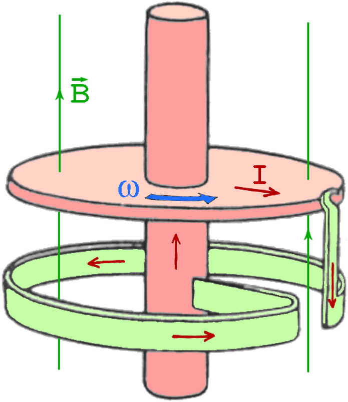
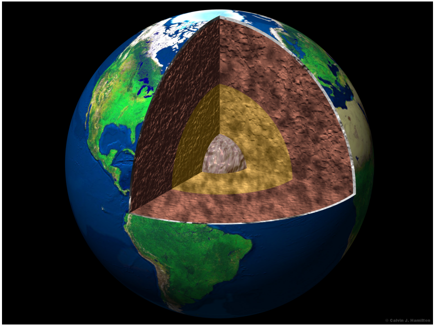
Dynamo (Bullard set-up)



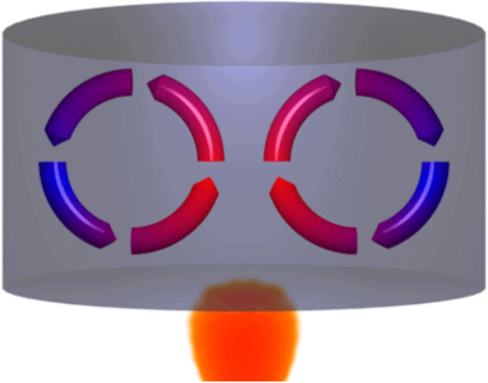
An example of magnetic field generated by motion.

Cores




Description of the core of the earth and the Sun.

Rayleigh-Bénard convection (heated from bellow)



Rayleigh-Bénard convective rolls.

Magnetic Reynolds number

Dimension

$$\partial_\tau \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mu_0 \sigma)^{-1} \Delta \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \tag{1}$$

$$\mathbf{v} = U\mathbf{u} \quad ; \quad X = Lx \quad ; \quad \tau = (L/U)\,t. \tag{2}$$

Dimensionless equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B} \quad \text{with} \quad Rm = \mu_0 \sigma U L. \tag{3}$$

Physical interpretation

- $Rm \ll 1$: viscous effects dominate
- $Rm \gg 1$: cinematic effects dominate (Sun: $Rm = 10^{11}$)

Symmetry: reversals explained

Dimensionless equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B} \quad \text{with} \quad Rm = \mu_0 \sigma U L. \tag{4}$$

Symmetry

$\mathbf{B} \to -\mathbf{B}$: leaves the equation unchanged.

Reversal of the magnetic field are possible.

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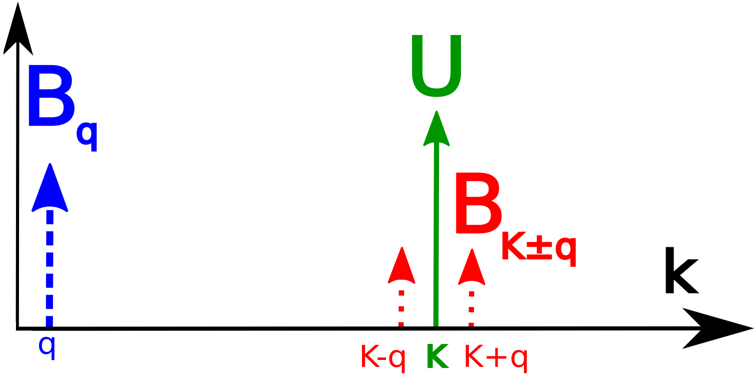
Three-mode interaction

Dimensionless equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B} \quad \text{with} \quad Rm = \mu_0 \sigma U L. \tag{5}$$



Instability generation

Dimensionless equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B} \quad \text{with} \quad Rm = \mu_0 \sigma U L. \tag{6}$$

Eigenvalue (ev) problem

Linear operator:

$$\mathscr{L}[\mathbf{B}] = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B}. \tag{7}$$

α-dynamo eigenvalue:

$$\mathscr{L}[\mathbf{B}_{ev}] \simeq (\alpha q - Rm^{-1} q^2) \mathbf{B}_{ev} \quad \text{with} \quad q = \frac{2\pi}{\ell}. \tag{8}$$

Methods to explain α-dynamos

Multi-scale expansion

- Theoretical demonstration of existence of the phenomenon
- Links small scales with large scales
- Valid at $Rm \ll 1$ (Sun $Rm = 10^{11}$)

Mean field

- Theoretical demonstration of existence of the phenomenon
- Dissociates large and small scales $(\alpha q - Rm^{-1} q^2)$
- Valid at large Rm

Floquet method

- Can only be performed with numeric simulations
- Dissociates large and small scales
- Valid at any Rm with *CFL* limitation

Our results: What is measured ?

Linear operator equation

$$\partial_t \mathbf{B} = \mathscr{L}[\mathbf{B}] \quad \text{with} \quad \mathscr{L}[\mathbf{B}] = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \Delta \mathbf{B} . \tag{9}$$

Eigenvalue (ev) problem

Temporal evolution:

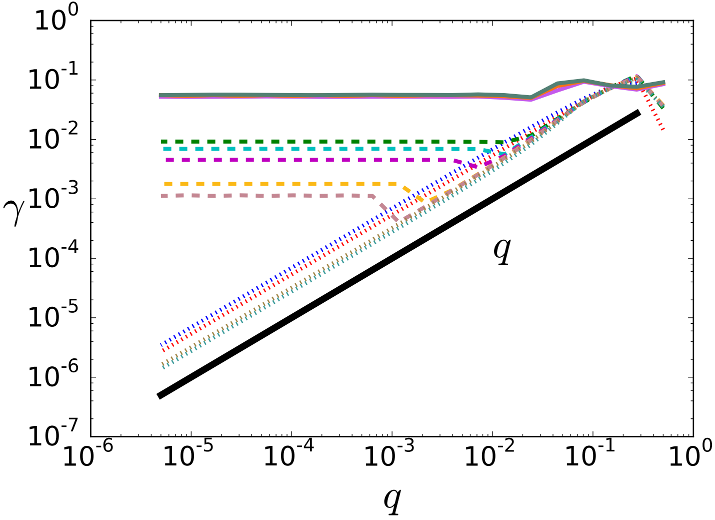
$$\mathscr{L}[\mathbf{B}_{ev}] = \lambda_{ev} \mathbf{B}_{ev} \quad \Longrightarrow \quad \mathbf{B}_{ev}(t) = \mathbf{B}_{ev}(t=0)\, e^{\lambda_{ev} t} . \tag{10}$$

Maximal eigenvalue: $\gamma = \max_i(\{\lambda^i_{ev}\})$.

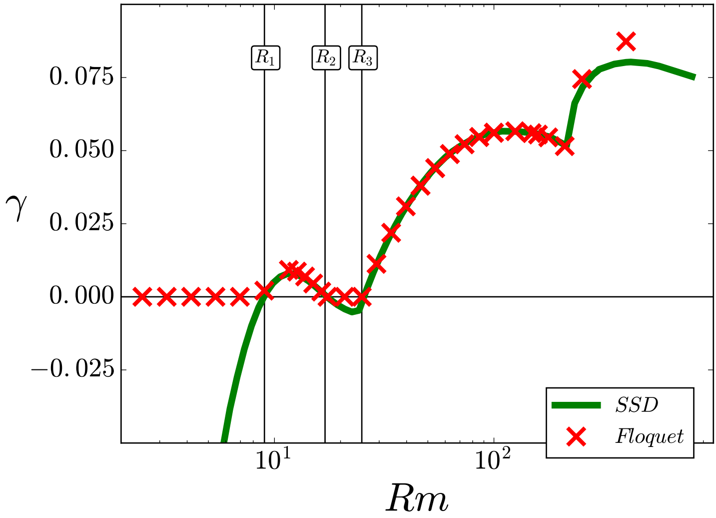
Random start

$$B(t=0) = \sum_i b_i B^i_{ev} \quad \Longrightarrow B(t) \underset{t \to \infty}{\simeq} b_\gamma B_\gamma e^{\gamma t} . \tag{11}$$

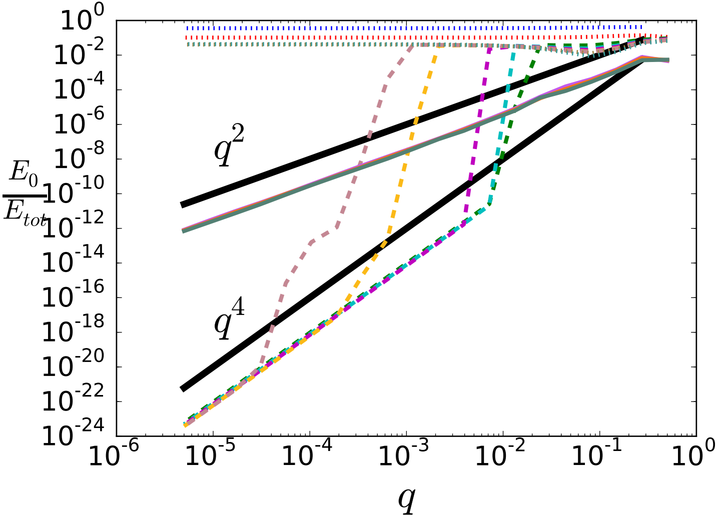
Our results: $\gamma(q)$



Our results: $\gamma(Rm)$



Our results: $E_0 / E_{tot}(q, Rm)$



Thank you for your attention

Multi-scale expansion (Method 1)

Use

- Theoretical demonstration of existence of the phenomenon
- Exact conditions on the velocity to have generate an instability
- Valid at $Rm \ll 1$ (Reminder: for the Sun $Rm = 10^{11}$)

Method

- $B(x_1, x_2, t_1, t_2)$
- $\partial_t \rightarrow \partial_{t_1} + Rm^{-4}\partial_{t_2}$
- $\partial_x \rightarrow \partial_{x_1} + Rm^{-2}\partial_{x_2}$

Mean field theory (Method 2)

Use

- Can compute numerical values of the α-coefficient
- Requires to know the statistic of the small scale b
- Dissociates large and small scales
- Valid at large Rm

Method

- $\mathbf{B} = \langle B \rangle + b$

Use

- Can compute numerical values of the α-coefficient
- Introduces an scale separation parameter $\mathbf{q}$
- <u>Links small scales with large scales</u>
- <u>Valid at any *Rm*</u>

Method

- $\mathbf{B} = \tilde{\mathbf{b}} \exp\left(-i\mathbf{q}\cdot\mathbf{r}\right) + c.c.$

Mean field

The induction equation governing the evolution of magnetic field:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \,. \tag{12}$$

Mean field theory decomposition: $\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$

$$\partial_t \langle \mathbf{B} \rangle \quad - \eta \nabla^2 \langle \mathbf{B} \rangle \quad = \nabla \times \mathscr{E} \,, \qquad \mathscr{E} = \langle \mathbf{u} \times \mathbf{b} \rangle \,, \tag{13}$$

$$\partial_t \mathbf{b} \quad - \eta \nabla^2 \mathbf{b} \quad = \nabla \times (\mathbf{u} \times \langle \mathbf{B} \rangle) + \nabla \times \mathbf{G} \,, \qquad \mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle \,. \tag{14}$$

When $\mathbf{G}$ can be neglected:

$$\mathscr{E}^i = \alpha^{ij} \langle B \rangle^j + \beta^{ijk} \nabla^j \langle B \rangle^k + \ldots \,. \tag{15}$$

Floquet analysis

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \,. \tag{16}$$

$$\mathbf{B} = \tilde{\mathbf{b}} \exp\left(-i\mathbf{q} \cdot \mathbf{r}\right) + c.c. \tag{17}$$

$$\partial_t \tilde{\mathbf{b}} = i\mathbf{q} \times (\mathbf{u} \times \tilde{\mathbf{b}}) + \nabla \times (\mathbf{u} \times \tilde{\mathbf{b}}) + \eta(\nabla + i\mathbf{q})^2 \tilde{\mathbf{b}} \,. \tag{18}$$

Lenz-Faraday law and Maxwell equation

Lenz-Faraday law

$$\mathcal{E} = \oint \mathbf{E} \cdot \mathbf{d}\boldsymbol{\ell} \quad ; \quad \Phi = \iint \mathbf{B} \cdot \mathbf{dS} \quad \text{and} \quad \mathcal{E} = -\frac{d}{dt}\Phi. \tag{19}$$

Without motion

$$\mathcal{E} = -\frac{d}{dt}\Phi \quad \Longleftrightarrow \quad \iint \nabla \times \mathbf{E} \cdot \mathbf{dS} = -\iint \partial_t \mathbf{B} \cdot \mathbf{dS}, \tag{20}$$

$$\boxed{\nabla \times \mathbf{E} = -\partial_t \mathbf{B}}. \tag{21}$$

With motion on the x axis

$$\partial_t(\mathbf{B} \cdot \mathbf{dS}) = \partial_t \mathbf{B} \cdot \mathbf{dS} + \mathbf{B} \cdot \partial_t(\mathbf{dx} \times \mathbf{dy}) = \partial_t \mathbf{B} \cdot \mathbf{dS} + (\mathbf{B} \times \mathbf{dv}) \cdot \mathbf{dy}, \tag{22}$$

$$\boxed{\nabla \times \mathbf{E} = -\partial_t \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})}. \tag{23}$$

Temporal evolution (1)

Maxwell's equations

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J} + \epsilon_0 \partial_t \mathbf{E}) \simeq \mu_0 \mathbf{J} \quad ; \quad \nabla \cdot \mathbf{B} = 0\,.$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})\,. \quad ; \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} = 0\,.$$

Ohm's law and approximation

$$\mathbf{J} = \sigma \mathbf{E} \quad ; \quad \mathbf{J} \gg \epsilon_0 \partial_t \mathbf{E}.$$

Temporal evolution (2)

Maxwell's equations

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = \mu_0 \sigma \mathbf{E} \quad ; \quad \nabla \cdot \mathbf{B} = 0\,.$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})\,. \quad ; \quad \nabla \cdot \mathbf{E} = 0\,.$$

Ohm's law and approximation

$$\mathbf{J} = \sigma \mathbf{E} \quad ; \quad \mathbf{J} \gg \epsilon_0 \partial_t \mathbf{E}\,.$$

Temporal evolution (3)

Maxwell's equations

$$\nabla \times \mathbf{B} = \mu_0 \sigma \mathbf{E} \quad ; \quad \nabla \cdot \mathbf{B} = 0\,.$$

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{v} \times \mathbf{B}) = -\nabla \times \mathbf{E}\,. \quad ; \quad \nabla \cdot \mathbf{E} = 0\,.$$

Ohm's law and approximation

$$\mathbf{J} = \sigma \mathbf{E} \quad ; \quad \mathbf{J} \gg \epsilon_0 \partial_t \mathbf{E}\,.$$

Evolution of the magnetic field

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{v} \times \mathbf{B}) = -\nabla \times \mathbf{E} = -\nabla \times (\mu_0 \sigma)^{-1} \nabla \times \mathbf{B}\,,$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mu_0 \sigma)^{-1} \Delta \mathbf{B} \quad \text{and} \quad \nabla \cdot \mathbf{B}\,.$$