Large scale effects in Turbulence

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Large-scale instabilities of helical flows

System

$$\partial_t \boldsymbol{u} = \nu \Delta \boldsymbol{u} + \boldsymbol{F} - \left((\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} + \boldsymbol{\nabla} P \right) \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 . \tag{1}$$

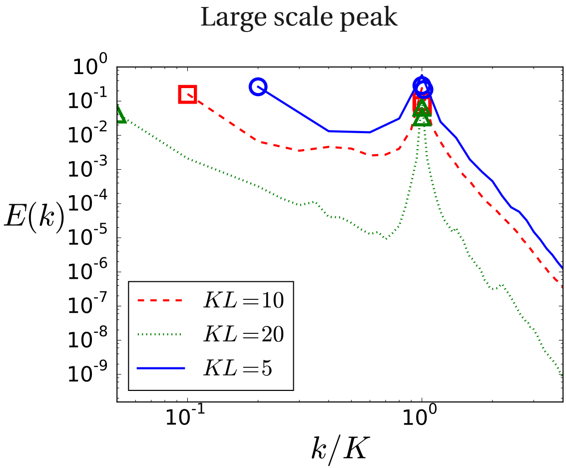
Variables

- $\boldsymbol{u}$: velocity field
- p: pressure field
- $\boldsymbol{F}$: force field
- ν: viscosity

Flow regime

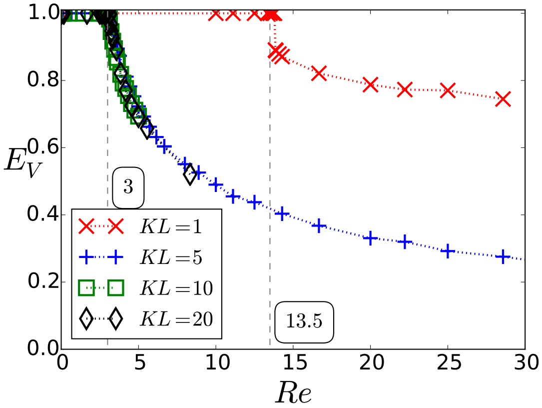
- Parameter: $Re = U\ell/\nu \sim 1$: laminar
- Helical flow consequence: $\left((\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} + \boldsymbol{\nabla} P \right) \ll \nu \Delta \boldsymbol{u}$

Non-linear: spectrum



Non-linear: bifurcation

Large scale bifurcation for sufficient scale separation



Linear Floquet transformation

Change of variable

$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{v} \quad \text{and} \quad \boldsymbol{v} = \widetilde{\boldsymbol{v}} e^{i\boldsymbol{q}\boldsymbol{r}} + c.c.. \tag{2}$$

Floquet evolution equation

$$\partial_t \widetilde{\boldsymbol{v}} = (\nabla \times \boldsymbol{U}) \times \widetilde{\boldsymbol{v}} + ((\imath \boldsymbol{q} + \nabla) \times \widetilde{\boldsymbol{v}}) \times \boldsymbol{U} - (\imath \boldsymbol{q} + \nabla)\widetilde{p} + \nu(-\boldsymbol{q}^2 + \Delta)\widetilde{\boldsymbol{v}} \tag{3}$$

$$0 = \imath \boldsymbol{q} \cdot \widetilde{\boldsymbol{v}} + \nabla \cdot \widetilde{\boldsymbol{v}}. \tag{4}$$

Linear Floquet: AKA theory

Property

- AKA: Anisotropic Kinetic Alpha effect
- Large scale instability happening at $Re \ll 1$.

Growth rate

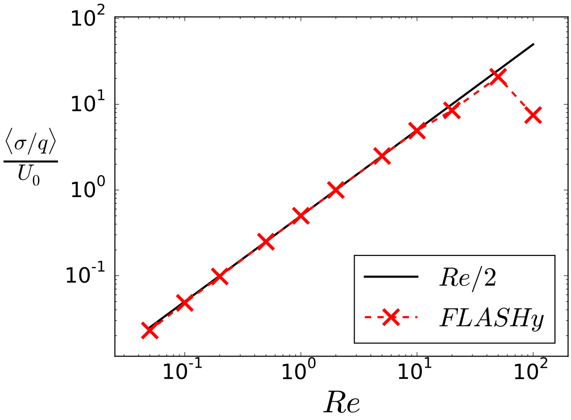
$$\widetilde{\boldsymbol{v}}(t) = \widetilde{\boldsymbol{v}}(0)e^{\sigma t} \quad \text{with} \quad \sigma = \alpha q - \nu q^2 \,, \tag{5}$$

AKA

$$\alpha \propto Re U_0 \quad \text{thus} \quad \frac{\langle \sigma / q \rangle}{U_0} \propto Re. \tag{6}$$

Linear Floquet results: AKA

Test on the flow from Frisch *et al.*, Physica D 1987



Linear Floquet results: Negative viscosity

Growth rate

$$\tilde{\boldsymbol{v}}(t) = \tilde{\boldsymbol{v}}(0)\, e^{\sigma t} \quad \text{with} \quad \sigma = \beta q^2 - \nu q^2 = (\beta - \nu) q^2 \,. \tag{7}$$

Measurement

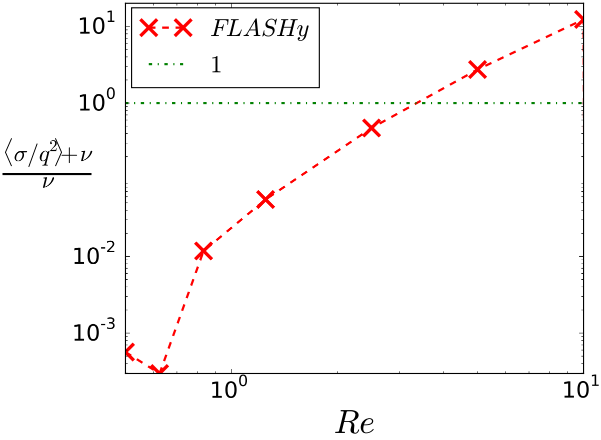
$$\beta \propto Re^2 \nu \quad \text{and} \quad \beta = \frac{\langle \sigma / q^2 \rangle + \nu}{\nu} \tag{8}$$

Threshold Independence on q

- All modes are unstable when $\beta > \nu$
- Identical critical Reynolds number for every scale separation

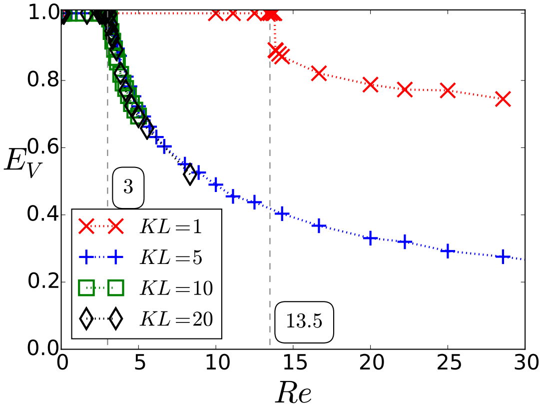
Linear Floquet results: ABC flow

Study of the ABC flow which is AKA-stable



Match linear and non-linear theory

Large scale bifurcation for sufficient scale separation



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Fate of Alpha Dynamos at Large Rm

Evolution equation

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \Delta \boldsymbol{B} \quad \text{and} \quad \nabla \cdot \boldsymbol{B} = 0 . \tag{9}$$

Variables

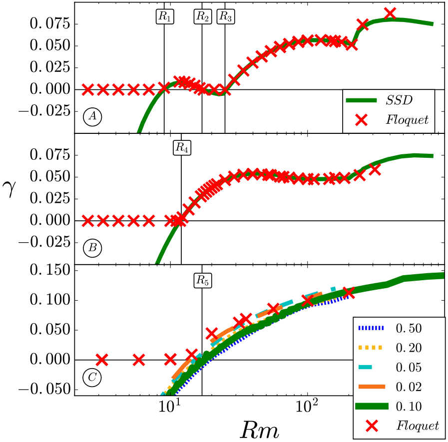
- $\boldsymbol{u}$: velocity field
- $\boldsymbol{B}$: magnetic field
- η: magnetic diffusivity

Flow regime

- Parameter: $Rm = U\ell/\eta \sim 1$
- Growth rate: $b(t) = b(t=0)e^{\gamma t}$

Growth rate results

Small Scale dynamo v. large scale dynamo



Growth rate explanation

Without scale separation

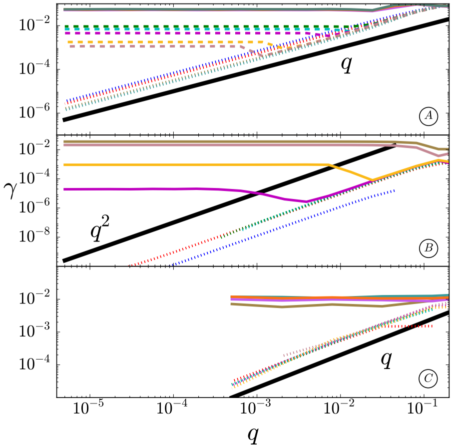
- Before small scale instability $Rm < Rm^c$: $\gamma_{SSD} \leq 0$
- After small scale instability $Rm^c < Rm$: $\gamma_{SSD} > 0$

With scale separation

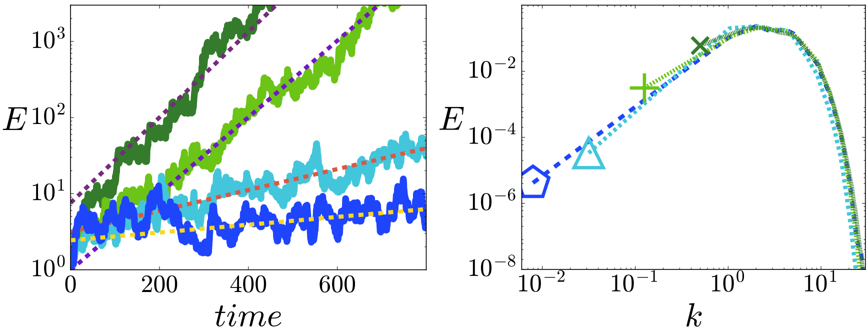
- Before small scale instability $Rm < Rm^c$: $\gamma = \alpha q - \eta q^2$
- After small scale instability $Rm^c < Rm$: $\gamma = \gamma_{SSD} > 0$

Growth rate

Small Scale dynamo v. scale separation



Method


10^3
E 10^2
10^1
10^0
0
200
400
600
$time$
10^{-2}
E 10^{-4}
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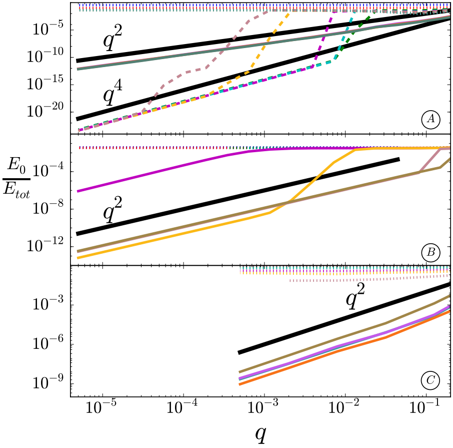
10^{-2}
10^{-1}
10^0
10^1
k

Energy distribution



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Overview

Name

- GHOST: **G**eophysical **H**igh **O**rder **S**uite for **T**urbulence
- TYGRES: **T**a**Y**lor-**GRE**en, **S**ymmetric
- FLASHy: **F**loquet **L**inear **A**nalysis of **S**pectral **Hy**drodynamics

Main features

- **Language**: Fortran 99
- **Paralization**: MPI, open MP
- **Type of DNS**: HD, MHD and GPE
- **Numeric method**: pseudo-spectral method with RK scheme
- **Geometry**: cubic
- **Boundary**: periodic

Equations

Navier-Stokes equation

$$\partial_t \boldsymbol{u} = \nu \Delta \boldsymbol{u} + \boldsymbol{F} - \left((\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} + \boldsymbol{\nabla} P \right) \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0. \tag{10}$$

- $\boldsymbol{u}$: velocity field.
- ν : viscosity.
- $\boldsymbol{F}$: force field.
- P : reduced pressure.

The real pressure is $p = \rho_0 P$ with ρ_0 the density.

Numeric implementation

Navier-Stokes equation

$$\partial_t \boldsymbol{u} = \nu \Delta \boldsymbol{u} + \boldsymbol{F} - \Big((\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} + \boldsymbol{\nabla} P \Big) \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 . \tag{11}$$

- Use Runge-Kutta method to perform the time evolution.
- Compute linear terms $\nu \Delta \boldsymbol{u} + \boldsymbol{F}$ in Fourier space.
- Compute non linear terms $\Big((\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} + \boldsymbol{\nabla} P \Big)$ in physical space.

Fourier Transform

Bottleneck

Back and Forth Fourier transform

Optimization

- Use Fast Fourier Transform package (FFTW): $\mathcal{O}(N \ln N)$.
- Reorder the data by slabs to compute.
- Use 1/3 truncation to avoid aliasing.

Data

Output

For a simulation in a cube with an $N \times N \times N$ resolution.

- **Global quantities**: energy, helicity, enstrophy, etc. . $\mathcal{O}(1)$
- **Spectral quantities**: energy spectrum, helicity spectrum. $\mathcal{O}(N)$
- **“Isotropic quantities”**: velocity sheets. $\mathcal{O}(N^2)$
- **Restart**: velocity block. $\mathcal{O}(N^3)$

Isotropic quantities: velocity sheets

Definition

Data coming from the plane at $k_x = 0$, $k_y = 0$, $k_z = 0$

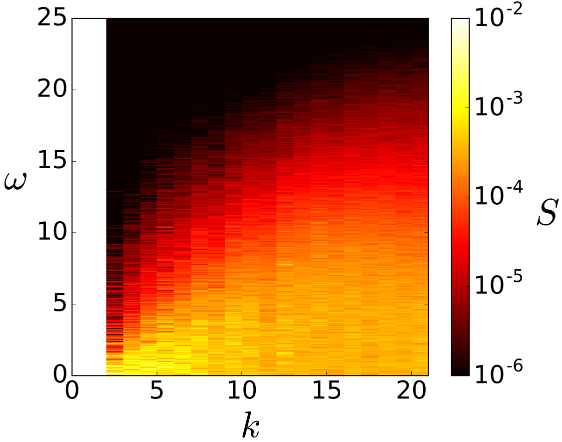
Purpose

If the quantity is isotropic, we can guess the value in the bulk.

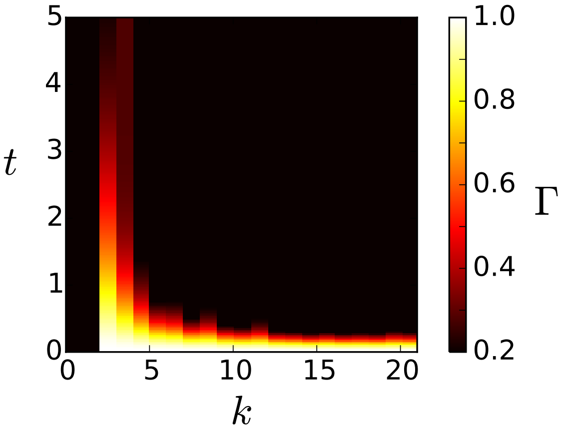
Application

Store the data at frequent time-steps
then compute spatio-temporal spectrum.
Measure temporal properties of flows.

Power-spectrum



Correlation function



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Conclusion

Beyond the AKA instability

- Results of the Fr87 flow at $Re \sim 1$.
- Analysis of AKA-stable flow.
- Observation of the non-linear consequence.

Property of dynamo instabilities at the small-scale threshold

- New model to describe the distribution of energy.
- Test on three flows (helical, non-helical, random).
- New description of the growth rate at large Rm.

Large scale effect in turbulence $Re > 1$

Large scales properties of turbulence

- Use power-spectrum to measure the correlation time of the modes of velocity.
- Build a semi-analytic model.
- Compare with existing theory.

Thanks you for your attention

Publication

- Large-scale instabilities of helical flows,
  Phys. Rev. Fluids 1, 063601
  – Published 3 October 2016
- Fate of Alpha Dynamos at Large Rm,
  Phys. Rev. Lett. 117, 205101
  – Published 8 November 2016
- The effect of helicity on the correlation time of large scale turbulent flows,
  arXiv:1705.05281

Thesis defense

Friday the 7^{th} of July at the Physics department of ENS in Conf IV.