# Spatio-temporal spectrum

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19th December, 2016

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# Non-linear physics group LPS ENS

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## École Normale Supérieure

Multiple départements (Mathématiques, Physique, Chimie, Géoscience, Biologie *etc.*)

## Département de Physique

• LPT • LKB • LPA • LRA

## Laboratoire de Physique Statistique

(11 groupes) Théorie non-linéaire des instabilités et Physique non-linéaire

# Permanent faculty members

## Numerical

- Marc-Étienne BRACHET
- Alexandros ALEXAKIS

## Experimental

- Stéphane FAUVE
- François PÉTRÉLIS
- Christophe GISSINGER

# Topic and codes

## Topic

HD, MHD, superfluids, wave-turbulence, instabilities, chaos

## Codes

- Sphere: Parody
- Cylinder: Heracles
- Box: GHOST, TYGRES

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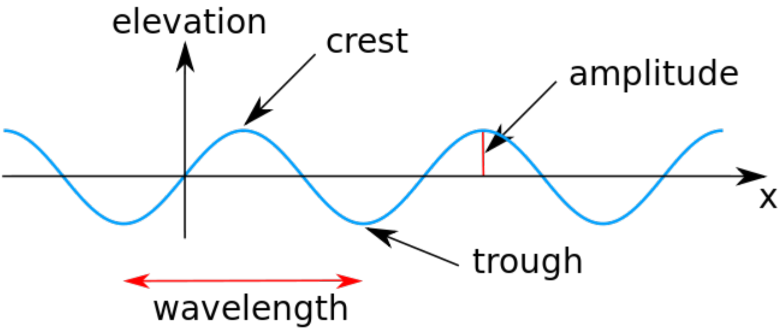
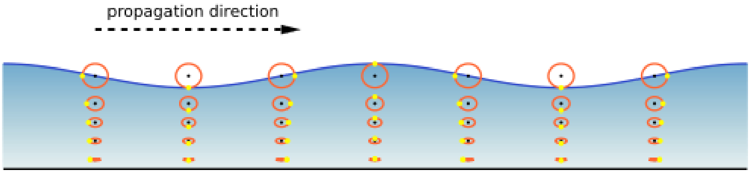
## Description



# Problem



# Equation

## Simplification

- Problem studied in the 2D $(x,z)$-plane.
- Potential flow: $\mathbf{u} = \boldsymbol{\nabla}\Phi\,.$

## Euler equation

- Incompressibility: $\Delta\Phi = \partial_x^2\Phi + \partial_z^2\Phi = 0\,.$
- Bernoulli: $\partial_t\Phi(z=\eta) + g\eta = 0\,.$

## Euler equation

- Bottom boundary conditions: $\partial_z\Phi(z=-h) = 0\,.$
- Top boundary conditions: $\partial_t\eta = \partial_z\Phi(z=\eta).$

## Solution

### Height

$$\eta = a\cos(kx - \omega t)\,. \tag{1}$$

### Velocity potential

$$\Phi = a\frac{\omega}{k}\frac{\cosh(k(z+h))}{\sinh(kh)}\sin(kx - \omega t)\,. \tag{2}$$

### Dispersion relation

$$\omega^2 = gk\tanh(kh)\,. \tag{3}$$

# Analyzing the dispersion relation

## Deriving the dispersion relation

$$\partial_t \Phi(z=\eta) + g\eta = 0 \quad \text{and} \quad \partial_t \eta = \partial_z \Phi(z=\eta)\,, \tag{4}$$

$$0 = \left(\omega^2 - gk\tanh(kh)\right)\Phi(k,\omega)\,. \tag{5}$$

## Necessary condition

- if $\Phi(k,\omega) = 0$, $\left(\omega^2 - gk\tanh(kh)\right) \neq 0$ can be true.
- If $\left(\omega^2 - gk\tanh(kh)\right) = 0$ and $\Phi(k,\omega) \neq 0$ can be true.

## Spatio-temporal spectrum technique

Plot $\Phi(k,\omega)$ to find the dispersion relation.

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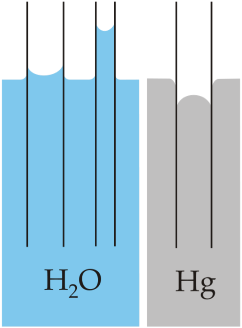
# Gravito-capillary dispersion relation

## Capillary effect

Energy related to surface tension: $E_c = \gamma S\,.$

## Dispersion relation

$$\omega^2 = \left(gk + \frac{\gamma}{\rho}k^3\right)\tanh(kh). \qquad (6)$$

# Reference

## Profilometry (ESPCI)

Pablo J. COBELLI, Agnès MAUREL, Vincent PAGNEUX, Philippe PETITJEANS
**Global measurement of water waves by Fourier transform profilometry.**
*Exp Fluids* 46:1037–1047 (2009) .

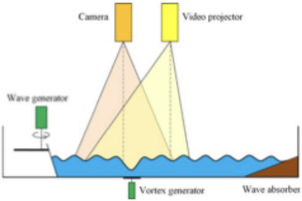
## Spatio-temporal spectrum (UBA)

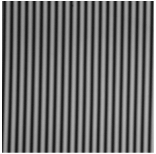
Patricio CLARK DI LEONI, Pablo J. COBELLI and Pablo D. MININNI
**The spatio-temporal spectrum of turbulent flows.**
*Eur. Phys. J. E* 38, 136 (2015).

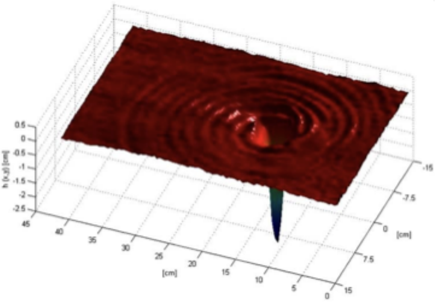
## Collaborator of University of Buenos Aires

- Patricio CLARK DI LEONI : PhD student of P. MININNI
- Pablo COBELLI : former post-doc at ESPCI (2 min. of LPS)
- Pablo MININNI : collaborator of A. ALEXAKIS and M.-É. BRACHET

# Profilometry set-up

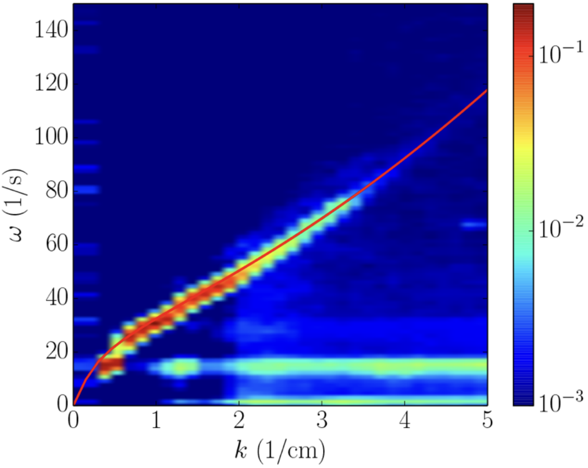


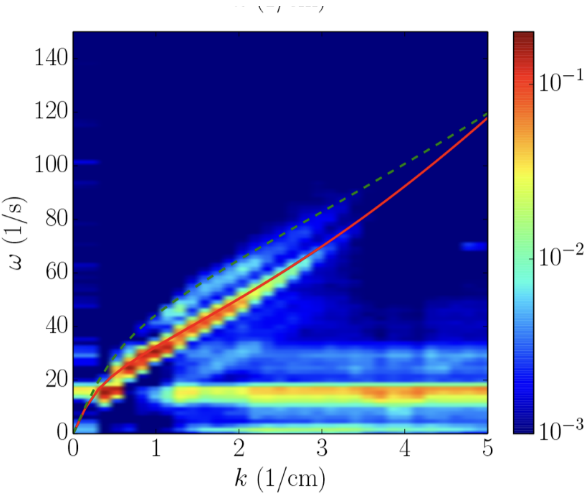


# Experimental set-up

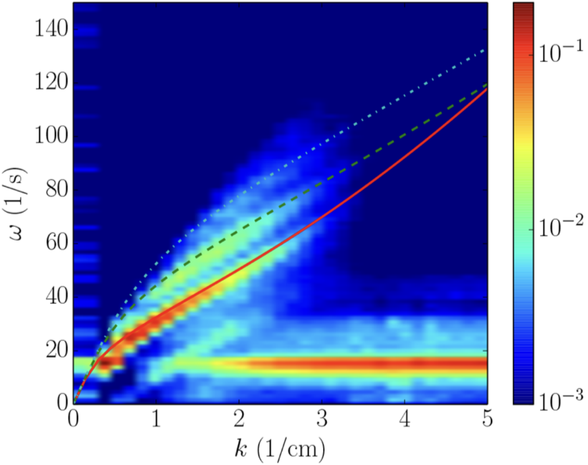
# Spatio-temporal spectrum



# Birth of bond waves



# Deep bond waves



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## Reference

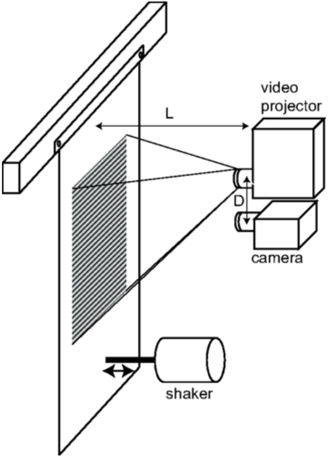
**Kolmogorov-Zakharov (LPS)**

Benjamin MIQUEL, Alexandros ALEXAKIS, Nicolas MORDANT

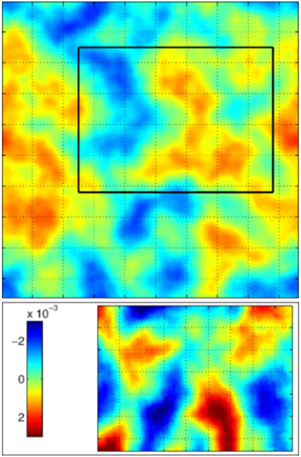
**Role of dissipation in flexural wave turbulence: From experimental spectrum to Kolmogorov-Zakharov spectrum.**

*Physical Review E* 89, 062925 (2014).

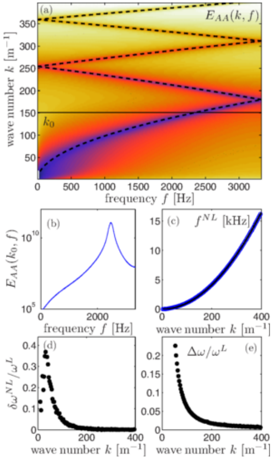
# Experimental set-up




# Thin plate deformation



# Dispersion relation



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# Navier-Stokes is non-linear

## Equations

Incompressibility condition: $\nabla \cdot \mathbf{v} = 0\,.$

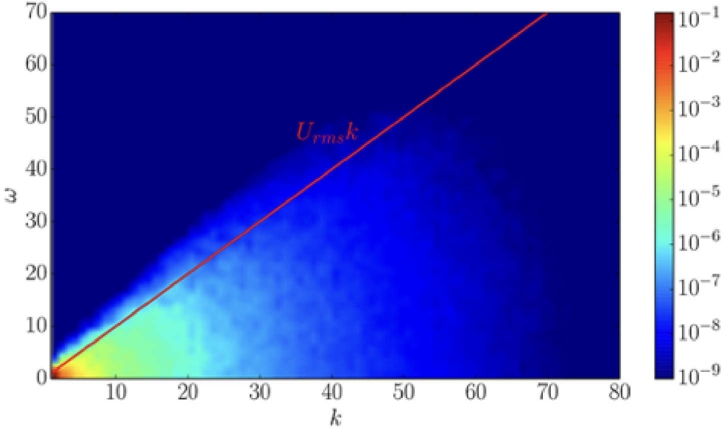
Equation of evolution, Navier-Stokes:

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P + Re^{-1} \Delta \mathbf{v} + \mathbf{F} \quad , \quad Re = \frac{VL}{\nu}\,. \tag{7}$$

## Variables

- $\mathbf{v}$ is the dimensionless velocity flow.
- $(\mathbf{v} \cdot \nabla)\mathbf{v}$ is the transport term.
- $\nabla P$ is the pressure term.
- $Re^{-1} \Delta \mathbf{v}$ is the viscous term.
- $\mathbf{F}$ is the dimensionless forcing.
- $Re$ is the Reynolds number.

# Turbulent spatio-temporal spectrum



# Wiener–Khinchin theorem

## Equation

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau}\, d\tau = F[R_X(\tau)]\,. \quad (8)$$

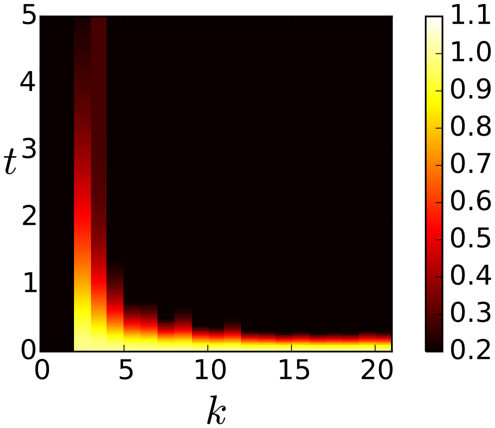
## Variables

- $R_X(\tau) = \langle X(t)X(t+\tau)\rangle_t$: auto-correlation function
- $S_X(f) = |F[X]|^2$: power spectral density

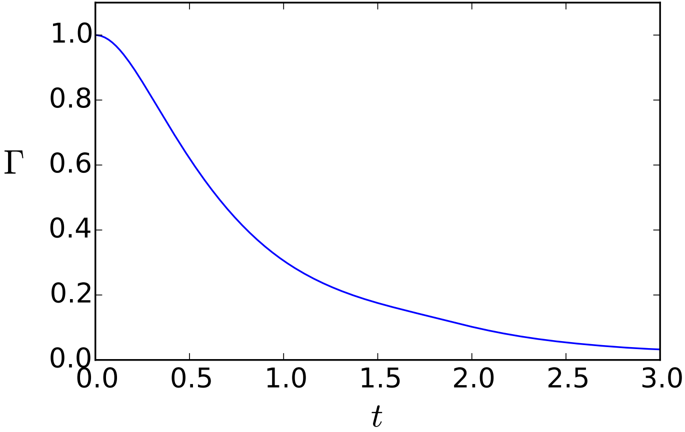
## Objectif

Quantify the correlation time in 3D HD turbulence

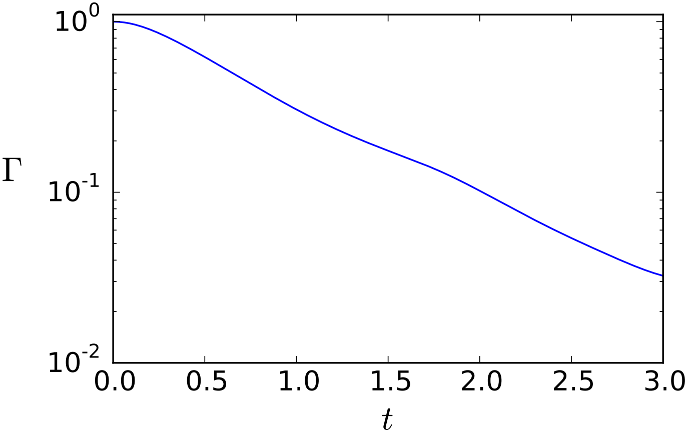
## Correlation time TE



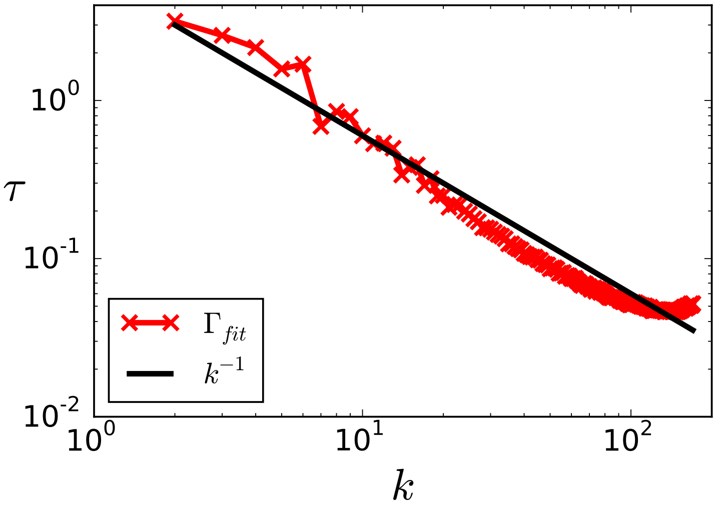
## Correlation function TE



# Correlation function TE (log)



## Scaling law



## Thank you for your attention