The fate of the α-dynamo at large Rm

Alexandre CAMERON†, Alexandros ALEXAKIS†
<`alexandre.cameron@ens.fr`>, <`alexakis@lps.ens.fr`>
† : LPS, Département de Physique, École Normale Supérieure, Paris, FRANCE

Introduction

At the heart of today's solar magnetic field evolution models lies the α-dynamo description. α-dynamos are based on the mechanism that velocity fields and magnetic fields of small wavenumber can interact and generate a large scale magnetic field [1]. Even if the analysis of the α-effect has been vastly studied at low Rm [2], a clear description of the effect at high Rm is still an open question in the scientific community.

Induction equation

The magnetic fields in a medium of magnetic diffusivity η ($Rm = UL/\eta$) follows the PDE:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \,. \tag{1}$$

Mean field description

Averaging over the large scales, the magnetic fields can be decomposed as $\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$. With this mean field decomposition, the induction equation now has two parts:

$$\partial_t \langle \mathbf{B} \rangle = \nabla \times \mathcal{E} + \eta \nabla^2 \langle \mathbf{B} \rangle \ , \ \mathcal{E} = \langle \mathbf{u} \times \mathbf{b} \rangle \,, \tag{2}$$

$$\partial_t \mathbf{b} - \eta \nabla^2 \mathbf{b} = \nabla \times (\mathbf{u} \times \langle \mathbf{B} \rangle) + \nabla \times \mathbf{G} \,, \tag{3}$$

where $\mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$. When $\mathbf{G}$ can be neglected, $\mathcal{E}$ can be expanded in the standard series of gradients of $\langle \mathbf{B} \rangle$:

$$\mathcal{E}^i = \alpha^{ij} \langle B \rangle^j + \beta^{ijk} \nabla^j \langle B \rangle^k + \ldots \,. \tag{4}$$

This decomposition has been used in many numerical models studying astro-dynamos [3].

Two-mode model objection

Linearly coupling the large scale mode b_q and the small scale b_k via an α-effect and a small scale dynamo growth rate γ_{SSD} gives:

$$\begin{aligned} \dot{b}_q &= -\eta q^2 \quad b_q \quad +\alpha q \quad b_k \,, \\ \dot{b}_k &= \alpha k \quad b_q \quad +\gamma_{SSD} \quad b_k \,. \end{aligned} \tag{5}$$

Computing the eigenvalue, two main regimes can be identified : (i) If $\gamma_{SSD} < 0$, the largest eigenvalue is $\gamma \simeq \alpha^2 kq / |\gamma_{SSD}|$ and $b_q/b_k \simeq (|\gamma_{SSD}|/\alpha k) = \mathcal{O}(1)$. (ii) If $\gamma_{SSD} > 0$, the largest eigenvalue is $\gamma \simeq \gamma_{SSD}$ and $b_q/b_k \simeq (\alpha q / |\gamma_{SSD}|)) = \mathcal{O}(q/k)$.

Floquet decomposition

Floquet decomposition [4] also gives access to the large scale mode using the *ansatz*:

$$\mathbf{B}(\mathbf{x}, t) = e^{i\mathbf{q}\cdot\mathbf{x}} \tilde{\mathbf{b}}(\mathbf{x}, t) + c.c. \,. \tag{6}$$

The induction eq. (1) can then be rewritten:

$$\partial_t \tilde{\mathbf{b}} = i\mathbf{q} \times (\mathbf{u} \times \tilde{\mathbf{b}}) + \nabla \times (\mathbf{u} \times \tilde{\mathbf{b}}) + \eta (\nabla + i\mathbf{q})^2 \tilde{\mathbf{b}} \,. \tag{7}$$

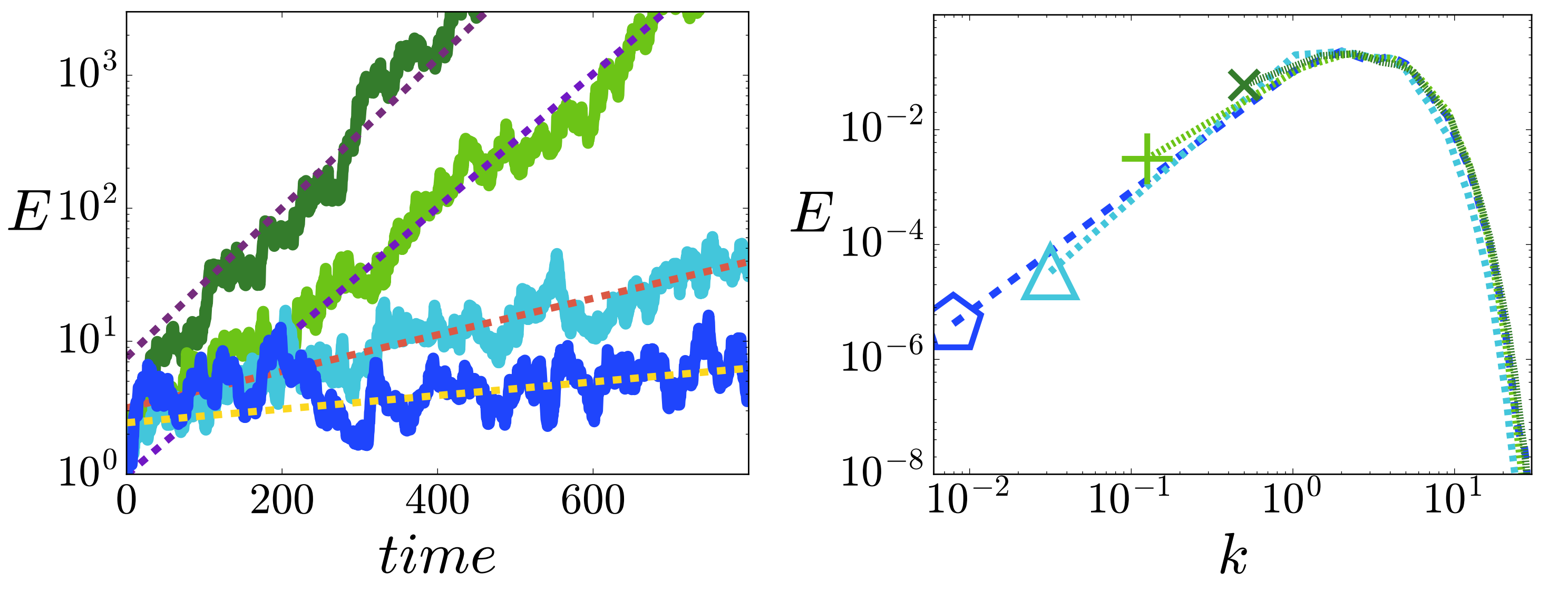
The Floquet wave-vector $\mathbf{q}$ introduced is related to the separation of scales. $\mathbf{q}$ is a major asset for numeric simulations because intermediate scales do not have to be solved anymore.

Acknowledgements

The present work benefited from the support of GENCI-TGCC-CURIE & GENCI-CINES-JADE (Project No. x2014056421, No. x2015056421 & No. x20162a7620) and MesoPSL financed by Region Ile de France and project EquipMeso (reference ANR-10-EQPX-29-01).

Analyzing Floquet results

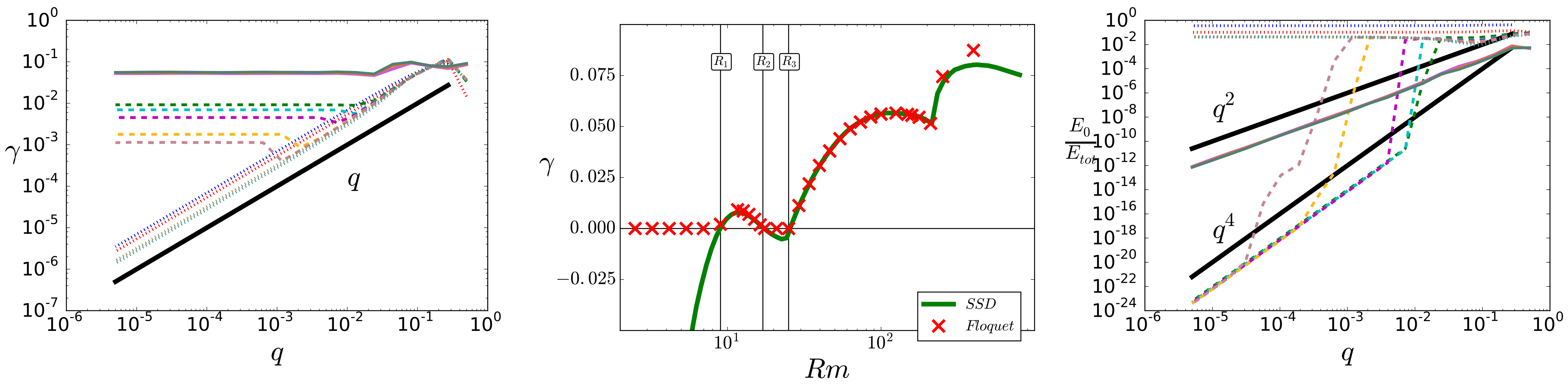
Standard characteristics of the magnetic fields can easily be computed on the Floquet field. Once processed, the temporal evolution of the energy gives access to the growth rate γ and the Fourier spectrum gives access the ratio of energy in the large scale E_0/E_{tot}.



ABC flow

The ABC flow was introduced to MHD studies by Childress [5] because its helical property: $\nabla \times \mathbf{u} = k\mathbf{u}$ makes it a good generator of dynamos. The ABC flow is defined by:

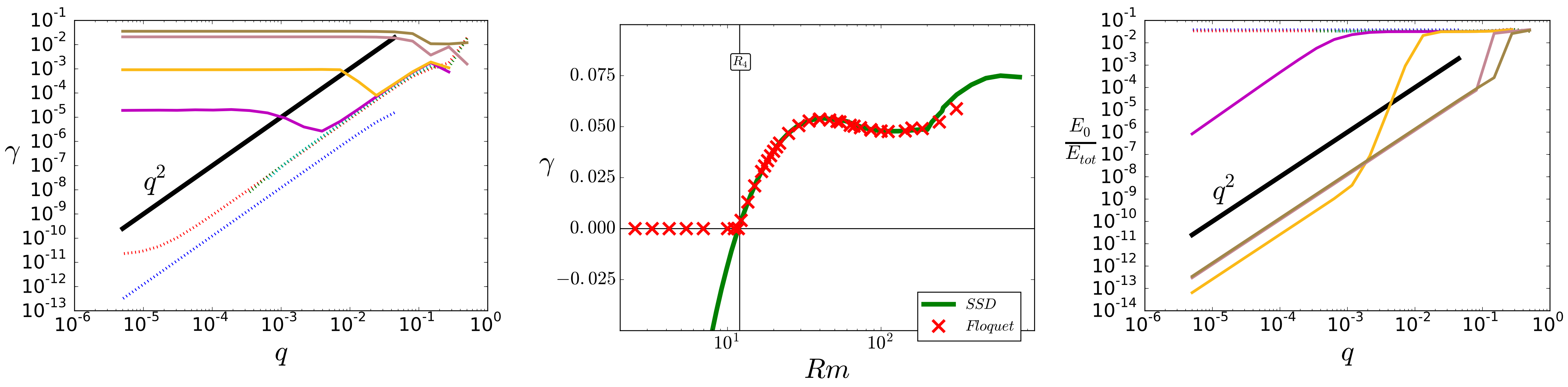
$$u_x = U_0 \sin(kz) + U_0 \cos(ky) \ , \ u_y = U_0 \sin(kx) + U_0 \cos(kz) \ , \ u_z = U_0 \sin(ky) + U_0 \cos(kx) \,. \tag{8}$$



Non-Helical ABC flow

The same transition occurs in the case of the following non-helical ABC flow:

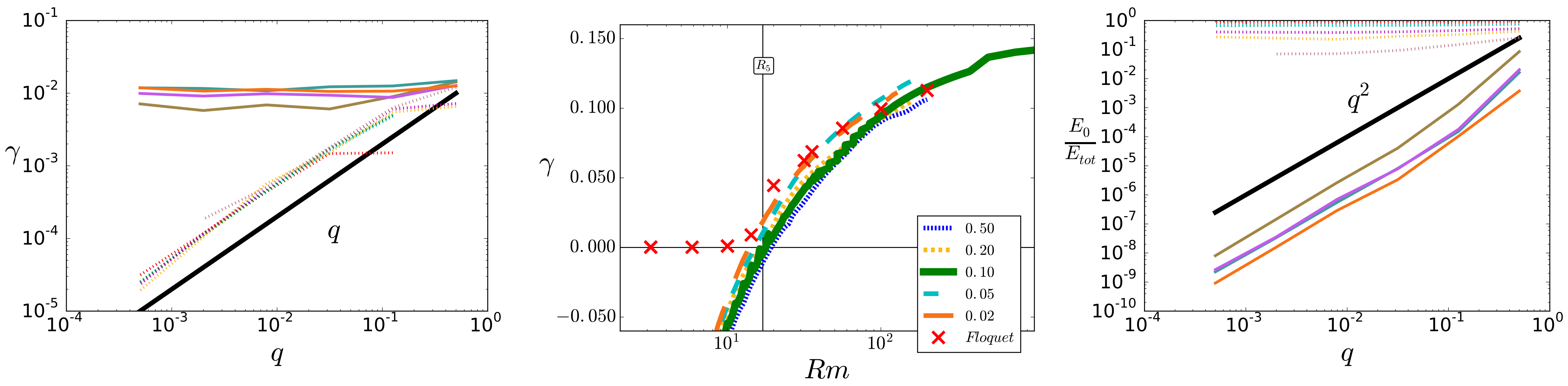
$$u_x = U_0 \left[\sin(kz) + \sin(ky)\right] \ , \ u_y = U_0 \left[\sin(kx) + \sin(kz)\right] \ , \ u_z = U_0 \left[\sin(ky) + \sin(kx)\right] \,. \tag{9}$$



Random ABC flow

The last set of simulations was carried out using an helical ABC flow with random phases $\phi_{x,y,z}$.

$$u_x = U_0 \left[\sin(kz + \phi_z) + \cos(ky + \phi_y)\right] \ , \ u_y = U_0 \left[\sin(kx + \phi_x) + \cos(kz + \phi_z)\right] \ , \ u_z = U_0 \left[\sin(ky + \phi_y) + \cos(kx + \phi_x)\right] \,. \tag{10}$$



Conclusion

As reported in [6], above the small scale dynamo onset, the large scales grow with the same growth rate as the small scales but the large scale projection of the field decreases with scale separation.

References

[1] M. Steenbeck, F. Krause, and K.-H. Rädler. Berechnung der mittleren Lorentz-Feldstärke für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflußter Bewegung. *Zeitschrift für Naturforschung A*, 21(4):369–376, 1966.

[2] Paul Charbonneau. Dynamo Models of the Solar Cycle. *Living Reviews in Solar Physics*, 7(1):1–91, September 2010.

[3] M. Schrinner, K.-H. Rädler, D. Schmitt, M. Rheinhardt, and U. Christensen. Mean-field view on rotating magnetoconvection and a geodynamo model. *Astronomische Nachrichten*, 326(3-4):245–249, April 2005.

[4] Gaston Floquet. Sur les équations différentielles linéaires à coefficients périodiques. *Annales scientifiques de l'Ecole normale superieure*, 12:47–88, 1883.

[5] Stephen Childress. New Solutions of the Kinematic Dynamo Problem. *Journal of Mathematical Physics*, 11(10):3063–3076, October 1970.

[6] Alexandre Cameron and Alexandros Alexakis. The fate of alpha dynamos at large Rm. *arXiv:1607.07294 [astro-ph, physics:physics], accepted in PRL*, July 2016. arXiv: 1607.07294.