Helicity effects on large scale correlation time in turbulence

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Introduction

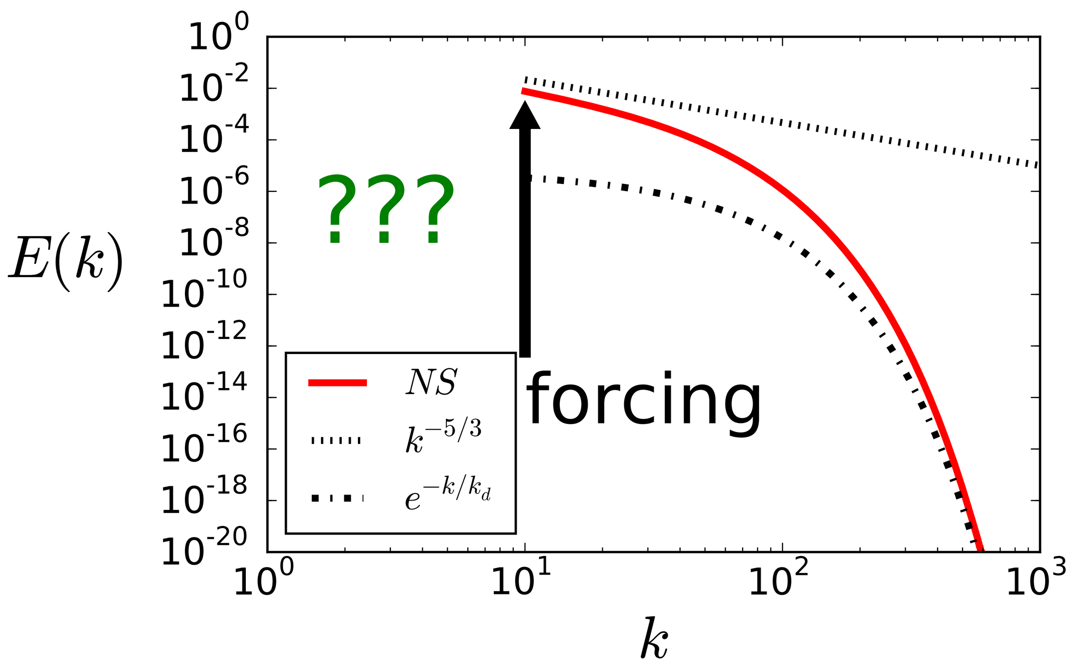
"Absolute equilibrium solutions seem highly unphysical in view of the approximately $k^{-5/3}$ spectrum of the three-dimensional turbulence. Actually, they are appropriate at the very smallest wave-numbers of turbulent flows maintained by forcing at intermediate wave-numbers."

— Uriel FISCH,

Turbulence : The Legacy of A. N. Kolmogorov, p. 209

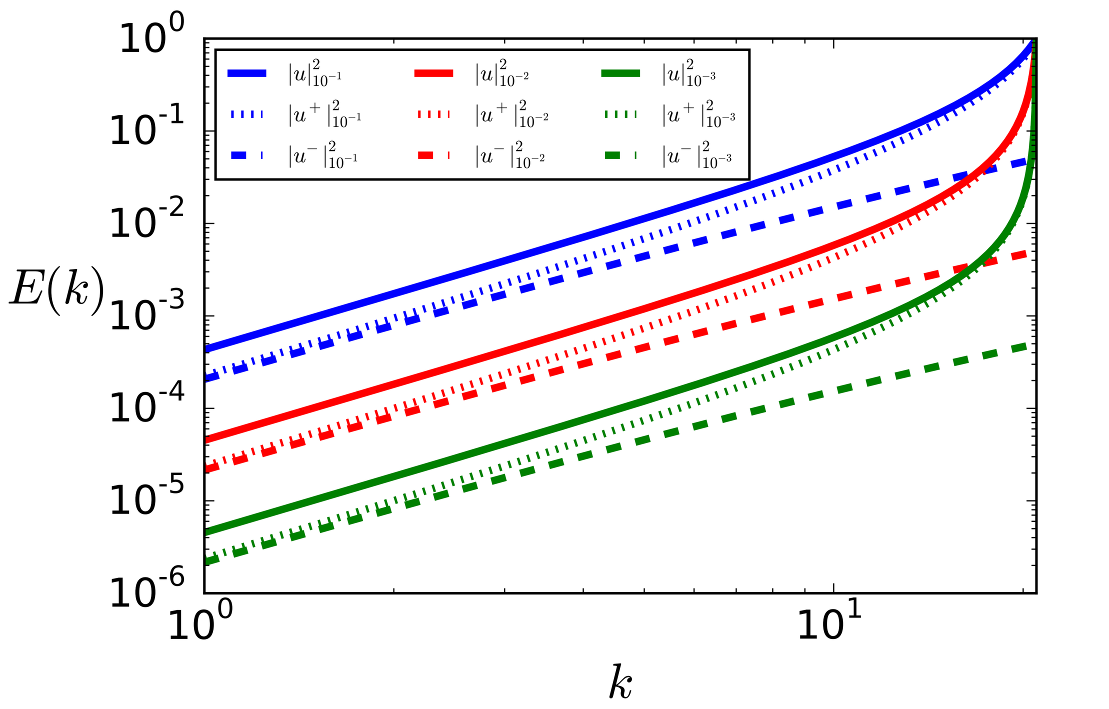
Navier-Stokes

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} P + \nu\Delta\boldsymbol{u} + \boldsymbol{F} \quad , \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$



Truncated Euler

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} P \quad , \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \quad , \quad \boldsymbol{u}_{k>k_M} = 0$$



The helical components of the flow can also be studied independently using

$$\boldsymbol{u}^{\pm}_{\boldsymbol{k}} = \boldsymbol{u}_{\boldsymbol{k}} \pm k^{-1}(\boldsymbol{\nabla}\times\boldsymbol{u})_{\boldsymbol{k}} \, .$$

They satisfy : $\boldsymbol{\nabla}\times\boldsymbol{u}^{\pm}_{\boldsymbol{k}} = \pm k \boldsymbol{u}^{\pm}_{\boldsymbol{k}}$.

Time-scale and correlation

- Governed by energy $\tau^E_k \propto \frac{1}{kU_{rms}} \propto \frac{1}{\sqrt{k^2 E}}$
- Governed by helicity $\tau^H_k \propto \frac{1}{\sqrt{kH}}$
- Parabolic time scale $\tau_k = \sqrt{\frac{\langle |\boldsymbol{u}_{\boldsymbol{k}}|^2\rangle}{\langle |\partial_t \boldsymbol{u}_{\boldsymbol{k}}|^2\rangle}}$

Measuring correlation times

1. Fourier transform: $\boldsymbol{u}(\boldsymbol{k},\omega) = \mathcal{DF}[\boldsymbol{u}(\boldsymbol{k}, n\Delta t)](\omega)$
2. Power spectrum: $s(\boldsymbol{k},\omega) = \sum_i |\boldsymbol{u}_i(\boldsymbol{k},\omega)|^2$
3. Shell average: $S(k,\omega) = \sum_{\boldsymbol{k}} \mathbf{1}(k - \frac{1}{2} < |\boldsymbol{k}| \leq k + \frac{1}{2}) s(\boldsymbol{k},\omega)$
4. Correlation function: $\gamma(k,t) = \mathcal{DF}^{-1}[S(k,\omega)](t)$
5. Normalized cor. func.: $\Gamma(k,t) = \gamma(k,t)/\gamma(k,0)$
6. Correlation time: $\tau(k) = Solve[t, \Gamma(k,t), 1/2]$

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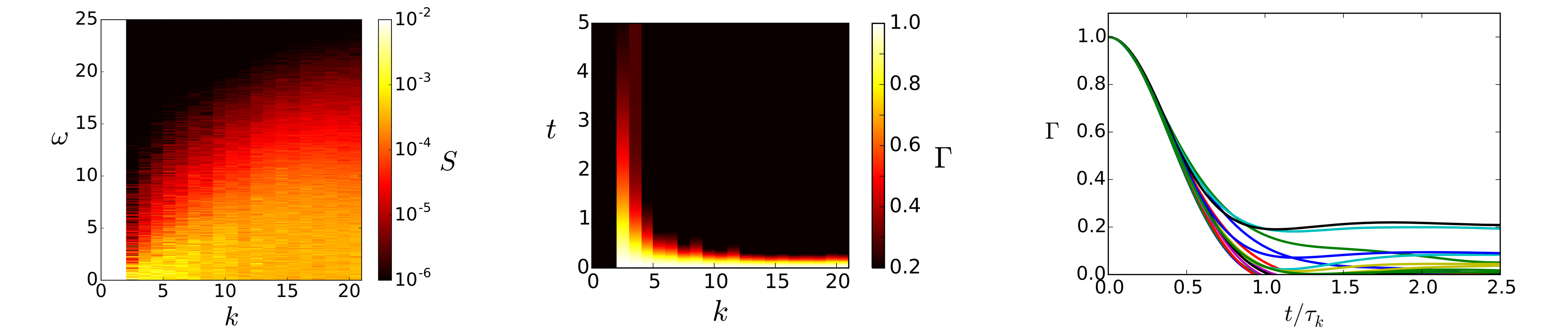
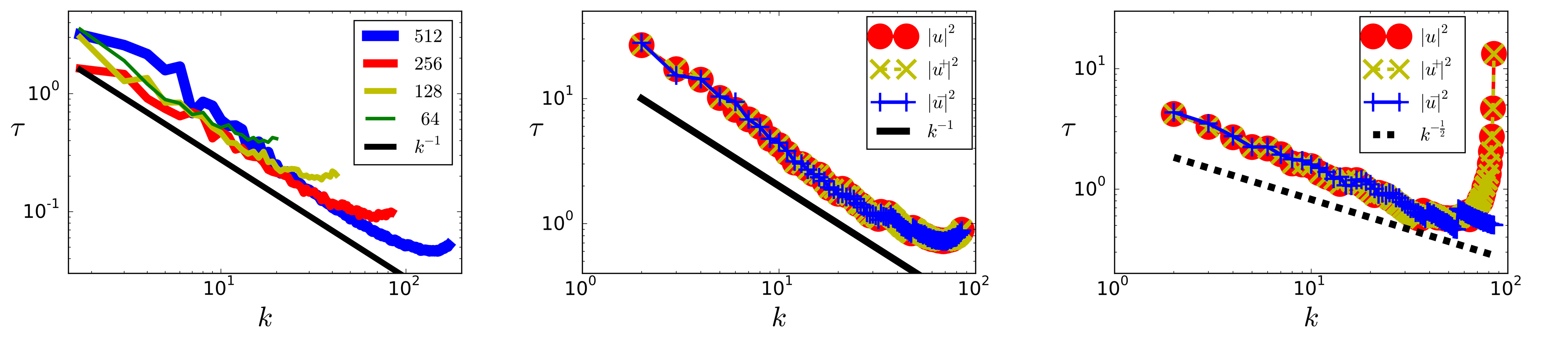
Steps of the correlation time procedure



(Left) Power spectrum $S(k,\omega)$, (Center) normalized correlation function $\Gamma(k,t)$, (Right) re-scaled correlation function $\Gamma(k,t/\tau_k)$ of the modes of Taylor-Green symmetric absolute equilibrium.

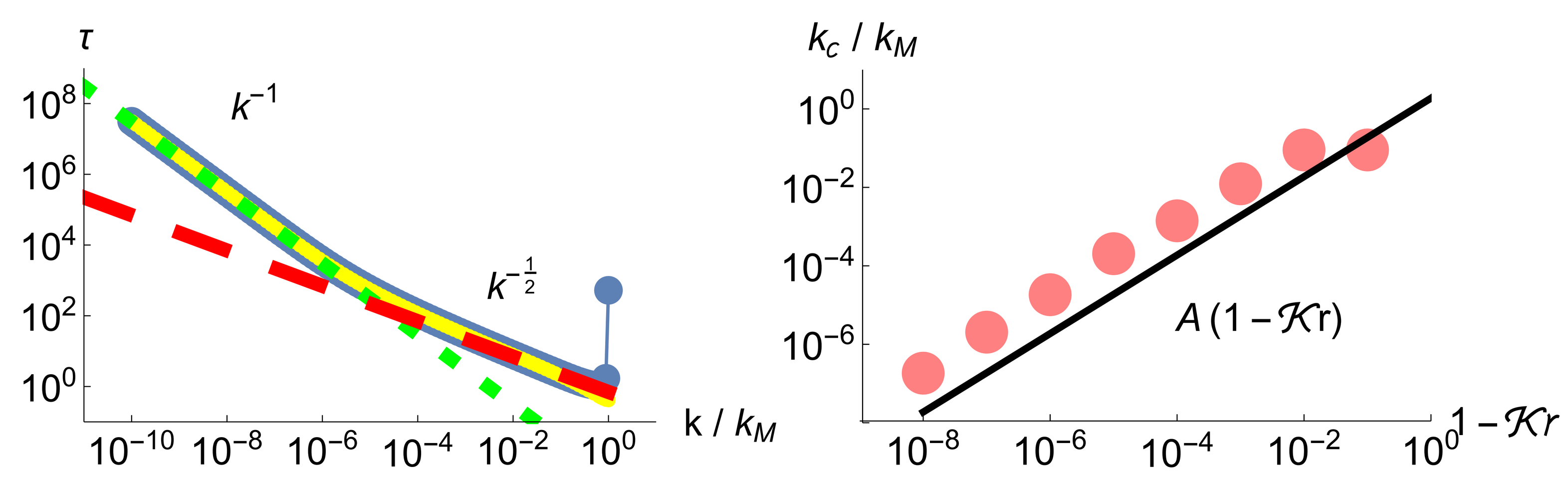
Truncated Euler DNS



Correlation time of: (Left) Taylor-Green symmetric absolute equilibrium, (Center) general non-helical absolute equilibrium, (Right) highly helical absolute equilibrium. Flows without helicity have a k^{-1}-power law characteristic of energy bases correlation times. Highly helical flows have a $k^{-\frac{1}{2}}$-power law characteristic of helicity bases correlation times.

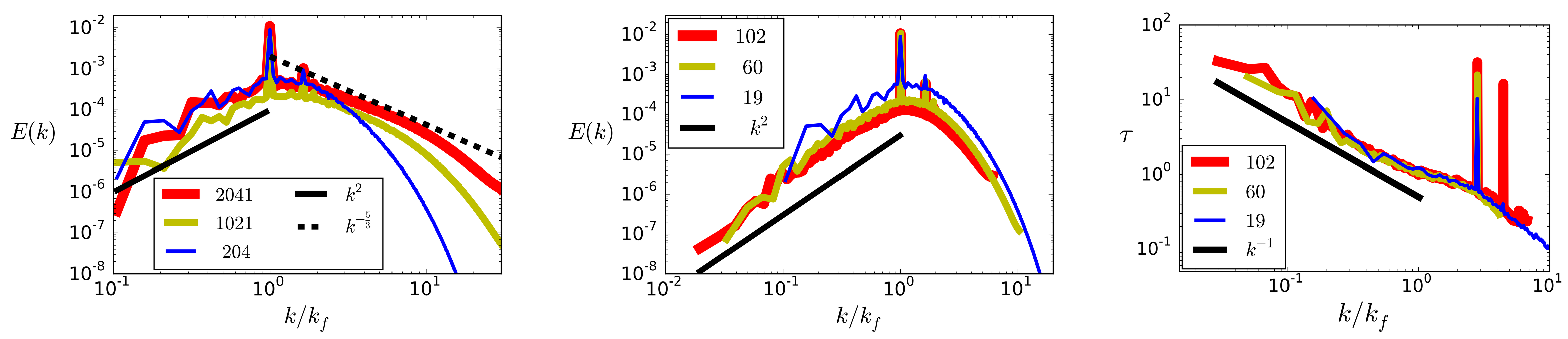
Truncated Euler model

Correlation time computed using the parabolic approximation of the correlation function. For highly helical flows, the correlation follows a k^{-1}-power law in the large scale, a $k^{-\frac{1}{2}}$-power law in the intermediate scale and diverges in the smallest scales.



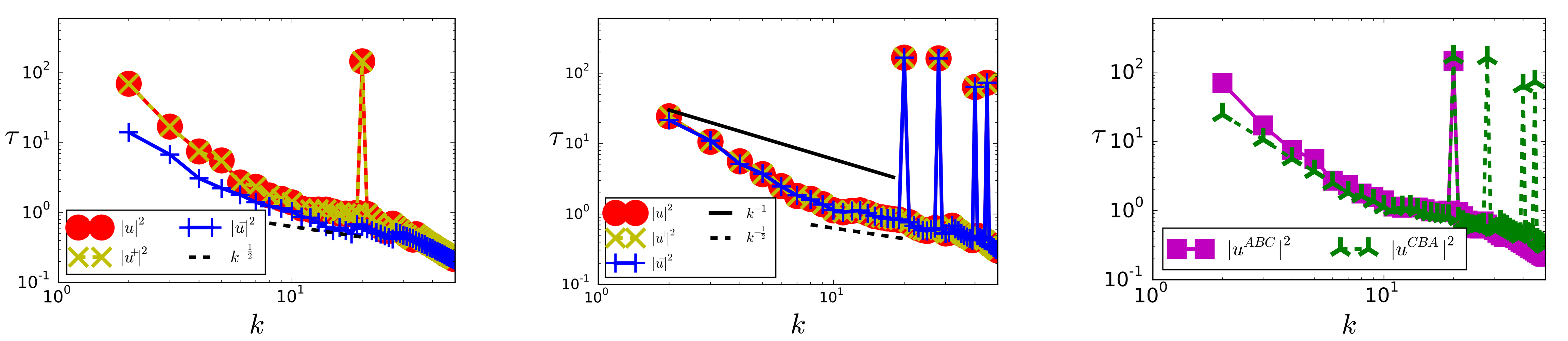
The wave-number where the correlation time changes scale, is represented as a function of $1-\mathcal{K}r$. $\mathcal{K}r$ measures the helicity of the flow. If $\mathcal{K}r = 1$, the energy is concentrated in the maximal wave-number.

Taylor-Green symmetric Navier-Stokes DNS



(Left) Energy spectrum for different Reynolds numbers. (Center) Energy spectrum for different scale separations at fixed ν/k_f. (Right) Correlation time for different scale separations at fixed ν/k_f.

Helical Navier-Stokes DNS



Correlation time of (Left) an ABC flow. (Center) a CBA flow. (Right) Comparison of the two flows.

References

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