Dipolar dynamos in spherical, anelastic simulations

Raphaël Raynaud, Ludovic Petitdemange, Emmanuel Dormy
raphael.raynaud@ens.fr

LERMA-LRA, Dép. de Physique, École normale supérieure, Paris

l'Observatoire de Paris — LERMA — Laboratoire d'Étude du Rayonnement et de la Matière en Astrophysique

École Normale Supérieure

Abstract

Dynamo action, i.e. the self-amplification of a magnetic field by the flow of an electrically conducting fluid, is considered to be the main mechanism for the generation of magnetic fields of stars and planets. Intensive and systematic parameter studies by direct numerical simulations using the Boussinesq approximation revealed fundamental properties of these models. However, this approximation considers an incompressible conducting fluid, and is therefore not adequate to describe convection in highly stratified systems like stars or gas giants. A common approach to overcome this difficulty is then to use the anelastic approximation, that allows for a reference density profile while filtering out sound waves for a faster numerical integration. We present the results of a systematic parameter study of spherical anelastic dynamo models, and compare them with previous results obtained in the Boussinesq approximation. We discuss the influence of the density stratification on the dipolar branch and its domain of stability. We also explain why the emergence of equatorial dipoles in anelastic simulations can result from the choice of a central mass distribution.

1. Anelastic MHD equations

- The reference state: decomposition of the thermodynamics variables into the sum of a steady variable corresponding to the reference atmosphere and a convective disturbance $f = f_a + f_c$. The reference state must be in mechanical and thermal quasiequilibrium, defined by :
  - hydrostatic balance $\boxed{-\nabla P_a + \rho_a \mathbf{g} = 0}$
  - "well-mixed" isentropic reference state $\boxed{\nabla S_a = 0}$

  Then, $\overline{P} = P_c\,\zeta^{n+1}$, $\overline{\varrho} = \varrho_c\,\zeta^n$, $\overline{T} = T_c\zeta$,

  with $\zeta = f\,(r, N_\varrho, n, \chi)$
- Navier-Stokes equation

$$D_t\mathbf{v} = Pm\Big[-\frac{1}{E}\nabla\frac{P'}{\zeta^n} + \frac{Pm}{Pr}Ra\frac{S}{r^2}\hat{\mathbf{r}} - \frac{2}{E}\hat{\mathbf{z}}\times\mathbf{v} + \mathbf{F}^\nu + \frac{1}{E\,\zeta^n}(\nabla\times\mathbf{B})\times\mathbf{B}\Big]$$

$$F_i^\nu = \zeta^{-n}\partial_j\left[\zeta^n\left((\partial_i v_j + \partial_j v_i) - \tfrac{2}{3}\delta_{ij}\partial_k v_k\right)\right]$$

- Induction equation

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla^2\mathbf{B}$$

- Heat transfer equation

$$D_t S = \zeta^{-n-1}\frac{Pm}{Pr}\nabla\cdot\left(\zeta^{n+1}\nabla S\right) + \frac{Di}{\zeta}\left[E^{-1}\zeta^{-n}(\nabla\times\mathbf{B})^2 + Q^\nu\right]$$

with $Q^\nu = 2\left[e_{ij}e_{ij} - \frac{1}{3}(\nabla\cdot\mathbf{v})^2\right]$

together with the constraints

$$\nabla\cdot(\zeta^n\mathbf{v}) = 0$$
$$\nabla\cdot\mathbf{B} = 0$$

Our numerical solver Parody reproduces the anelastic dynamo benchmark (Jones et al. 2011).

2. Control parameters

The system involves 7 dimensionless numbers:

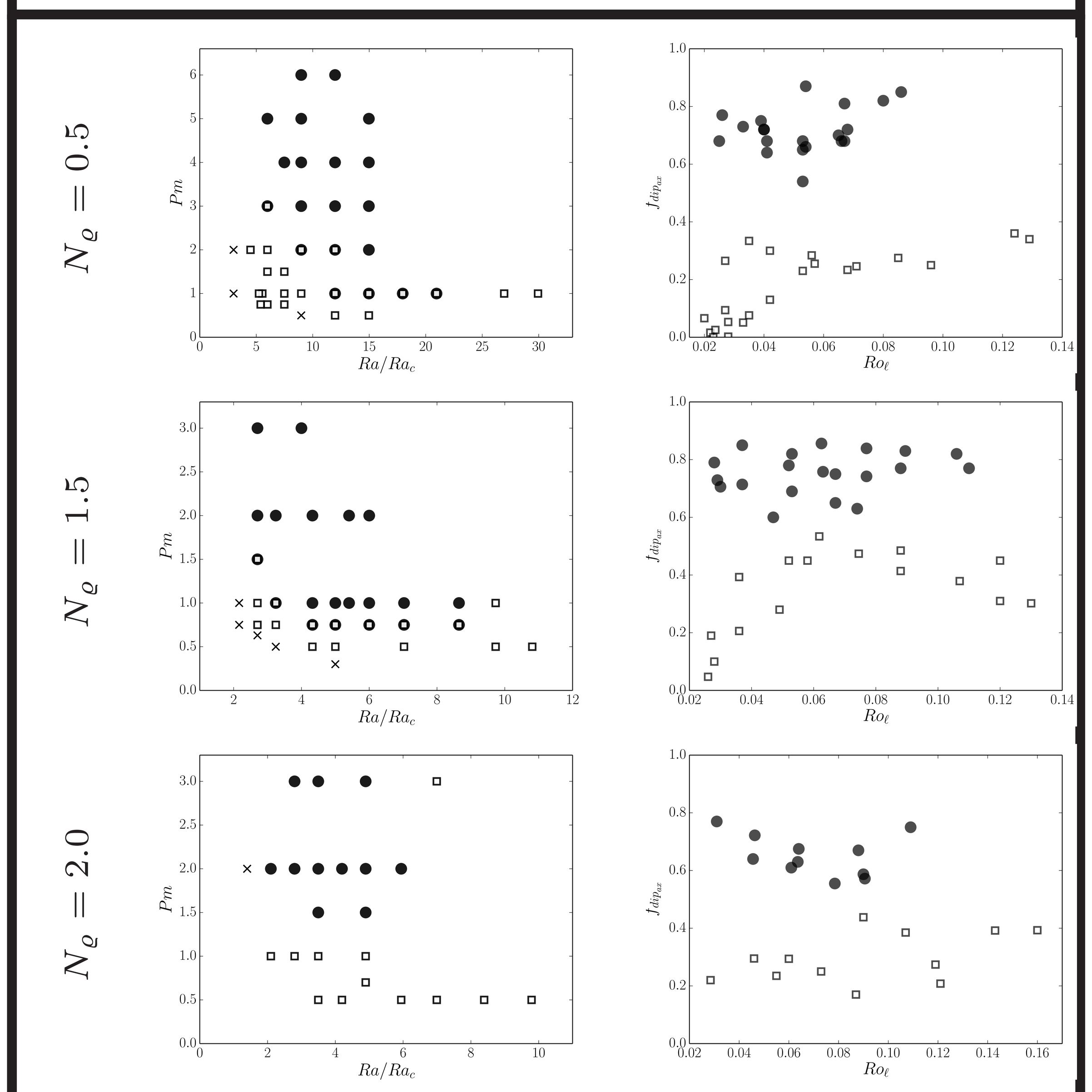
$$Ra = \frac{GMd\Delta S}{\nu\kappa c_p} \in [10^4, 10^7]\,,\quad Pr = \frac{\nu}{\kappa} \in [1,2]\,,$$

$$Pm = \frac{\nu}{\eta} \in [1,5]\,,\quad E = \frac{\nu}{\Omega d^2} \in [10^{-3}, 10^{-5}]\,,$$

$$N_\varrho = \ln\left(\frac{\varrho_i}{\varrho_o}\right) \in [0.1, 3.5]\,,$$

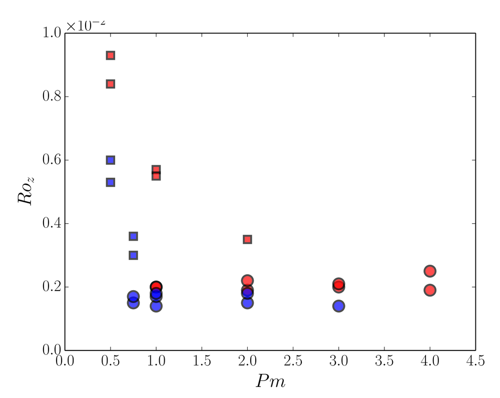
$$n = 2,\quad \chi = \frac{r_i}{r_o} \in [0.35, 0.60]\,.$$

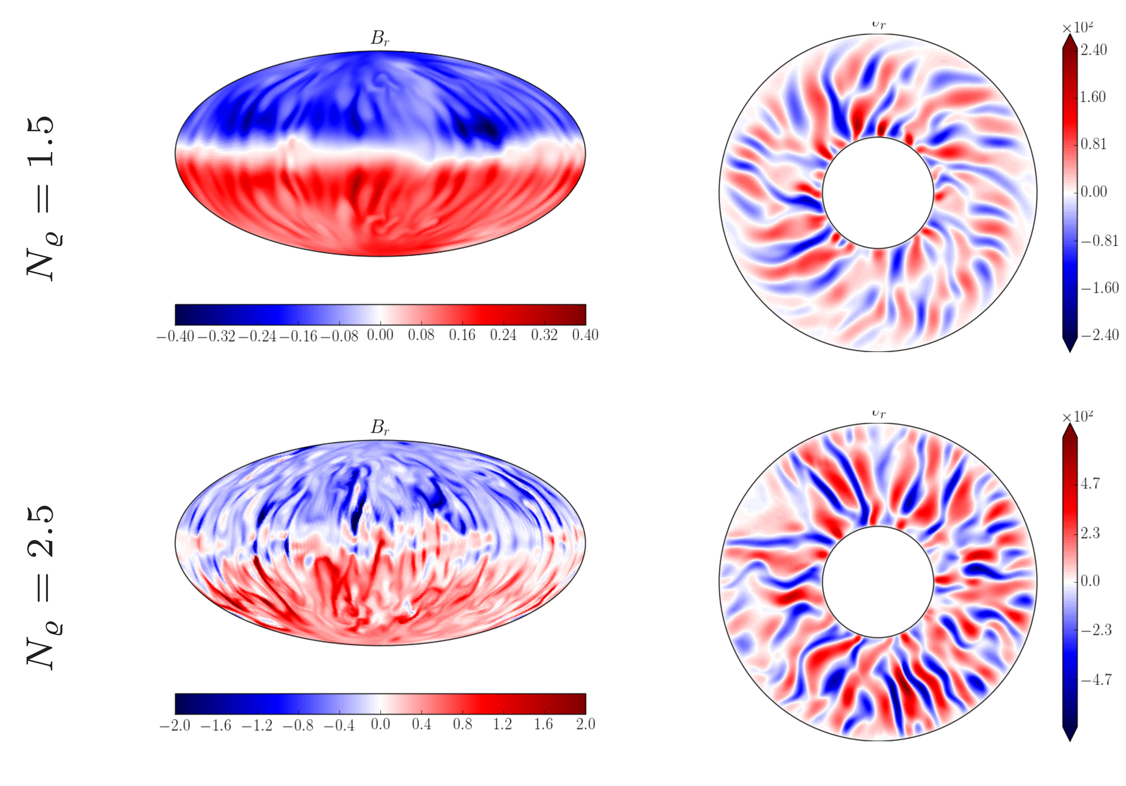
3. Bistability



Regime diagram $f_{dip_{ax}}$ vs. Ro_ℓ

Zonal wind:
Ro_z vs. Pm
(red: $N_\varrho = 0.5$
blue: $N_\varrho = 1.5$)

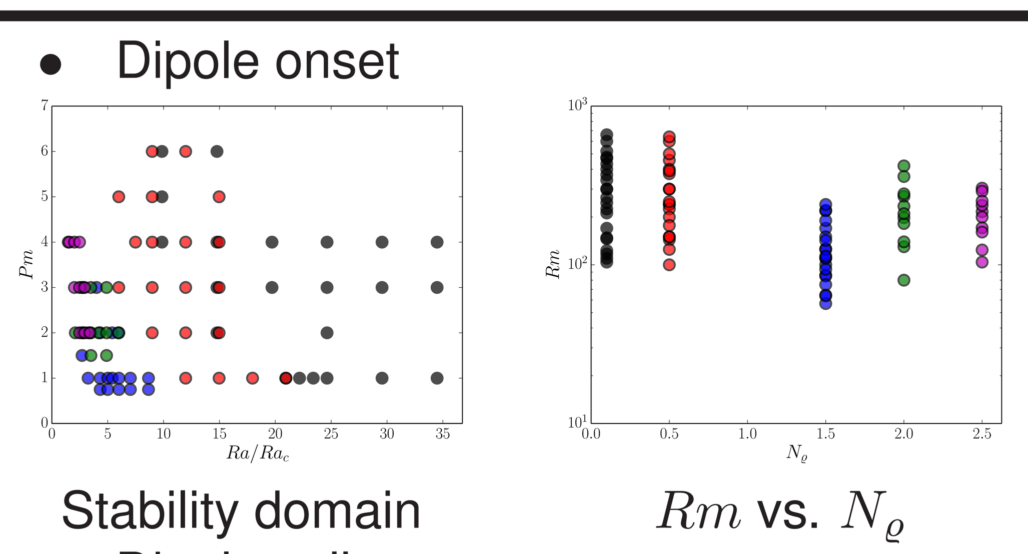




$B_r\,(r = r_o)$ v_r

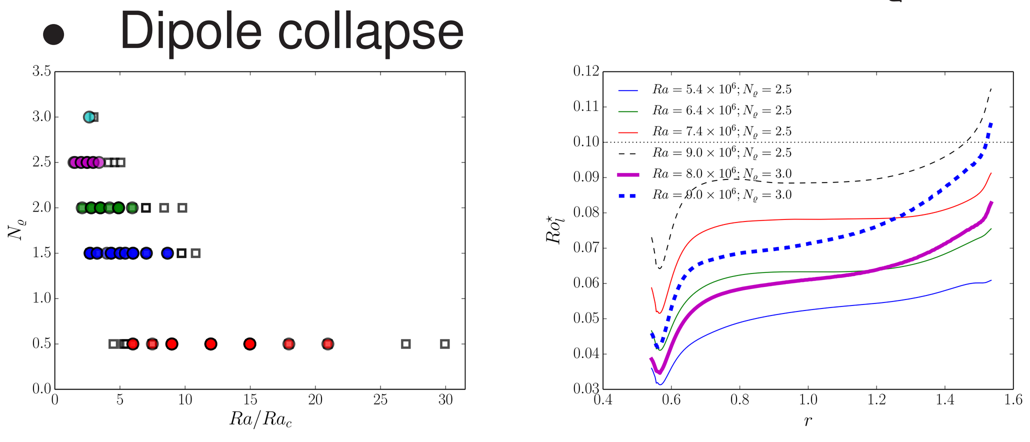
4. Dipole onset & collapse

- Dipole onset



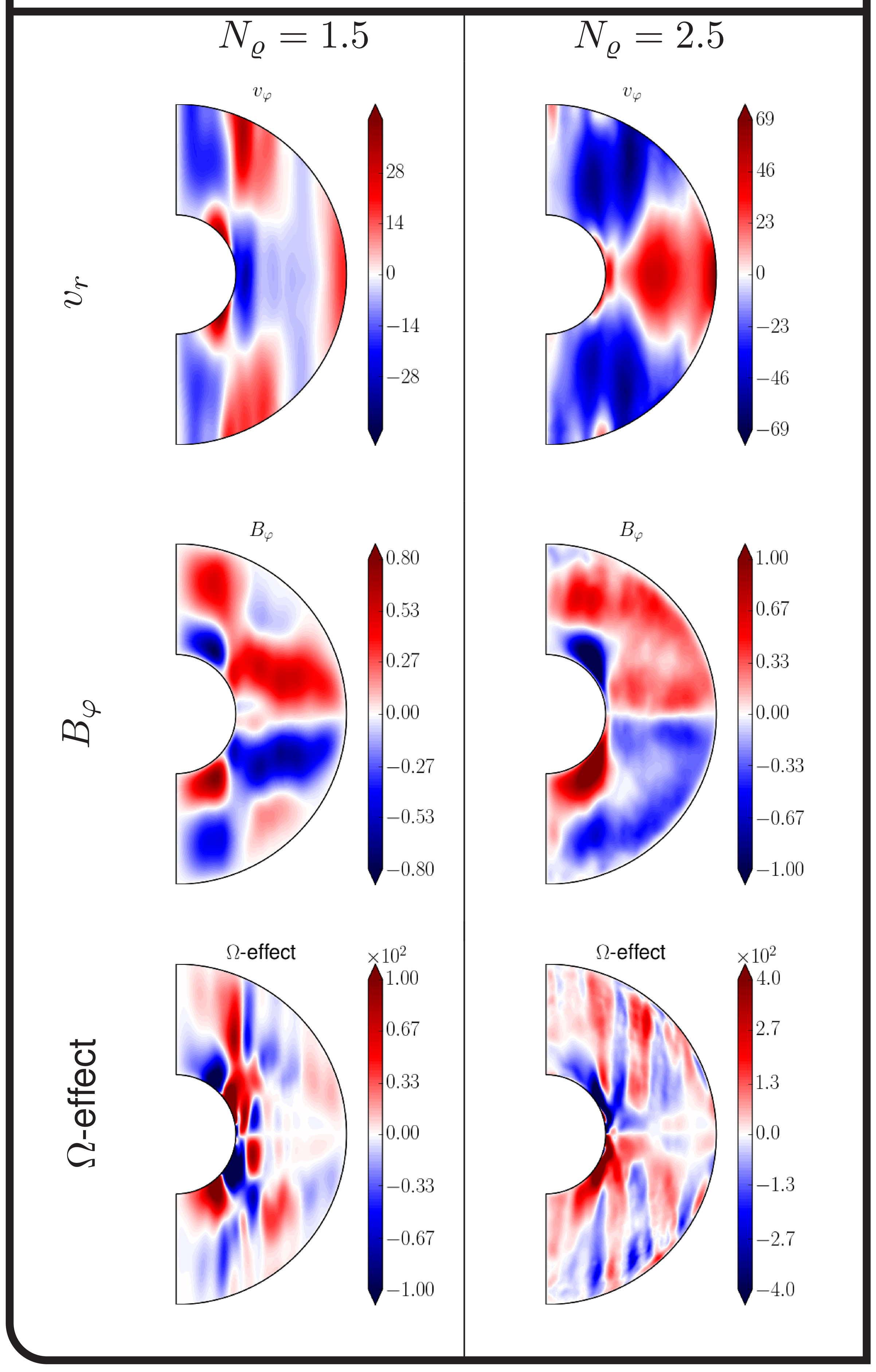
Stability domain Rm vs. N_ϱ

- Dipole collapse

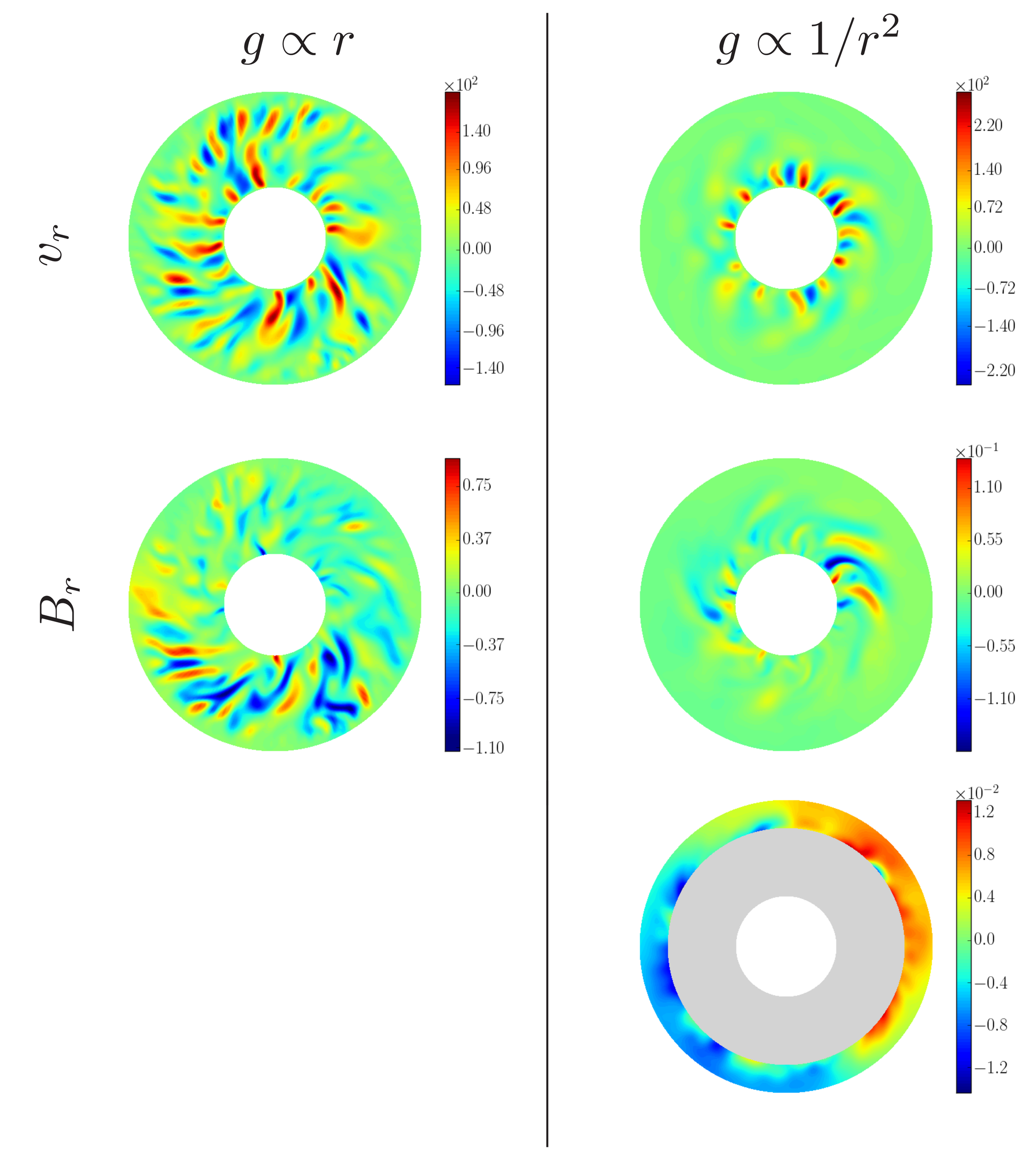


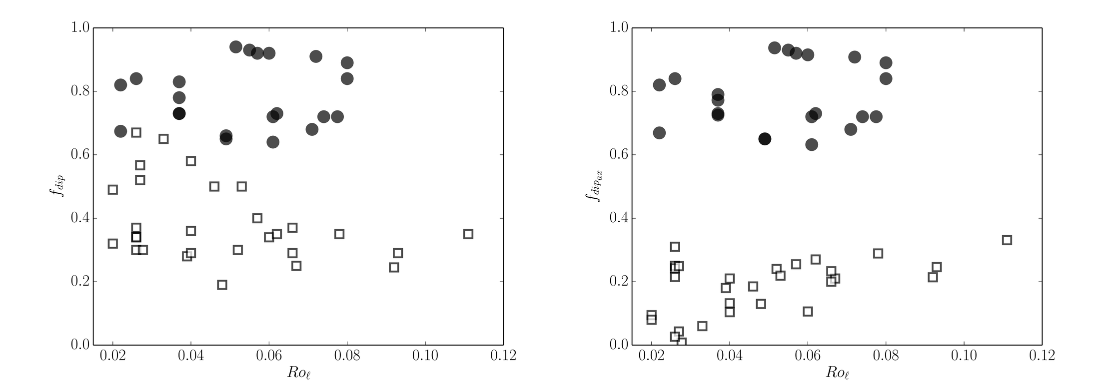
Ra/Ra_c vs. N_ϱ $Ro_\ell^\star(r)$

5. Dynamo mechanisms



6. Equatorial dipole





f_{dip} vs. Ro_ℓ $f_{dip_{ax}}$ vs. Ro_ℓ

Conclusions

1. dichotomy between dipolar and multipolar dynamos and their transition mechanisms extend to anelastic simulations
2. equatorial dipole not related to large N_ϱ
3. towards observations: M dwarfs (bistability), equatorial dipole

Publications:

- Raynaud et al., 2014, A&A, 567, A107
- Raynaud et al., 2014, MNRAS, in prep.
- Schrinner et al., 2012, APJ, 752, 121
- Schrinner et al., 2014, A&A, 564, A78