

Feuille de TD 1
Calcul d'intégrales simples

Exercice 1 :

Considérons l'intégrale

$$I = \int_0^{\ln 2} \sqrt{e^x - 1} dx.$$

Effectuer le changement de variable $u = \sqrt{e^x - 1}$ et calculer I . (Résultat : $I = 2 - \frac{\pi}{2}$).

Exercice 2 (Calculer les primitives) :

$$(1) \int \frac{1}{\sqrt{2+x} + (2+x)^{\frac{1}{5}}} dx \quad \left(t = (2+x)^{\frac{1}{5}}\right). \quad (2) \int (\arcsin x)^2 dx.$$

$$(3) \int \frac{1}{((x-1)^2 - 4)^2} dx \quad \left(\frac{x-1}{2} = \tan u \text{ ou } \coth u\right). \quad (4) \int x^2 \sqrt{1+x^3} dx.$$

Exercice 3 (Calculer les primitives) :

$$(1) \int e^x \cos x dx \quad (2) \int \frac{\ln x}{x^n} dx, n \in \mathbb{N} \quad (3) \int x \arctan x dx \quad (4) \int (x^2 + x + 1)e^x dx$$

Exercice 4 (Calculer les primitives) :

$$(1) \int (\cos x \cos 2x + \sin x \sin 3x) dx \quad (2) \int \cos x \sin^4 x dx \quad (3) \int \cos^6 x dx$$

$$(4) \int \sin^3 x \cos x dx \quad (5) \int \sin^4 x dx \quad (6) \int \sin^3 x \cos^2 x dx$$

$$(7) \int \operatorname{ch}^2 x \operatorname{sh}^2 x dx \quad (8) \int \operatorname{sh} x \operatorname{ch}^3 x dx \quad (9) \int \operatorname{ch} x \operatorname{sh}^3 x dx$$

Exercice 5 (Décomposer les fractions rationnelles ; en calculer les primitives) :

$$(1) \frac{1}{a^2 + x^2} \quad (2) \frac{1}{(1+x^2)^2} \quad (3) \frac{x^3}{x^2 - 4} \quad (4) \frac{4x}{(x-2)^2}$$

$$(5) \frac{1}{x^2 + x + 1} \quad (6) \frac{1}{(t^2 + 2t - 1)^2} \quad (7) \frac{3t + 1}{(t^2 - 2t + 10)^2} \quad (8) \frac{3t + 1}{t^2 - 2t + 10}$$

$$(9) \frac{1}{t^3 + 1} \quad (10) \frac{x^3 + 2}{(x+1)^2} \quad (11) \frac{x+1}{x(x-2)^2} \quad (12) \frac{(x^2-1)(x^3+3)}{2x+2x^2}$$

$$(13) \frac{x^2}{(x^2+3)^3(x+1)} \quad (14) \frac{x^7 + x^3 - 4x - 1}{x(x^2+1)^2} \quad (15) \frac{3x^4 - 9x^3 + 12x^2 - 11x + 7}{(x-1)^3(x^2+1)}$$

Exercice 6 (Calculer les intégrales des fractions rationnelles) :

$$\begin{array}{lll}
 (1) \int_0^1 \frac{dx}{x^2 + 2} & (2) \int_{-1/2}^{1/2} \frac{dx}{1 - x^2} & (3) \int_0^3 \frac{x^4 + 6x^3 - 5x^2 + 3x - 7}{(x - 4)^3} dx \\
 (4) \int_2^3 \frac{2x + 1}{x^2 + x - 3} dx & (5) \int_0^2 \frac{x dx}{x^4 + 16} & (6) \int_{-1}^1 \frac{2x^4 + 3x^3 + 5x^2 + 17x + 30}{x^3 + 8} dx \\
 (7) \int_{-2}^0 \frac{dx}{x^3 - 7x + 6} & (8) \int_2^3 \frac{4x^2}{x^4 - 1} dx & (9) \int_1^2 \frac{2x^8 + 5x^6 - 12x^5 + 30x^4 + 36x^2 + 24}{x^4(x^2 + 2)^3} dx \\
 (10) \int_0^2 \frac{dx}{x^4 + 1} & (11) \int_{-1}^0 \frac{x^3 + 2x + 1}{x^3 - 3x + 2} dx &
 \end{array}$$

Calculer

$$\int_0^a \frac{-2x^2 + 6x + 7}{x^4 + 5x^2 + 4} dx \quad \text{pour } a \in \mathbb{R}.$$

Y a-t-il une limite lorsque $a \rightarrow +\infty$?

Exercice 7 (Calculer les primitives) :

$$\begin{array}{lll}
 (1) \int \frac{\cos^3 x}{\sin^5 x} dx & (2) \int \frac{\sin^3 x}{1 + \cos x} dx & (3) \int \frac{dx}{\cos^4 x + \sin^4 x} \\
 (4) \int \frac{\cos x}{1 + \sin 2x} dx & (5) \int \frac{\tan x - \tan \alpha}{\tan x + \tan \alpha} dx & (6) \int \frac{\operatorname{sh} x \operatorname{ch} x}{\operatorname{sh}^4 x + \operatorname{ch}^4 x} dx
 \end{array}$$

Exercice 8 (Calculer les primitives suivantes) :

$$(1) \int \frac{dx}{x + \sqrt{x - 1}} \quad (2) \int \frac{dx}{x\sqrt{x^2 + x + 1}} \quad (3) \int \frac{x dx}{\sqrt{9 + 4x^4}} \quad (4) \int \frac{x + 1}{\sqrt{-4x^2 + 4x + 1}} dx$$

Exercice 9 (Calculer les primitives) :

$$\begin{array}{llll}
 (1) \int \cos^5 t dt & (2) \int e^{\sin^2 x} \sin 2x dx & (3) \int \cosh^3 t dt & (4) \int \cos^4 t dt \\
 (5) \int \sinh^4 t dt & (6) \int x^3 e^x dx & (7) \int \ln x dx & (8) \int x \ln x dx \\
 (9) \int \arcsin x dx & (10) \int \cosh t \sin t dt & (11) \int \frac{dx}{\sin x} & (12) \int \sqrt{a^2 - x^2} dx \\
 (13) \int \frac{e^{2x}}{\sqrt{e^x + 1}} dx & (14) \int e^{ax} \cos bx dx & (15) \int e^{ax} \sin bx dx & (16) \int \sqrt{\frac{x}{(1-x)^3}} dx \quad (0 < x < 1) \\
 (17) \int \frac{x^2}{\sqrt{1 - x^2}} dx & (18) \int \frac{dx}{\cos x + 2 \sin x + 3} & (19) \int \frac{\cosh x dx}{\cosh x + \sinh x} & (20) \int \frac{\sqrt{x} dx}{\sqrt{a^3 - x^3}} \quad (0 < x < a)
 \end{array}$$