Université Paris 7 - Denis Diderot — Cours de Mathématiques-MP4
Licence de Physique , L2 — 2010-2011, 2^{nd} semestre

Feuille de TD 1
Calcul d'intégrales simples

Exercice 1 :

Considérons l'intégrale

$$I = \int_0^{\ln 2} \sqrt{e^x - 1}\, dx.$$

Effectuer le changement de variable $u = \sqrt{e^x - 1}$ et calculer I. (Résultat : $I = 2 - \frac{\pi}{2}$).

Exercice 2 (Calculer les primitives) :

(1) $\displaystyle\int \frac{1}{\sqrt{2+x} + (2+x)^{\frac{1}{6}}}\, dx \quad \left(t = (2+x)^{\frac{1}{6}}\right).$ (2) $\displaystyle\int (\arcsin x)^2\, dx.$

(3) $\displaystyle\int \frac{1}{((x-1)^2-4)^2}\, dx \quad \left(\frac{x-1}{2} = \tan u \text{ ou } \coth u\right).$ (4) $\displaystyle\int x^2\sqrt{1+x^3}\, dx.$

Exercice 3 (Calculer les primitives) :

(1) $\displaystyle\int e^x \cos x\, dx$ (2) $\displaystyle\int \frac{\ln x}{x^n}\, dx, n \in \mathbb{N}$ (3) $\displaystyle\int x \arctan x\, dx$ (4) $\displaystyle\int (x^2+x+1)e^x\, dx$

Exercice 4 (Calculer les primitives) :

(1) $\displaystyle\int (\cos x \cos 2x + \sin x \sin 3x)\, dx$ (2) $\displaystyle\int \cos x \sin^4 x\, dx$ (3) $\displaystyle\int \cos^6 x\, dx$

(4) $\displaystyle\int \sin^3 x \cos x\, dx$ (5) $\displaystyle\int \sin^4 x\, dx$ (6) $\displaystyle\int \sin^3 x \cos^2 x\, dx$

(7) $\displaystyle\int \mathrm{ch}^2 x\, \mathrm{sh}^2 x\, dx$ (8) $\displaystyle\int \mathrm{sh}\, x\, \mathrm{ch}^3 x\, dx$ (9) $\displaystyle\int \mathrm{ch}\, x\, \mathrm{sh}^3 x\, dx$

Exercice 5 (Décomposer les fractions rationelles ; en calculer les primitives) :

(1) $\dfrac{1}{a^2+x^2}$ (2) $\dfrac{1}{(1+x^2)^2}$ (3) $\dfrac{x^3}{x^2-4}$ (4) $\dfrac{4x}{(x-2)^2}$

(5) $\dfrac{1}{x^2+x+1}$ (6) $\dfrac{1}{(t^2+2t-1)^2}$ (7) $\dfrac{3t+1}{(t^2-2t+10)^2}$ (8) $\dfrac{3t+1}{t^2-2t+10}$

(9) $\dfrac{1}{t^3+1}$ (10) $\dfrac{x^3+2}{(x+1)^2}$ (11) $\dfrac{x+1}{x(x-2)^2}$ (12) $\dfrac{(x^2-1)(x^3+3)}{2x+2x^2}$

(13) $\dfrac{x^2}{(x^2+3)^3(x+1)}$ (14) $\dfrac{x^7+x^3-4x-1}{x(x^2+1)^2}$ (15) $\dfrac{3x^4-9x^3+12x^2-11x+7}{(x-1)^3(x^2+1)}$

Exercice 6 (Calculer les intégrales des fractions rationnelles) :

(1) $\displaystyle\int_0^1 \frac{dx}{x^2+2}$ (2) $\displaystyle\int_{-1/2}^{1/2} \frac{dx}{1-x^2}$ (3) $\displaystyle\int_0^3 \frac{x^4+6x^3-5x^2+3x-7}{(x-4)^3}\,dx$

(4) $\displaystyle\int_2^3 \frac{2x+1}{x^2+x-3}\,dx$ (5) $\displaystyle\int_0^2 \frac{x\,dx}{x^4+16}$ (6) $\displaystyle\int_{-1}^1 \frac{2x^4+3x^3+5x^2+17x+30}{x^3+8}\,dx$

(7) $\displaystyle\int_{-2}^0 \frac{dx}{x^3-7x+6}$ (8) $\displaystyle\int_2^3 \frac{4x^2}{x^4-1}\,dx$ (9) $\displaystyle\int_1^2 \frac{2x^8+5x^6-12x^5+30x^4+36x^2+24}{x^4(x^2+2)^3}\,dx$

(10) $\displaystyle\int_0^2 \frac{dx}{x^4+1}$ (11) $\displaystyle\int_{-1}^0 \frac{x^3+2x+1}{x^3-3x+2}\,dx$

Calculer

$$\int_0^a \frac{-2x^2+6x+7}{x^4+5x^2+4}\,dx \qquad \text{pour } a\in\mathbb{R}.$$

Y a-t-il une limite lorsque $a\to+\infty$?

Exercice 7 (Calculer les primitives) :

(1) $\displaystyle\int \frac{\cos^3 x}{\sin^5 x}\,dx$ (2) $\displaystyle\int \frac{\sin^3 x}{1+\cos x}\,dx$ (3) $\displaystyle\int \frac{dx}{\cos^4 x+\sin^4 x}$

(4) $\displaystyle\int \frac{\cos x}{1+\sin 2x}\,dx$ (5) $\displaystyle\int \frac{\tan x-\tan\alpha}{\tan x+\tan\alpha}\,dx$ (6) $\displaystyle\int \frac{\operatorname{sh} x\operatorname{ch} x}{\operatorname{sh}^4 x+\operatorname{ch}^4 x}\,dx$

Exercice 8 (Calculer les primitives suivantes) :

(1) $\displaystyle\int \frac{dx}{x+\sqrt{x-1}}$ (2) $\displaystyle\int \frac{dx}{x\sqrt{x^2+x+1}}$ (3) $\displaystyle\int \frac{x\,dx}{\sqrt{9+4x^4}}$ (4) $\displaystyle\int \frac{x+1}{\sqrt{-4x^2+4x+1}}\,dx$

Exercice 9 (Calculer les primitives) :

(1) $\displaystyle\int \cos^5 t\,dt$ (2) $\displaystyle\int e^{\sin^2 x}\sin 2x\,dx$ (3) $\displaystyle\int \cosh^3 t dt$ (4) $\displaystyle\int \cos^4 t\,dt$

(5) $\displaystyle\int \sinh^4 t\,dt$ (6) $\displaystyle\int x^3 e^x\,dx$ (7) $\displaystyle\int \ln x\,dx$ (8) $\displaystyle\int x\ln x\,dx$

(9) $\displaystyle\int \arcsin x\,dx$ (10) $\displaystyle\int \cosh t\sin t\,dt$ (11) $\displaystyle\int \frac{dx}{\sin x}$ (12) $\displaystyle\int \sqrt{a^2-x^2}dx$

(13) $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx$ (14) $\displaystyle\int e^{ax}\cos bx\,dx$ (15) $\displaystyle\int e^{ax}\sin bx\,dx$ (16) $\displaystyle\int \sqrt{\frac{x}{(1-x)^3}}\,dx$ $(0<x<1)$

(17) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ (18) $\displaystyle\int \frac{dx}{\cos x+2\sin x+3}$ (19) $\displaystyle\int \frac{\cosh x\,dx}{\cosh x+\sinh x}$ (20) $\displaystyle\int \frac{\sqrt{x}\,dx}{\sqrt{a^3-x^3}}$ $(0<x<a)$