Composition of service curves in Network Calculus

Anne Bouillard
ENS / INRIA
45 rue d'Ulm
75005 Paris, France
anne.bouillard@ens.fr

ABSTRACT

In this article, we study the composition of simple and strict service curves in network calculus for two operations: the concatenation of servers and the residual service curves. Whereas strict service curves enable more operations, the strict character of the curves is not stable by those two operations. We show that, beyond the already known results about stability for those two classes of service, no stable class of service curve can be defined in between. We compare this result to some classes of service curves that has been defined in the literature.

Categories and Subject Descriptors

C.4 [**PERFORMANCE OF SYSTEMS**]: Modeling techniques; G.m [**Mathematics of Computing**]: Miscellaneous—*Queueing theory*; C.3 [**SPECIAL-PURPOSE AND APPLICATION-BASED SYSTEMS**]: Real-time and embedded systems

General Terms

Theory

1. INTRODUCTION

Network Calculus is a theory that has been developed to study deterministic worst-case performance bounds in communication networks [5, 9] and has been applied in various domains of embedded networks (the Avionic Full Duplex (AFDX), [4] and Ethernet networks [8] for example). Network calculus uses functions (named curves) to describe constraints on system. More precisely, *arrival curves* shape the incoming traffic by bounding the amount of data that can arrive during any interval of time and *service curves* give some guarantee about the minimal amount of data that is served. Its first developments were purely based on the (min,plus) algebra [6, 7], and can be seen as a tropical version of the filtering theory, making intensive use of the (min,plus) convolution: the output process can be computed as the convolution of the input process by the service curve.

The main advantages if this notion of service curve are:

- the computation of the worst-case performances (delay and backlog) is purely algebraic, algorithmically efficient and can be performed with the arrival and the service curves only;
- the concatenation of servers is simple: due to the associativity of the (min,plus) convolution, servers in tandem can be replaced by one system whose service curve is the (min,plus)-convolution of the service curves of the servers.

But this notion has also some drawbacks:

- multiple flow networks cannot be handled this way: for many service policies (arbitrary [11], fixed priorities [12]), it is not possible to compute individual service curves for each flow [9];
- the physical interpretation is not so clear: the output process depends too strongly on the input process's past. Indeed, even if at time t the system is empty, the output process after time t will depend on the input process before time t. This usually does not happen for real systems, where the output process at time t depends only on the last busy period.

To overcome these drawbacks, some other classes of service curves have been defined. One can cite the *variable capacity nodes*, the *strict service curves*, the *adaptive service curves* [9], the *weakly strict service curves* [3] or the *sufficiently strict service curves* [10]. Among them the *strict service curves*, where the output process only depends on the current backlogged period (interval of time during which the system is never empty) are the most widely used (this is more or less equivalent to the notion of service curves in *real-time calculus* [12]. On each backlogged period, the service is at least equal to that of the curve. Despite having a better physical interpretation and the ability to handle with multiple flows, strict service curves is not a stable class:

- the strictness property is lost by the convolution (hence the concatenation);
- individual service curves can be computed by removing the maximal arrival of the cross traffic (*i.e.* its arrival curve) from the service curve. In some cases (arbitrary multiplexing for example), the individual service curves that are computed are not strict.

This induces a careful use of the results of the network calculus and prevents to make advantage of the strict service curves by using basics results in cascade.

The questions we answer in this article are the following.

1. Is it possible to compute individual strict service curves?
2. It it possible to define a new operation of concatenation that preserves the strictness of the service curve?
3. Is there a class of service curve (between the service curves and the strict service curves) that is stable by the concatenation and the individual service curves?

We will answer by Yes to the first question and No to the others. In this view, we will also compare those results with some of classes enumerated above.

The remaining of the paper is organized as follows. In Section 2, we will recall the Network Calculus framework and the notations. Then, Section 3 deals with the problem of individual service curves and Section 4 deals with the problem of the concatenation. Finally, in Section 5, we comment on these results and the other classes of service curves. We conclude with Section 6.

2. NETWORK CALCULUS FRAMEWORK

In network calculus, flows of data are represented by non-decreasing functions and left-continuous functions, that model the cumulative processes. More precisely, if A represents a flow at a certain point in the network, $A(t)$ is the amount of data of that flow that crossed that point until time t, with the convention $A(0) = 0$. More formally, let

$$\mathcal{F} = \{f : \mathbb{R}_+ \to \mathbb{R}_{\min} \mid f(0) = 0, f \text{ non-decreasing and left-continuous}\},$$

where $\mathbb{R}_{\min} = \mathbb{R} \cup \{+\infty\}$. The structure $(\mathbb{R}_{\min}, \min, +)$ is a dioid and called the (min,plus) dioid.

Let us define the following operations on $\mathcal{F}$: let $f, g \in \mathcal{F}$, $\forall t \in \mathbb{R}_+$,

- *minimum:* $f \wedge g(t) = \min(f(t), g(t))$;
- *(min,plus)-convolution:* $f * g(t) = \inf_{0\leq s\leq t} f(s) + g(t-s)$;
- *(min,plus)-deconvolution:* $f \oslash g(t) = \sup_{u\in\mathbb{R}_+} f(t+u) - g(u)$.

The (min,plus) dioid can be lifted to the space of functions into the $(\mathcal{F}, \wedge, *)$ dioid and $\oslash$ is the residuation operator of the convolution (see [1] for more details about residuation).

The following lemma is a small generalization of Theorem 3.1.8 in [9] will be useful later.

Lemma 1. *Let $f, g \in \mathcal{F}$. Then, $\forall t \in \mathbb{R}_+$, $\exists s \in [0, t]$ such that $f * g(t) = f(s) + g(t - s)$.*

Proof. Fix $t \in \mathbb{R}_+$ and define $F : s \mapsto f(s) + g(t - s)$. Let (u_n) a sequence converging to $u \in [0, t]$. One can assume without loss of generality (by taking a sub-sequence and if needed exchanging the role of f and g by replacing (u_n) by $(t - u_n)$ that (u_n) is increasing.

As f is left-continuous, $\lim_{n\to\infty} f(u_n) = f(u)$ and, as g is left-continuous and non-decreasing, $\lim_{n\to\infty} g(u_n) \geq g(u)$. Then $\lim_{n\to\infty} F(u_n) \geq F(u)$. As a consequence, F reaches its minimum on $[0, t]$. □

A system $\mathcal{S}$ is a non-deterministic relations between some input flows and output flows, where the number of inputs is the same as the number of outputs: $\mathcal{S} \subseteq \mathcal{F}^m \times \mathcal{F}^m$ and there is a one-to-one relation between the inputs and the outputs of the system, such that to each input flow corresponds one and only one output flow that is causal - no data is created inside one flow in the system - meaning that for $((A_i)_{i=1}^m, (B_i)_{i=1}^m) \in \mathcal{S}$, $\forall i \in \{1, \ldots, m\}$, $A_i \geq B_i$. The vector $((A_i)_{i=1}^m, (B_i)_{i=1}^m)$ is an *(admissible) trajectory* of $\mathcal{S}$ if $((A_i)_{i=1}^m, (B_i)_{i=1}^m) \in \mathcal{S}$.

In order to compute the performances, one must give the characteristics of the system. Those characteristics concern two objects, the flows and the servers, and are respectively named the *arrival curves* and the *service curves*.

Arrival curves.

The notion of arrival curve is quite simple: the amount of data that arrived during an interval of time is a function of the length of this interval. More formally, let $\alpha \in \mathcal{F}$. A flow is constrained by the arrival curve α, or is α-constrained if $\forall s, t \in \mathbb{R}_+, s \leq t$,

$$A(t) - A(s) \leq \alpha(t - s).$$

Let α be an arrival curve. We call $\mathcal{A}$ the single input/single output system of the α-constrained arrivals:

$$\mathcal{A}(\alpha) = \{(A, B) \in \mathcal{F} \times \mathcal{F} \mid A \geq B \text{ and } A \text{ is } \alpha\text{-constrained}\}.$$

A typical example of such arrival curves is the pseudo-affine functions: $\alpha_{\sigma,\rho} : 0 \mapsto 0;\ t \mapsto \sigma + \rho t$, if $t > 0$, where σ can be interpreted as the maximal amount of data that can arrive simultaneously and ρ as a maximal long-term arrival rate.

Service curves.

As mentioned in the introduction, several notions of service curve coexist. The role of the service curve is to constrain the relation between the input of a system and its output. Let A be an arrival cumulative process in a system and B its departure cumulative process. We say that an interval I is a backlogged period for $(A, B) \in \mathcal{F} \times \mathcal{F}$ if $\forall t \in I$, $A(t) > B(t)$. Let $t \in \mathbb{R}_+$. The start of the backlogged period of t is $start(t) = \sup\{u \leq t \mid A(u) = B(u)\}$. As A and B are left-continuous, $A(start(t)) = B(start(t))$, and $]start(t), t]$ is a backlogged period.

Let us define the following systems:

- *Simple service curve:* $\mathcal{S}_{simple}(\beta) = \{(A, B) \in \mathcal{F} \times \mathcal{F} \mid A \geq B \geq A * \beta\}$;
- *Strict service curve:* $\mathcal{S}_{strict}(\beta) = \{(A, B) \in \mathcal{F}\times\mathcal{F} \mid A \geq B$, and $\forall$ backlogged period $]s, t], B(t) - B(s) \geq \beta(t - s)\}$.

We say that a system $\mathcal{S}$ offers (or guarantees) a simple (resp. strict) service curve β if $\mathcal{S} \subseteq \mathcal{S}_{simple}(\beta)$ (resp. $\mathcal{S} \subseteq \mathcal{S}_{strict}(\beta)$).

It is well-known that $\mathcal{S}_{strict}(\beta) \subseteq \mathcal{S}_{simple}(\beta)$, but that the reverse is not true. A more precise study about the comparisons between simple and strict service curves can be found in [3]. In particular, those two notions are not equivalent and a simple service curve cannot be expressed as a strict service curve. In addition, we only study in the next two sections those two main definitions of service curves. Variable capacity nodes have been proved to be equivalent to strict service curves in most of the cases and the study of other notions is postponed to Section 5.

Two common families of service curves are

- the *pure delay* curve $\delta_T : t \mapsto 0$ if $t \in [0, T]$; $t \mapsto +\infty$ otherwise. For simple service curves, this means that the sojourn time of each bit of data is at most T. For strict service curves, this means that each backlogged period lasts at most T;
- the *guaranteed rate* curve $\lambda_\rho : t \mapsto \rho t$. For every arrival process, the minimal admissible trajectory is such that either the service rate is exactly ρ or the server is empty. The interpretation is the same for simple or strict service curves. The only difference is that for strict service curves, the service rate is guaranteed, whereas for simple service curves, serving data at a faster rate allows can be compensate by a slower rate service.

Finally, we say that a trajectory (A, B) satisfy the *exact service* β if $B = A * \beta$ for a simple service curve or if, for a strict service curve, there is a start of a backlogged period as soon as possible, and during a backlogged period, $B(t) = A(start(t)) + \beta(t - start(t))$ (note that β should then be super-additive, but β can always be replaced by its super-additive closure, see [3]).

Performance guarantees.

THEOREM 1. *[5] Let $(A, B) \in \mathcal{S}_{simple}(\beta)$ and suppose that A is α-constrained. Then*

(i) $\forall t \in \mathbb{R}_+$, the backlog of the trajectory at time t is bounded by $A(t) - B(t) \leq \alpha \oslash \beta(0)$ (the vertical maximal distance between α and β);

(ii) $\forall t \in \mathbb{R}_+$ the delay of data arrived at time t, assuming a FIFO per flow service policy, is bounded by $\sup\{d \in \mathbb{R}_+ \mid A(t) \geq B(t + d)\} \leq \sup\{d \in \mathbb{R}_+ \mid \forall t \in \mathbb{R}_+ \alpha(t) \geq \beta(t + d)\}$ (the horizontal maximal distance between α and β);

(iii) B is $\alpha \oslash \beta$-constrained.

This result only concerns the case of a single server with a single input and output. In the next sections, we will detail results for multiple inputs/outputs and the concatenation of servers. We will often use the following notation: $x_+ = \max(x, 0)$ and $f_\uparrow : \mathbb{R}_+ \to \mathbb{R}_{\min}$; $t \mapsto \sup_{0 \leq s \leq t}(f(s)_+)$, the non-negative and non-decreasing closure of f.

3. INDIVIDUAL SERVICE CURVES

This section is dedicated to systems with multiple input/output flows. Consider a system with m input flows and m output flows. Let $\mathcal{S} \in \mathcal{F}^m \times \mathcal{F}^m$. The aggregated system if $\mathcal{S}$ is $Ag(\mathcal{S}) = \{(\sum_{i=1}^m A_i, \sum_{i=1}^m B_i) \mid ((A_i)_{i=1}^m, (B_i)_{i=1}^m) \in \mathcal{S}\}$ and the projection of this server on $I \subseteq \{1, \ldots, m\}$ is $P_I(\mathcal{S}) = \{((A_i)_{i \in I}, (B_i)_{i \in I}) \mid ((A_i)_{i=1}^m, (B_i)_{i=1}^m) \in \mathcal{S}\}$. Those notations are illustrated on Figure 1.

We say that the system $\mathcal{S} \in \mathcal{F}^m \times \mathcal{F}^m$ offers a simple (resp. strict) service curve β if $Ag(\mathcal{S}) \in \mathcal{S}_{simple}(\beta)$ (resp. $Ag(\mathcal{S}) \in \mathcal{S}_{strict}(\beta)$). In the following, we only consider the case $m = 2$ as for more general cases it suffices to consider $Ag(P_I)$ and $Ag(P(I^c))$.

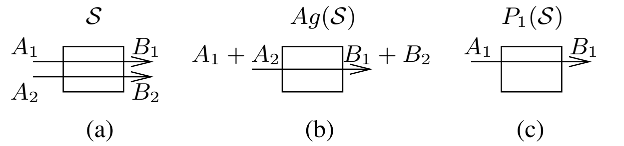

Figure 1: (a) system with two input and two output flows; (b) aggregated system; (c) projection of the system on the first input/output.

THEOREM 2. *Let $\mathcal{S} \in \mathcal{F}^2 \times \mathcal{F}^2$ and $\beta \in \mathcal{F}$. The following statements hold.*

(i) If $\exists T >$ and $\beta(T) = 0$, $Ag(\mathcal{S}) \subseteq \mathcal{S}_{simple}(\beta) \not\Rightarrow \exists \beta' \neq 0$ such that $P_1(\mathcal{S}) \subseteq \mathcal{S}_{simple}(\beta')$;

(ii) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{strict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2) \Rightarrow P_1(\mathcal{S}) \in \mathcal{S}_{simple}((\beta - \alpha_2)_\uparrow)$;

(iii) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{strict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2) \not\Rightarrow P_1(\mathcal{S}) \subseteq \mathcal{S}_{strict}((\beta - \alpha_2)_\uparrow)$;

(iv) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{strict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2)$ and $P_1(\mathcal{S}) \subseteq \mathcal{A}(\alpha_1) \Rightarrow P_1(\mathcal{S}) \subseteq \mathcal{S}_{strict}((\beta - (\alpha_2 \oslash (\beta - \alpha_1)_\uparrow))_\uparrow)$.

By definition of an aggregate and a projection, we here deal with *arbitrary*, or *blind* multiplexing. That is, we make no assumption about the service policy and consider the worst-case of them. More precise results exist when dealing with a precise service policy, for example, it is possible to define an infinite family of individual service curves in FIFO systems when the initial service curve is simple, or when static priorities are considered, the strict character of the service curves is preserved in fluid systems (otherwise, the maximum size of a packet has to be considered).

PROOF. The first three statements are now classical however, for sake of completeness, we give the proof or examples to invalidate the implications.

Let $A_1, B_1, A_2, B_2 \in \mathcal{F}$ such that $(A_1 + A_2, B_1 + B_2) \in \mathcal{S}_{simple}(\beta)$. For every $t \in \mathbb{R}_+$, there exist $s \leq t$ such that $B_1(t) + B_2(t) \geq A_1(s) + A_2(s) + \beta(t - s)$. Then,

$$B_1(t) \geq A_1(s) + [A_2(s) - B_2(t)] + \beta(t - s). \qquad (1)$$

Suppose that $\beta(t) = 0$, $\forall t \in [0, T]$. Then, $\beta \leq \delta_T$. Take $A_2(t) = \rho t$ and $A_1(t) = \rho T$ $\forall t > 0$. A possible output for the aggregated server is $B_1(t) + B_2(t) = (A_1 + A_2) * \delta_T(t) = (A_1 + A_2)((t - T)_+) \leq A_2(t)$. Then, a possible trajectory for system $\mathcal{S}$ is $(A_1, A_2, 0, (A_1 + A_2) * \delta_T)$, and the only service offered to flow A_1 is 0. This example can be found in [9], Section 7.2.

If the server offers a strict service curve, then Equation (1) is valid for $s = start(t)$ and we conclude by remarking that $A_2(s) - B_2(t) \leq \alpha_2(t - s)$ and $B_1(t) \geq B_1(s) = A_1(s)$.

Now, let us give an example of trajectory to show that the individual service curve is not a strict service curve. Consider a server offering a strict service curve $\beta(t) = 3t$. Let two flows cross this node with respective cumulative arrival functions $A_1(t) = t + 2$ and $A_2(t) = t$. An arrival curve of Flow 2 is $\alpha_2(t) = t$. Since the sum $A_1(t) + A_2(t) = 2t + 2$, in the interval $[0, 2[$, it is possible to choose $B_1(t) + B_2(t) = 3t$ and the system is backlogged. Figure 2 shows the trajectory of the system for the following policy: at first, for $0 \leq t \leq 1$, Flow 1 is given top priority, then for $t > 1$, Flow 2 is given top priority. But $(\beta - \alpha_2)_+(t) = 2t$ is not a *strict* service curve: during the period $1 \le t \le 2$, Flow 1 has some data backlogged in the node but this data is not served at all. A similar example can be found in [2].

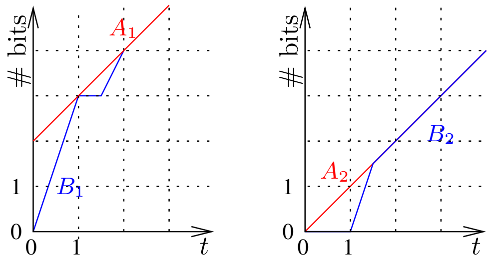

Figure 2: Residual service curves are not necessarily strict.

Let $s < t \in R_+$ in the same backlogged period as regards arrival process A_2, and thus as regards $A_1 + A_2$. As the server offers a strict service curve β,

$$B_2(t) \ge B_2(s) - [B_1(t) - B_1(s)] + \beta(t - s).$$

Applying Theorem 1 and statement *(ii)* of this proof leads to

$$B_1(t) - B_1(s) \le \alpha_1 \oslash (\beta - \alpha_2)_\uparrow(t - s),$$

hence the result. □

Using the example in the proof : $\beta(t) = 3t$, $\alpha_1(t) = 2 + t$ and $\alpha_2(t) = t$, one gets $(\beta - \alpha_1)_\uparrow(t) = (t-1)_+$ and $\alpha_2 \oslash (\beta - \alpha_1)_\uparrow = t + 1$ and finally $(\beta - (\alpha_2 \oslash (\beta - \alpha_1)_\uparrow))_\uparrow(t) = 2(t - 1/2)_+$. For this example, it seems that it is the best possible curve, as it is reached on the interval $[1, 2]$. Strict service curves obtained this way are not tight for more general cases. Also remark that this kind of strict residual service curve should only be used when necessary, as there is no chance that the delays and backlogs computed with it are tight.

4. COMPOSITION OF SERVICE CURVES

In the previous section, we proved that it is possible to define an individual service curve that preserves the strict character of the service curves. We now attempt to get the same kind of results for the composition. We will see that this is not possible, either by changing the operation of composition, either by defining a new notion of service curve (and keeping the convolution as the composition).

4.1 Composition of strict service curves

We first consider consider two servers in tandem and define the composition of two servers: let $\mathcal{S}_1, \mathcal{S}_2 \in \mathcal{F} \times \mathcal{F}$, the concatenation of $\mathcal{S}_1$ and $\mathcal{S}_2$ is $\mathcal{S}_2 \circ \mathcal{S}_1 = \{(A, C) \mid \exists B \in \mathcal{F}$ such that $(A, B) \in \mathcal{S}_1$ and $(B, C) \in \mathcal{S}_2\}$. Set $\mathcal{S}^1 = \mathcal{S}$ and $\forall n \ge 1$, $\mathcal{S}^{n+1} = \mathcal{S} \circ \mathcal{S}^n$.

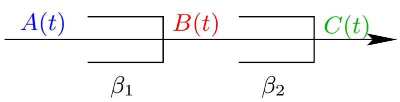

Figure 3: servers in tandem

Theorem 3. *The following statements hold.*

(i) $\mathcal{S}_{simple}(\beta_2) \circ \mathcal{S}_{simple}(\beta_1) \subseteq \mathcal{S}_{simple}(\beta_1 * \beta_2)$*;*

(ii) Let β_1 *and* β_2 *such that there exist* $T_1, T_2 > 0$ *with* $\beta_1(T_1) = 0$ *and* $\beta_2(T_2)$*=0. Then* $\nexists \beta > 0$ *such that* $\mathcal{S}_{strict}(\beta_2) \circ \mathcal{S}_{strict}(\beta_1) \subseteq \mathcal{S}_{strict}(\beta)$*.*

Proof. Statement *(i)* of this theorem is a classical property of the concatenation of servers, which can be found in [9, 5].

However, this inclusion may be strict: Let $\beta_1 = \delta_3$ and $\beta_2 = \lambda_1$, $A = \alpha_{2,1/2}$ and $C : 0 \mapsto 0; t \in]0, 4] \mapsto 2; t > 4 \mapsto A(t)$. It is easy to check that $C \ge A * \beta_1 * \beta_2$. Now, given A and C the minimal admissible departure process from the first server is $B = \max(C, A * \beta_1)$. But, $B * \beta_2(4) = 2.5 > 2 = C(4)$. Those functions are depicted on Figure 4. As the convolution is a non-decreasing operator, one can conclude that $\nexists B \in \mathcal{F}$ such that $(A, B) \in \mathcal{S}_{simple}(\beta_1)$ and $(B, C) \in \mathcal{S}_{simple}(\beta_2)$.

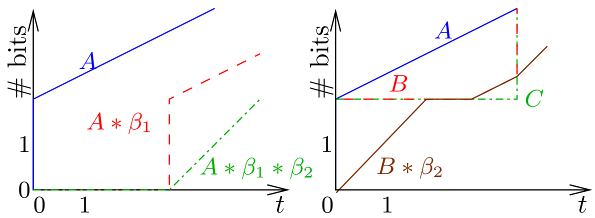

Figure 4: An example for which $\mathcal{S}_{simple}(\beta_2) \circ \mathcal{S}_{simple}(\beta_1) \subsetneq \mathcal{S}_{simple}(\beta_1 * \beta_2)$. On the left, A, $A * \beta_1$ and $A * \beta_1 * \beta_2$; one the right, A, a possible output C for the system with service curve $\beta_1 * \beta_2$. Then, the smallest possible B is $B = \max(C, A * \beta_1)$. Between times on the interval $]3, 4[$, $C(t) < B * \beta_2$.

Let us focus on statement *(ii)*. We restrict ourselves to the family of pure delay curves and try to compute a strict service curve for the concatenation of those two curves. Indeed, if $\beta \ge \beta'$, then $\mathcal{S}_{strict}(\beta) \subseteq \mathcal{S}_{strict}(\beta')$ there is no need to consider more general cases.

Consider two servers in tandem with respective strict service curves δ_{T_1} and δ_{T_2} and the arrival cumulative process A defined by : fix $\sigma > 0$ and $T \in]\max(T_1, T_2), T_1 + T_2[$, then $\forall t \in \mathbb{R}_+$,

$$A(t) = \begin{cases} 0 & \text{if } t = 0, \\ k\sigma & \text{if } \exists k \in \mathbb{N},\ t \in](k-1)T, kT]. \end{cases}$$

Note that $A(t)$ is constrained by $\alpha_{\sigma,\sigma/T}$.

Now let us compute a departure process from the first server, B, and then a departure process from the second server, C.

Since $T > T_1$, a possible departure process is $B(t) = A((t - T_1)_+)$, $\forall t \in \mathbb{R}_+$. Then, since $T > T_2$, a possible departure process is $C(t) = (B(t - T_2)_+) = A((t - T_1 - T_2)_+)$, $\forall t \in \mathbb{R}_+$. The trajectory is depicted on Figure 5.

We now prove that the system is always backlogged: $\forall t \in \mathbb{R}_+$, $A(t) - C(t) = A(t) - A((t - T_1 - T_2)_+) > 0$ since by definition, $A(t) > A((t - T)_+) \ge A((t - T_1 + T_2)_+)$.

So, if β is a strict service curve for the system, $\beta \le C \le A \le \alpha_{\sigma,\sigma/T}$. When T is fixed, this must hold for all $\sigma > 0$. Then, the only possible strict service curve is $\beta = 0$. □

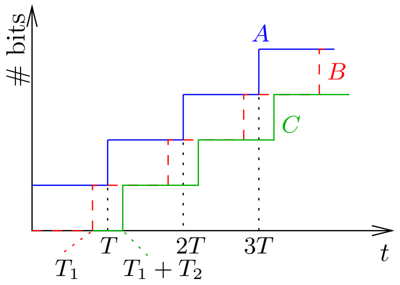

Figure 5: No strict service curve for two servers in tandem.

4.2 A new notion of service curve?

Let us recall the properties that our service should satisfy to be preserved by concatenation (we denote by $\hat{\mathcal{S}}(\beta)$ the relation between trajectories that must be satisfied for an intermediate service curve β):

1. $\mathcal{S}_{strict}(\beta) \subseteq \hat{\mathcal{S}}(\beta) \subseteq \mathcal{S}_{simple}(\beta)$;
2. $\hat{\mathcal{S}}(\beta_2) \circ \hat{\mathcal{S}}(\beta_1) = \hat{\mathcal{S}}(\beta_1 * \beta_2)$.

Let us first define the closure of a systems: Let $\mathcal{S} \subseteq \mathcal{F} \times \mathcal{F}$. The *closure* of $\mathcal{S}$ is

$$\overline{\mathcal{S}} = \{(A, B') \in \mathcal{F} \times \mathcal{F} \mid \forall \varepsilon > 0,\ \exists (A, B) \in \mathcal{S} \text{ such that } \forall t \in \mathbb{R}_+,\ A(t) \geq B'(t) \geq B(t - \varepsilon)\}.$$

$\overline{\mathcal{S}}$ is the smallest system of $\mathcal{F} \times \mathcal{F}$ containing $\mathcal{S}$, closed and such that $\forall A \in \mathcal{F}$, if $(A, B) \in \mathcal{S}$, then $\forall B' \geq B$, $(A, B') \in \overline{\mathcal{S}}$. With this definition, the problem of the strict inclusion of the composition exposed in Theorem 3 is avoided: $\overline{\mathcal{S}_{simple}(\beta_2) \circ \mathcal{S}_{simple}(\beta_1)} = \mathcal{S}_{simple}(\beta_1 * \beta_2)$.

LEMMA 2. *Let $T \in \mathbb{R}_+$. Then,*

$$\overline{\bigcup_{n \in \mathbb{N}} (\mathcal{S}_{strict}(\delta_{T/n}))^n} = \mathcal{S}_{simple}(\delta_T).$$

PROOF. This case is a special case of Lemma 3, but gives a good intuition of it.

Fix $n \in \mathbb{N}$ and $T \in \mathbb{R}_+$. Consider the system composed of n servers in tandem, each of them offering a service curve $\delta_{T/n}$. Let $A : \mathbb{R}_+ \to \mathbb{R}_{\min}$ be a cumulative arrival process in that system. We denote by A_i the cumulative departure process of the i-th server when the service is strict and exact with the convention $A_0 = A$. When the service offered are simple and exact, the global cumulative departure process is $B = A * (\delta_{T/n})^n = A * \delta_T = A((. - T)_+)$.

Now, let us compute the delay of each bit of data when the services are strict and exact. After the first server, each bit of data has a delay that lays between 0 and T/n: $\forall t \in \mathbb{R}_+$, $A_0(t) \geq A_1(t) \geq A_0((t - T/n)_+)$. Indeed, each backlogged period is of length exactly T/n. The cumulative process A_1 is purely made of bursts, and the inter-arrival time between two bursts is at least T/n. Then, every bit of data in the second server has a delay exactly T/n: $\forall t \in \mathbb{R}_+$, $A_2(t) = A_1((t - T/n)_+)$. The same holds for the next $n - 2$ servers: $\forall i \in [2, n]$, $\forall t \in \mathbb{R}_+$, $A_i(t) = A_{i-1}((t - T/n)_+)$. So, $\forall t \in \mathbb{R}_+$,

$$A(t - (n-1)T/n) = B(t + T/n) \geq A_n(t) \geq A(t - T) = B(t).$$

Figure 6 illustrates the proof for $n = 1$ and $n = 4$. $\square$

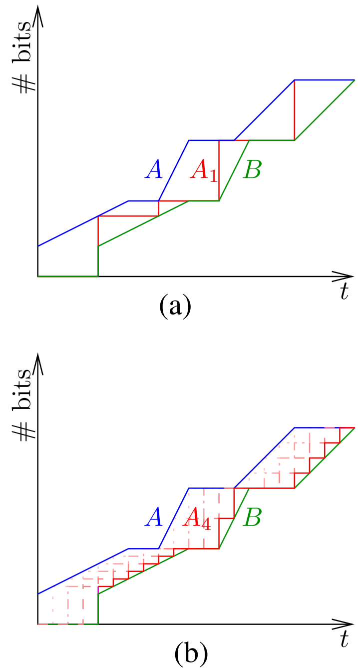

Figure 6: Example of output service curve after 1 and 4 servers in tandem. (a) one server: the arrival process A (blue) and departure processes for strict A_1 (red) and non strict B (green) service curves. (b) Arrival A and departure process for non strict service curve B. From light to dark, the departure process after each server with strict service curve $\delta_{T/4}$. A_1, A_2 and A_3 are dashed and A_4, the output process of the system is plain.

For $\lambda, \rho \geq 0$, let $\beta_{\rho,T} = \max(\delta_T, \lambda_\rho)$.

Lemma 3. *Let $\rho, T \in \mathbb{R}_+$. Then,*

$$\overline{\bigcup_{n\in\mathbb{N}} (\mathcal{S}_{strict}(\beta_{\rho,T/n}))^n} = \mathcal{S}_{simple}(\beta_{\rho,T}).$$

Proof. First, remark that, as in the previous lemma, $\beta_{\rho,T} = \beta^n_{\rho,T/n}$. Let A be a cumulative arrival process. Set $B = A * \beta_{\rho,T}$ as the cumulative departure process of the system when the servers offer simple service curves.

Now, consider the system of n servers in tandem where each server offers a strict and exact service curve $\beta_{\rho,T}$, and denote by B_i the output departure process after the i-th server.

Let us focus on B_1. Each backlogged period is of length at most T/n, and for $s \leq t$ in the same backlogged period, $B(t) - B(s) = \rho(t-s)$. A backlogged period is of length less than T/n when $\exists t$ such that $A(t) - A(start(t^-)) = \rho(t - start(t^-))$. Let $\mathcal{R} = \{t \in \mathbb{R}_+ \mid A(t) = B_1(t)\}$ be the set of the start of backlogged period (of possibly null length). We have

$$B_1(t) = \min_{s\in\mathcal{R}\cap[0,t]} A(s) + \beta_{\rho,T/n}(t-s).$$

Note that $\forall t \in \mathbb{R}_+$, $\exists s \leq t$ such that $t - s < T/n$ and $s \in \mathcal{R}$. In addition, on each interval $]s,t[$ where B_1 is continuous, B_1 is derivable and $\frac{dB_1}{dt}(t) \leq \rho$. Figure 7 gives an illustration of the different possible behaviors of B_1.

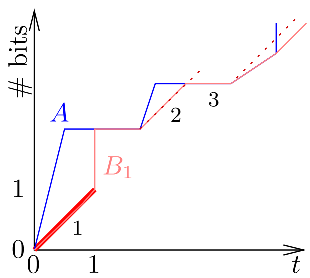

Figure 7: Different cases for elementary segment strict service curves $\beta_{1,1}$, represented in bold. 1: the backlogged period is exactly d/n, the slope of the departure process during this period is λ and there is a burst at the end of the backlogged period. 2: the backlogged period is strictly less than d/n, as the segment of slope λ and of length d/n intersects A. After the intersection, the segment is above the curves or segments that begin the next backlogged period. 3: the slope of the arrival process is less than λ at the end of a backlogged period, then, the departure process is not delayed.

Now focus on B_2. The only beginning of backlogged period in the second server is when there is a burst (or discontinuity) in B_1. Every backlogged period is of length exactly T/n except when there exist t such that $B_1(t) - B_1(start(t^-)) = \rho(t - start(t^-))$. But this happens only when $\frac{dB_1}{dt} < \rho$: then $B_1(t) = A(t)$. As a consequence, we have

$$B_2(t) = \min_{s\in\mathcal{R}\cap[0,t]} A(s) + \beta_{\rho,2T/n}(t-s).$$

The same argument, we have $\forall i \leq n$,

$$B_i(t) = \min_{s\in\mathcal{R}\cap[0,t]} A(s) + \beta_{\rho,iT/n}(t-s)$$

and

$$B_n(t) = \min_{s\in\mathcal{R}\cap[0,t]} A(s) + \beta_{\rho,T}(t-s).$$

Fix $t \in \mathbb{R}_+$. From Lemma 1, there exists $s \leq t$ such that $B(t) = A(s) + \beta_{\rho,T}(t-s)$. Let $u = \max\{v \in \mathcal{R} \cap [0,s]\}$. We know that $0 \leq s - u < T/n$ and $B_n(t - T/n) \leq A(u) + \beta_{\rho,T}(t - T/n - u)$. But then

$$\begin{aligned} B_n(t-T/n) - B(t) &\leq A(u) - A(s) \\ &\quad + \beta_{\rho,T}(t - T/n - u) - \beta_{\rho,T}(t-s) \\ &\leq A(u) - A(s) \\ &\quad + \rho(t - T/n - u) - \rho(t-s) \\ &\leq 0. \end{aligned}$$

□

Theorem 4. *To each convex and piecewise affine function β in $\mathcal{F}$, associate $\hat{\mathcal{S}}(\beta)$ a system such that $\mathcal{S}_{strict}(\beta) \subseteq \hat{\mathcal{S}}(\beta) \subseteq \mathcal{S}_{simple}(\beta)$. If $\forall \beta_1, \beta_2$ convex piecewise affine functions $\hat{\mathcal{S}}(\beta_2) \circ \hat{\mathcal{S}}(\beta_1) \subseteq \hat{\mathcal{S}}(\beta_1 * \beta_2)$, then $\forall \beta$, $\overline{\hat{\mathcal{S}}(\beta)} = \mathcal{S}_{simple}(\beta)$.*

Proof. We prove the theorem when $\hat{\mathcal{S}} = \mathcal{S}_{strict}$, and the general result will follow by inclusion. The idea is to decompose one server with a service curve β into a sequence of servers with of strict service curves β_i, $i \in I$, where I is a finite set such that $\beta = *_i \beta_i$, and to compare the departure processes when servers are used as strict service curves or as simple service curves using Lemmas 2 and 3.

Let β be a convex and piece-wise affine service curve, composed of k finite segments of respective slope ρ_i (possibly $+\infty$) and length T_i and of one semi-infinite segment of slope ρ. We know that $\beta = *_{i=1}^k \beta_{\rho_i,T_i} * \lambda_\rho$. One can decompose that service curve into $k+1$ systems, with respective global service curve: λ_ρ, and β_{ρ_i,T_i}. Fix $\varepsilon > 0$. System 0 is composed of 1 server of service curve λ_ρ. For $i \in \{1, \ldots, k\}$, system i is composed of n_i servers in tandem, each of them of service curve $\beta_{\rho_i,T_i/n_i}$. From Lemmas 2 and 3 (and the proofs), for any arrival cumulative process, the departure processes when the services are simple and exact, A, and when the services are strict and exact, B satisfy the relation $B \geq A \geq B(. - T_i/n_i)$. Then, one can set n_i such that $n_i \geq kT_i/\varepsilon$. The system of servers in tandem is depicted on Figure 8. Let A be an arrival process in the system. We denote by A_0 (resp. B_0) the departure process after the first server when the service offered is simple (resp. strict), A_i (resp. B_i) the departure process after system i when the service is simple (resp. strict). One has $A_0 = B_0$.

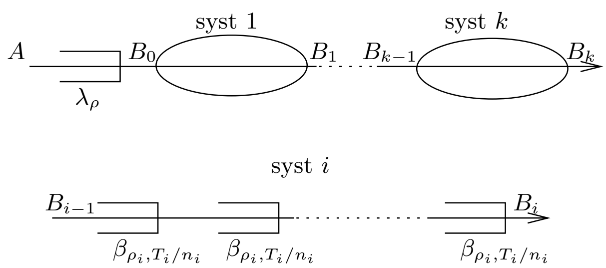

Figure 8: Decomposition of a server with convex service curve into a concatenation of elementary servers.

We now prove by induction on i that $B_i \geq A_i \geq B_i(. - i\varepsilon/k)$. We have $A_0 = B_0$. Suppose that the result is true for i. We prove it for $i+1$ segments using the following properties of the convolution:

- for $f, g \in \mathcal{F}$, for every $d \in \mathbb{R}_+$, $[f(.-d) * g](t) = [f * g](t-d)$ (with the convention that for all $t < 0$, $f(t) = \infty$);
- For $f_1, f_2, g \in \mathcal{F}$, if $f_1 \geq f_2$, then $f_1 * g \geq f_2 * g$.

We have

$$\begin{aligned} A_{i+1} &= A_i * \beta_{\rho_i, T_i} \\ &\geq B_i(.-i\varepsilon/k) * \beta_{\rho_{i+1}, T_{i+1}} \\ &\geq [B_i * \beta_{\rho_{i+1}, T_{i+1}}](.-i\varepsilon/k) \\ &\geq B_{i+1}(.-(i+1)\varepsilon/k). \end{aligned}$$

The last inequality holds considering B_i as the arrival process in system $i+1$. When $i = k$, on gets $B_k \geq A_k \geq B_k(.-\varepsilon)$, hence the result. $\square$

5. CONSEQUENCES FOR OTHER MODELS OF SERVICE CURVES

In the previous paragraphs, we dealt only with two notions of service curves, that can somewhat be considered as extremal: usually, simple service curves is the weakest model whereas strict service curves are the strongest model. In order to avoid the drawbacks describes above, there has been several attempts to define intermediate service curves. We will focus here on *adaptive* service curves [9], on *weakly strict* service curves [3] and on *sufficiently strict* service curves [10].

5.1 Adaptive service curves

Adaptive service curves have been defined in [9], Section 7.4, in order to deal with both strict and simple service curves with only one definition. Let $\beta, \tilde{\beta} \in \mathcal{F}$.

- $\mathcal{S}_{asc}(\beta, \tilde{\beta}) = \{(A, B) \in \mathcal{S} \times \mathcal{S} \mid A \geq B$ and $\forall t \in \mathbb{R}_+,\ \forall s \leq t,\ B(t) \geq (B(s) + \tilde{\beta}(t-s)) \wedge \inf_{s \leq u \leq t} A(u) + \beta(t-u)\}$.
- $\mathcal{S}_{asc}(\beta) = \{(A, B) \in \mathcal{S} \times \mathcal{S} \mid A \geq B$ and $\forall t \in \mathbb{R}_+,\ \forall s \leq t,\ B(t) \geq (B(s) + \beta(t-s)) \wedge \inf_{s \leq u \leq t} A(u) + \beta(t-u)\}$.

Until now, we have only considered servers described by only one curve, so we may here favor the second definition.

PROPOSITION 1. *Let $\mathcal{S} \in \mathcal{F}^2 \times \mathcal{F}^2$, let $\beta, \tilde{\beta} \in \mathcal{F}$ and suppose that there exist $T, \tilde{T}$ such that $\beta(T) = 0$ and $\tilde{\beta}(\tilde{T}) = 0$. Then $Ag(\mathcal{S}) \subseteq \mathcal{S}_{asc} \not\Rightarrow \exists \beta' \neq 0$ such that $P_1 \subseteq (\beta')$.*

PROOF. We only need to prove this result for $\beta = \tilde{\beta} = \delta_{T'}$ with $T' = \min(T, \tilde{T})$ and the rest will follow by inclusion. From [9], Example 1, p. 203, this adaptive service curve coincide with the pure delay $\delta_{T'}$, which is a simple service curve. So the example given in the proof of Theorem 2 can be applied in this case. $\square$

This notion of service curve cannot be a candidate for our notion of intermediate service curve. Furthermore, Theorem 4 implies that $\overline{\mathcal{S}_{asc}(\beta)} = \mathcal{S}_{simple}(\beta)$. We shall indeed now prove that in many cases $(A, A * \beta) \in \mathcal{S}_{asc}(\beta)$.

PROPOSITION 2. *Let $\beta \in \mathcal{F}$ be a piecewise affine convex function. Then $\forall A \in \mathcal{F}$, $(A, A * \beta) \in \mathcal{S}_{asc}(\beta)$.*

PROOF. The output process is at least $A * \beta$: with $s = 0$, an output process B is such that $B(t) \geq B(0) + \beta(t) \wedge A * \beta(t) = A * \beta(t)$.

We prove that $\forall t \in \mathbb{R}_+$, $\forall s \leq t$, $A * \beta(t) \leq A * \beta(s) + \beta(t-s) \wedge \inf_{s \leq u \leq t} A(u) + \beta(t-u)$. Let $v \leq t$ such that $A * \beta(t) = A(v) + \beta(t-v)$. If $s \leq v$, the inequality trivially holds. If $s > v$, let $u \leq s$ such that $A * \beta(s) = A(u) + \beta(s-u)$. Suppose that $A * \beta(t) < A * \beta(s) + \beta(t-s)$. Then, $A(v) + \beta(t-v) < A(u) + \beta(s-u) + \beta(t-s)$. Moreover, by definition of u, $A(u) + \beta(s-u) \leq A(v) + \beta(s-v)$. Then, $\beta(s-u) - \beta(s-v) \leq A(v) - A(u) < \beta(s-u) + \beta(t-s) - \beta(t-v)$ and $\beta(t-v) < \beta(t-s) + \beta(s-v)$. This is impossible since β is convex with $\beta(0) = 0$, so $A * \beta(t) \geq A * \beta(s) + \beta(t-s)$. $\square$

5.2 Weakly strict service curves

A natural intermediate class of service curves to consider would be the following: for $\beta \in \mathcal{F}$:

- *weakly strict service curve:* $\mathcal{S}_{wstrict}(\beta) = \{(A, B) \in \mathcal{F} \times \mathcal{F} \mid \forall t \in \mathbb{R}_+,\ A(t) \geq B(t) \geq A(start(t)) + \beta(t - start(t))\}$.

In this paragraph, we see that this notion does not bring much to the theory, concerning the issue of this article.

Indeed, Theorem 4 asserts this class of service curves is not stable by composition. Second, the proof of Theorem 3 can be transposed to weakly service curves, thanks to Theorem 3 in [3]: the class of weakly strict and strict service curves coincide on pure delay curves.

Finally, Theorem 2 can be adapted:

THEOREM 5. *The following statements hold:*

(i) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{wstrict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2) \Rightarrow P_1(\mathcal{S}) \in \mathcal{S}_{simple}((\beta - \alpha_2)_\uparrow)$;

(ii) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{wstrict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2) \not\Rightarrow P_1(\mathcal{S}) \subseteq \mathcal{S}_{wstrict}((\beta - \alpha_2)_\uparrow)$;

(iii) $Ag(\mathcal{S}) \subseteq \mathcal{S}_{wstrict}(\beta)$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2)$ and $P_1(\mathcal{S}) \subseteq \mathcal{A}(\alpha_1) \Rightarrow P_1(\mathcal{S}) \subseteq \mathcal{S}_{wstrict}(((\beta - \alpha_2)\overline{\oslash}\alpha_1))_\uparrow)$, where $\forall t > 0$, $f\overline{\oslash}g(t) = \inf_{u \geq 0} f(t+u) - g(t)$.

PROOF. The proof of the two first statements is the same as in Theorem 2.

Equation (1) is valid for $s = start(t)$. Let $u = \sup\{v \leq t \mid A_1(v) = B_1(v)\}$ the start of the backlogged period for t regarding Flow 1 only. Then, $u \geq s$ and

$$\begin{aligned} B_1(t) &\geq A_1(u) - [A_1(u) - A_1(s)] - [B_2(t) - A_2(s)] \\ &\quad + \beta(t-s) \\ &\geq A_1(u) - \alpha_1(u-s) - \alpha_2(t-s) + \beta(t-s) \\ &\geq A_1(u) + \inf_{0 \leq s \leq u} \beta(t-s) - \alpha_2(t-s) - \alpha_1(u-s) \\ &\geq A_1(u) + \inf_{v \geq 0} (\beta - \alpha_2)(t-u+v) - \alpha_1(v). \end{aligned}$$

One can now conclude by remarking that $A_1(u) = B_1(u) \leq B_1(t)$ and that B_1 is non-decreasing. $\square$

With the same example as in Section 3: $\beta(t)1 = 3t$, $\alpha_1(t) = 2 + t$ and $\alpha_2(t) = t$, $((\beta - \alpha_2)\overline{\oslash}\alpha_1))_\uparrow = 2(t-1)_+$.

5.3 Sufficiently strict service curves

Recently a new notion of service curves has been defined, in order to deal with non-FIFO flows and their performance evaluation. Those *sufficiently strict service curves* are defined as follows.

- $\mathcal{S}_{s3c}(\beta) = \{(A, B) \in \mathcal{F} \times \mathcal{F} \mid \forall t \in \mathbb{R}_+,\ A(t) \geq B(t) \geq A(t - D(t)) + \beta(D(t))\}$,

where $D(t)$ is the *maximum achievable dwell period* (MADP) at time t, that is, $D(t) = t - t_0(t)$ where t_0 is the arrival time of the oldest bit of data in the system at time t under *all possible processing orders*. Under our assumptions, and for a simple server, $t_0(t) = start(t)$. Then this definition seems to be the same as weakly strong service curves. However, the authors showed in [10], Theorem 5, that their curve has the concatenation property whereas weakly strict service curves do not have that property.

In appearance, this notion of service curve is in contradiction with Theorem 4, and appears as a good candidate for intermediate service curves. In fact, this is not, because of the definition of the MADP, that depends on how the system is made: when a system is composed of servers in tandem, then the overall service curve is the concatenation of the services, but the MADP of the system does not coincide with the start of the backlogged period of a server that guaranties the sufficiently strict overall service curve. So, because of that, $\mathcal{S}_{s3c}(\beta_2) \circ \mathcal{S}_{s3c}(\beta_2) \not\subseteq \mathcal{S}_{s3c}(\beta_1 * \beta_2)$.

On the other hand, Theorem 4 suggests that the description of a system by a service curve is not precise enough to describe a system, and lots of information is lost by the concatenation and by computing individual service curves. Then, adding the information contained in the MADP may lead to the right notion: let $\mathcal{D} : \mathcal{S}_{s3c}(\beta) \to \mathcal{P}(\mathcal{F})$; $(A, B) \mapsto \mathcal{D}((A, B)) \subseteq \{D \in \mathcal{F} \mid D$ is a possible dwell period for $(A, B)\}$ (a *possible* dwell period must be such that $\forall t \in \mathbb{R}_+$, $D(t) \leq t - start(t)$). Then, one can define another class of service curve:

- $\mathcal{S}(\beta, \mathcal{D}) = \{(A, B) \in \mathcal{F} \times \mathcal{F} \mid \exists D \in \mathcal{D}(A, B),\ \forall t \geq 0,\ A(t) \geq B(t) \geq A(t - D(t))\}$.

Then the following theorem is straightforward:

THEOREM 6. *Let $\beta, \beta_1, \beta_2 \in \mathcal{F}$, $\mathcal{D} : \mathcal{S}_{s3c}(\beta) \to \mathcal{P}(\mathcal{F})$, $\mathcal{D}_1 : \mathcal{S}_{s3c}(\beta_1) \to \mathcal{P}(\mathcal{F})$ and $\mathcal{D}_2 : \mathcal{S}_{s3c}(\beta_2) \to \mathcal{P}(\mathcal{F})$ be possible dwell periods for respectively β,β_1 and β_2. Then,*

- *$\mathcal{S}_{s3c}(\beta_2, \mathcal{D}_2) \circ \mathcal{S}_{s3c}(\beta_1, \mathcal{D}_1) \subseteq \mathcal{S}(\beta_1 * \beta_2, \mathcal{D}')$ with $\forall t$, $\mathcal{D}'(A, C) = \{D \mid \exists B \in \mathcal{F}, \exists D_1 \in \mathcal{D}_1(A, B), D_2 \in \mathcal{D}_2(B, C),$ such that $D_2(t) + D_1(t - D_2(t))\}$;*
- *If $\mathcal{S} \in \mathcal{F}^2 \times \mathcal{F}^2$, then $Ag(\mathcal{S}) \subseteq \mathcal{S}_{s3c}(\beta, \mathcal{D})$ and $P_2(\mathcal{S}) \subseteq \mathcal{A}(\alpha_2) \Rightarrow P_1(\mathcal{S}) \subseteq \mathcal{S}_{s3c}((\beta - \alpha_2)_\uparrow, \mathcal{D})$.*

PROOF. The first statement is a reformulation of Theorem 5 in [10] and the second statement is straightforward from Equation (1). □

This notion of service curve is an intermediate between simple and strict service curves, with the nice properties we targeted. But, if this curve is obtained after several steps of computations, the topology of the network being studied is hidden in $\mathcal{D}$. Moreover, up to our knowledge, there is no nice representation for the MADP.

6. CONCLUSION

In this article we compared the behavior of strict and simple service curves with two operations: individual service curves and composition. It seems that no more restrictive class than that of simple service curve can be stable with the composition, whereas, it is possible to define strict individual service curve. Following the example of sufficiently strict service curves, any notion of service curve that is stable with composition and individual service curves must contain more information that only one function.

Acknowledgments

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