Individual Service Curves for Bandwidth-Sharing Policies using Network Calculus

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***Abstract*—Bandwidth-sharing policies guarantee a proportion of the service offered by a server to each class of traffic. These policies include GPS (Generalized Processor Sharing) and DRR (Deficit Round Robin). In this paper, we compute service curves for each class of traffic for such scheduling policies using network calculus. These individual service curves take into account the characteristics of the cross-traffic and thus improve the state-of-the-art performance bounds like delay or backlog.**

***Index Terms*—Network calculus, Deficit round robin, scheduling, performance evaluation.**

I. Introduction

RECENT communication networks, like 5G, have stronger requirements in terms of end-to-end latency and reliability. In order to have more control on the network, more elaborate schedulers have been defined to guarantee delays of critical classes of traffic. Among them, the bandwidth-sharing policies have gained popularity. The idealized version of such policies is GPS, where each class of traffic is guaranteed a service rate. Several implementations have been proposed. One can cite the Round-Robin policies where each class is served in rounds. For example, in DRR, each class is allocated a quantum, *i.e.* the maximum amount of data that can be served in a round.

Network calculus [1], [5], [6] is a theory used to compute deterministic performance bounds, by abstracting data flows and servers by curves bounding the maximal or minimal amount of data that can arrive or be served in each time interval. It emerges as a pertinent tool to analyze networks with strong latency and reliability requirements. GPS was among the first policies to be analyzed in this framework [7], and recent works concern the modeling DRR [3]. These works only model the *sharing* part, and do not take into account the characteristics of the incoming traffic. Indeed, when a class has no data to transmit, its share of the bandwidth is shared among the other flows. This phenomenon has recently been considered for the GPS policy [4].

In this paper, we propose to generalize this result to bandwidth-sharing policies. As a consequence, we both improve the individual service curves for the DRR and similar policies, and generalize the result of [4]. To this end, we will follow the steps of the alternative proof for GPS in [1, Theorem 7.8], that has some inaccuracies, thus correct it.

In Section II, we recall the network calculus model. In Section III, we present and prove our main result. Finally, in Section IV, we apply the results to GPS and DRR, and compare them with the state of the art.

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II. Model of Bandwidth-Sharing Policies

In this paper, we use the notations $(x)_+ = \max(0, x)$ and $1_{\geq\tau} : t \mapsto 1$ if $t \geq \tau$ and 0 otherwise.

Consider a server crossed by n flows. We assume that the system is empty at time 0. For all $t \geq 0$ and $i \in \{1, \dots, n\}$, we denote by $A_i(t)$ (resp. $D_i(t)$) the cumulative amount of data of flow i that arrived (resp. departed) in the time interval $[0, t)$. Then, for all flow i, $A_i(0) = D_i(0) = 0$.

We also denote by $C(t)$ the cumulative amount of service that is offered to the flows by the server.

For the sake of concision, we will use bivariate functions, and $A_i(s, t)$ represents the amount of data arrived during the time interval $(s, t]$: $A_i(s, t) = A_i(t) - A_i(s)$, and similarly for D_i and C. Note that for all these processes are additive: $\forall t \geq u \geq s \geq 0$, $A_i(s, u) + A_i(u, t) = A_i(s, t)$ and similarly for D_i and C.

Let M be a subset of flows. The interval $(s, t]$ is called a *backlogged period* for flows in M if $\forall u \in (s, t]$, $\sum_{i\in M} A_i(u) > \sum_{i\in M} D_i(u)$. The start of backlogged period of t for flows in M is $start_M(t) = \inf\{s \leq t \mid \sum_{i\in M} A_i(s) = \sum_{i\in M} D_i(s)\}$.

Definition 1. *The server has a bandwidth-sharing policy if there exist positive numbers $(\phi_i)_{1\leq i\leq n}$ and non-negative numbers $(H_{i,j})_{1\leq i,j\leq n}$ such that for all $i \neq j$, for all backlogged period $(s, t]$ of flow i, $\phi_j D_i(s, t) \geq \phi_i (D_j(s, t) - H_{i,j})_+$.*

Informally, $\phi_i / (\sum_{j=1}^n \phi_j)$ represents the share of the bandwidth guaranteed for flow i, and $H_{i,j}$ a tolerance regarding this guarantee (*e.g.*, due to packetization).

Network calculus [1], [5], [6] is a theory that models flows and servers by curves that bound the number of arrivals or service for each time interval. Note that these curves can always be assumed to be non-decreasing and null at 0.

- α_i is an arrival curve for flow i if for all $t \geq s \geq 0$, $A_i(s, t) \leq \alpha_i(t - s)$;
- β is a variable capacity node (VCN) for a server if for all $t \geq s \geq 0$, $C(s, t) \geq \beta(t - s)$.

We also need to define *strict service curves* that are quasi-equivalent to VCN.

Denote $D = \sum_i D_i$ the aggregate departure process of the server. The server offers a *strict service curve* β if, whenever $(s, t]$ is a backlogged period, $D(s, t) = C(s, t) \geq \beta(t - s)$. It has been proved in [2] that VCN and strict service curves are equivalent if the asymptotic growth rate of β is finite, which we assume from now on. This is not restrictive since bandwidths have physical limitations.

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We will also use the following classical results for strict service curves: a) if β_1 and β_2 are two strict service curves for a server, so is $\max(\beta_1, \beta_2)$; b) suppose $\beta_1 \geq \beta_2$. If β_1 is a strict service curve for a server, so is β_2.

III. A New Residual Strict Service Curve for Bandwidth-Sharing Policies

In this section, we assume a server offering VCN β and having a bandwidth-sharing policy with parameters $(\phi_i)_{1\leq i\leq n}$ and $(H_{i,j})_{1\leq,i,j\leq n}$ crossed by n flows with respective arrival curves α_i. Our aim is to prove the following theorem.

Theorem 1. *If β is convex and α_i concave for all i, there exist non-negative numbers H_M, $M \subseteq \{1, \dots, n-1\}$ such that*

$$\beta_n = \sup_{M\subseteq\{1,\dots,n-1\}} \frac{\phi_n}{\sum_{j\notin M}\phi_j}\Big(\beta - \sum_{i\in M}\alpha_i - H_M\Big)_+ \quad (1)$$

is a strict service curve for flow n.

The proof is divided in several steps. First, we show how to compute an individual strict service curve from a strict service curve for a subset of flows: the bandwidth-sharing property is inherited for subsets of flows. Second, we show how to compute a residual strict service curve when removing one flow from the aggregate flow. Last, these two steps will be used inductively to prove the result and compute $(H_M)_{M\subseteq\{1,\dots,n-1\}}$.

A. Individual service curves

Suppose that we are able to compute the residual strict service curve β^M for a subset M of flows. Lemma 1 shows how to derive a residual service curve for each flow $i \in M$.

Lemma 1. *If β^M is a strict service curve for flows in M, then for all $i \in M$,*

$$\beta_i^M = \frac{\phi_i}{\sum_{j\in M}\phi_j}\Big(\beta^M - H_i\Big)_+ \quad (2)$$

is a strict service curve for flow i and $H_i = \sum_{j\in M} H_{i,j}$.

Proof. For all $s,t \in \mathbb{R}_+$ such that $(s,t]$ is a backlogged for flow i, for all $j \in M$, $\phi_j D_i(s,t) \geq \phi_i(D_j(s,t) - H_{i,j})$. Therefore, $\sum_{j\in M}\phi_j D_i(s,t) \geq \sum_{j\in M}\phi_i(D_j(s,t) - H_{i,j})$, which can be rewritten as

$$(\sum_{j\in M}\phi_j)D_i(s,t) \geq \phi_i(\sum_{j\in M}(D_j(s,t) - H_{i,j})). \quad (3)$$

Since $(s,t]$ is also a backlogged period for the aggregate flows in M, $\sum_{j\in M} D_j(s,t) \geq \beta^M(t-s)$ and then $D_i(s,t) \geq \frac{\phi_i}{\sum_{j\in M}\phi_j}(\beta^M(t-s) - H_i)$. As D_i is not decreasing, we also know that $D_i(s,t) \geq 0$, hence the result. □

B. Residual strict service curve when removing one flow

Let us first prove a preliminary lemma.

Lemma 2. *Let β be a convex and non-decreasing function with $\beta(0) = 0$ and α be a concave, non-negative function. Then, there exists $\tau \in \mathbb{R}_+ \cup \{\infty\}$ such that $(\beta-\alpha)1_{\geq\tau}$ is convex, non-negative and non-decreasing. We denote by $(\beta-\alpha)^{-1}(0)$ the minimal value of such τ .*

Proof. $\beta - \alpha$ is the difference of a convex by a concave function: it is convex. Since $(\beta-\alpha)(0) \leq 0$, three cases can occur. Either (a) $\beta-\alpha$ is non-negative and non-decreasing, and the results holds with $\tau = 0$, or (b) $\forall t \geq 0$, $(\beta-\alpha)(t) \leq 0$, and the results holds with $\tau = \infty$, or (c) there exists $t > 0$ such that $\beta - \alpha$ is increasing on $[t, \infty)$, and $(\beta-\alpha)(t) < 0$. Then there exists a unique $\tau > 0$ such that $(\beta-\alpha)(\tau) = 0$. Therefore, $(\beta-\alpha)1_{\geq\tau} = (\beta-\alpha)_+$ is the maximum of two convex functions, and the rest follows straightforwardly. □

Suppose that β is convex, non-negative and non-decreasing and set $t_1 = (\frac{\phi_1}{\Phi}(\beta - H_1) - \alpha_1)^{-1}(0)$, where $H_1 \geq \sum_{i=1}^n H_{1,i}$ and $\Phi = \sum_{i=1}^n \phi_i$.

Lemma 3. *If $\beta 1_{\geq t_0}$ is a variable capacity node for flows $\{1,\dots,n\}$, and $t_1 \geq t_0$, then $(\beta - \alpha_1 - \frac{\phi_1}{\Phi}H_1)1_{\geq t_1}$ is a strict service curve for flows $\{2,\dots,n\}$.*

Proof. Let s and t be such that $(s,t]$ is a backlogged period for flows $\{2,\dots,n\}$ and $t - s \geq t_1$. We have

$$D(s,t) = C(s,t) \geq \beta(t-s)1_{t-s\geq t_0} = \beta(t-s). \quad (4)$$

Let $p = start_1(s)$ be the start of backlogged period of s for flow 1. On the one hand, from Lemma 1 applied to $M = \{1,\dots,n\}$,

$$D_1(p,s) \geq (\phi_1/\Phi)(C(p,s) - H_1). \quad (5)$$

On the other hand, $D_1(p,t) = D_1(t) - D_1(p) = D_1(t) - A_1(p) \leq A_1(t) - A_1(p) = A_1(p,t) \leq \alpha_1(t-p)$.

Set $D_{-1} = \sum_{i\neq 1} D_i$ and $\Phi_{-1} = \sum_{i\neq 1}\phi_i$. Combining the two previous inequalities, we obtain

$$\begin{aligned} D_{-1}(s,t) &= C(s,t) - D_1(s,t) \\ &= C(s,t) - D_1(p,t) + D_1(p,s) \\ &\geq C(s,t) - \alpha_1(t-p) + (\phi_1/\Phi)(C(p,s) - H_1) \\ &\geq \frac{\Phi_{-1}}{\Phi}C(s,t) - \alpha_1(t-p) + \frac{\phi_1}{\Phi}(C(p,t) - H_1) \\ &\geq \frac{\Phi_{-1}}{\Phi}\beta(t-s) - \alpha_1(t-p) + \frac{\phi_1}{\Phi}(\beta(t-p) - H_1). \end{aligned} \quad (6)$$

As β is convex and non-decreasing and α is concave and non-negative, $(\phi_1/\Phi)(\beta(\cdot) - H_1) - \alpha_1(\cdot)$ is non-decreasing from t_1, and $[(\phi_1/\Phi)(\beta - H_1) - \alpha_1](t-p) \geq [(\phi_1/\Phi)(\beta - H_1) - \alpha_1](t-s)$. Therefore,

$$\begin{aligned} D_{-1}(s,t) &\geq (\Phi_{-1}/\Phi)\beta(t-s) + [(\phi_1/\Phi)(\beta - H_1) - \alpha_1](t-s) \\ &\geq \beta(t-s) - \alpha_1(t-s) - (\phi_1/\Phi)H_1. \end{aligned} \quad (7)$$

We conclude by noticing that $\forall s \leq t$, $D_{-1}(s,t) \geq 0$. □

We can use Lemmas 1 and 3 to deduce residual service curves for the individual flows, and the maximum of these is still a strict service curve.

Example 1. *Consider a server offering the strict service curve $\beta(t) = 8(t-1)_+$ to two classes of flows with respective arrival curves $\alpha_1(t) = 2 + t$ and $\alpha_2(t) = 6 + 3t$, and suppose that $\phi_1 = \phi_2 = 1/2$ and $H_{1,2} = H_{2,1} = 1$.*

From Lemma 1, $\beta_2(t) = \frac{1}{2}(8(t-1)_+ - 1)_+ = 4(t - \frac{9}{8})_+$ is a strict service curve for flow 2. To apply Lemma 3, we compute

$t_1 = (\frac{1}{2}(\beta - 1) - \alpha_1)^{-1}(0) = \frac{13}{6}$. *Then* $\beta_2'(t) = (8(t-1)_+ - (2+t) - \frac{1}{2})_{t \geq \frac{13}{6}} = 7(t - 3/2)_{t \geq \frac{13}{6}}$ *is also a strict service curve for flow 2, as well as* $\max(\beta_2, \beta_2')$*, depicted in Figure 1 (left).*

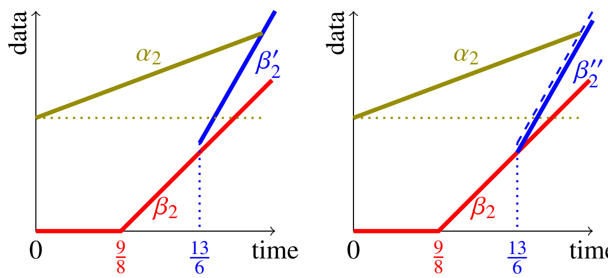

Fig. 1. (Left): Example of residual strict service curves computed with Lemmas 1 and 3. The maximum of the curves is also a strict service curve. The delay reads as the horizontal distance α_2 and $\max(\beta_2, \beta_2')$ (Right): Relaxing the $H_{i,j}$ parameters: the new service curve is smaller but continuous.

We remark on Figure 1 that the curves obtained are not continuous, and could be difficult to compute when more classes of traffic are involved. In order to deal with continuous service curves only, we will later show how to relax the service curves obtained by slightly increasing the values $H_{i,j}$.

The next step of the proof is to apply the Lemmas 1 and 3 inductively. For this we have to carefully handle the order in which the operations are performed.

C. A good order on the operations

Let M be a subset of flows. Let us proceed by induction to define a residual service curve for flows not in M. Without loss of generality, set $M = \{1, \dots, k\}$.

Initially, we define $\tau_0 = 0$, $\beta_0 = \beta$, $\Phi_{\geq 0} = \Phi_{\geq 1} = \sum_{j=1}^n \phi_j$ and $\forall j \in M$, $H_j^{\geq 0} = 0$.

For the induction step, suppose that τ_i, $\Phi_{\geq i+1}$ and $H_j^{\geq i}$ have been defined for $i < k$. One defines β_{i+1}, τ_{i+1}, $\Phi_{\geq i+2}$ and $H_j^{\geq i+1}$ as follows:

- $\forall j \in \{i+1, \dots, k\}$, $H_j^{\geq i+1} = \max(\sum_{k=i+1}^n H_{j,k}, \frac{\Phi_{\geq i+1}}{\Phi_{\geq i}} H_j^{\geq i})$;
- $\forall j \in \{i+1, \dots, k\}$, $t_j^{i+1} = (\frac{\phi_j}{\Phi_{\geq i+1}}(\beta_i - H_j^{\geq i+1}) - \alpha_j)^{-1}(0)$, and define $\tau_{i+1} = \min_{j \in \{i+1, \dots, k\}} t_j$. Suppose without loss of generality, by renumbering the flows, that $\tau_{i+1} = t_{i+1}^{i+1}$;
- $\Phi_{\geq i+2} = \Phi_{\geq i+1} - \phi_{i+1}$ (with the new numbering of flows);
- $\beta_{i+1} = \beta - \sum_{j=1}^{i+1} \alpha_j - \sum_{j=1}^{i+1} \frac{\phi_j}{\Phi_{\geq j}} H_j^{\geq j}$ (likewise).

Lemma 4. *With the construction above,* $\forall i < k$, $\tau_{i+1} \geq \tau_i$.

Proof. We proceed by contradiction and suppose that $\tau_{i+1} < \tau_i$. Let $t \in (\tau_{i+1}, \tau_i)$. From Lemma 2,

- $t < \tau_i \leq t_{i+1}^i$, so that $\frac{\phi_{i+1}}{\Phi_{\geq i}}(\beta_{i-1} - H_{i+1}^{\geq i})(t) < \alpha_{i+1}(t)$;
- $t > \tau_{t+i} = t_{i+1}^{i+1}$, so that $\frac{\phi_{i+1}}{\Phi_{\geq_{i+1}}}(\beta_i - H_{i+1}^{\geq i+1})(t) \geq \alpha_{i+1}(t)$;
- $t < \tau_i = t_i^i$, so that $\frac{\phi_i}{\Phi_{\geq i}}(\beta_{i-1} - H_i^{\geq i})(t) < \alpha_i(t)$.

Combining two first items leads to

$$\frac{\phi_{i+1}}{\Phi_{\geq i}}(\beta_{i-1} - H_{i+1}^{\geq i})(t) < \frac{\phi_{i+1}}{\Phi_{\geq i+1}}(\beta_i - H_{i+1}^{\geq i+1})(t), \quad (8)$$

which after rewriting is equivalent to

$$\Phi_{\geq i+1}(\beta_{i-1} - H_{i+1}^{\geq i})(t) < \Phi_{\geq i}(\beta_{i-1} - \alpha_i - \frac{\phi_i}{\Phi_{\geq i}} H_i^{\geq i} - H_{i+1}^{\geq i+1})(t) \quad (9)$$

and

$$\Phi_{\geq i} H_{i+1}^{\geq i+1} - \Phi_{\geq i+1} H_{i+1}^{\geq i} < \phi_i(\beta_{i-1}(t) - H_i^{\geq i}) - \Phi_{\geq i}\alpha_i(t). \quad (10)$$

By definition of $H_{i+1}^{\geq i+1}$, the inequality $\Phi_{\geq i} H_{i+1}^{\geq i+1} - \Phi_{\geq i+1} H_{i+1}^{\geq i} \geq 0$ holds. Therefore,

$$\phi_i(\beta_{i-1} - H_i^{\geq i}) - \Phi_{\geq i}\alpha_i > 0, \quad (11)$$

which is in contradiction with the third inequality and proves that $\tau_{i+1} \geq \tau_i$. □

Lemma 5. *For all* $i \in \{0, \dots, k\}$, $\beta_i 1_{\geq \tau_i}$ *is a strict service curve for flows* $i+1, \dots, n$.

Proof. We prove the result by induction on i. We have $\beta_0 = \beta 1_{t \geq 0} = \beta$, so the result holds for $i = 0$.

Assume now that for $i < k$, $\beta_i 1_{\geq \tau_i}$ is a strict service curve for flows $i+1, \dots, n$. From Lemma 4, $\tau_{i+1} \geq \tau_i$, and Lemma 3 can be applied. As $\sum_{k \geq i+1} H_{i+1,k} \leq H_{i+1}^{\geq i+1}$, the function $(\beta_i - \alpha_{i+1} - \frac{\phi_{i+1}}{\Phi_{\geq i+1}} H_{i+1}^{\geq i+1})_{t \geq \tau_{i+1}} = \beta_{i+1} 1_{\geq \tau_{i+1}}$ is a strict service curve for flows $i+2, \dots, n$. □

From Lemmas 5 and 1, one can deduce that $\forall i \in \{0, \dots, k\}$, $\frac{\phi_n}{\Phi_{\geq i+1}}(\beta_i 1_{\geq \tau_i} - \sum_{j \geq i+1} H_{n,j})_+$ is a strict service curve for flow n. Therefore,

$$\max_{i \in \{0, \dots, k\}} \frac{\phi_n}{\Phi_{\geq i+1}} \Big(\beta_i 1_{\geq \tau_i} - \sum_{j \geq i+1} H_{n,j}\Big)_+. \quad (12)$$

is a strict service curve for flow n. We now show that the formula can be simplified by removing the $1_{\geq \tau_i}$ parts. This is done by relaxing $\sum_{j \geq i+1} H_{n,j}$ and replacing it by $H_n^{\geq i+1}$.

Lemma 6. *For all* $j \geq i+1$*, for all* $t \leq \tau_i$, $\frac{\phi_j}{\Phi_{\geq i+1}}(\beta_i(t) - H_j^{\geq i+1}) \leq \frac{\phi_j}{\Phi_{\geq i}}(\beta_{i-1} - H_j^{\geq i})(t)$.

Proof. We prove the contraposition:

$$\frac{\phi_j}{\Phi_{\geq i+1}}(\beta_i - H_j^{\geq i+1})(t) > \frac{\phi_j}{\Phi_{\geq i}}(\beta_{i-1} - H_j^{\geq i})(t) \quad (13)$$

is equivalent to (by rewriting β_i as $\beta_{i-1} - \alpha_i - \frac{\phi_i}{\Phi_{\geq i}} H_i^{\geq i}$)

$$\Phi_{\geq i}(\beta_{i-1} - \alpha_i - \frac{\phi_i}{\Phi_{\geq i}} H_i^{\geq i} - H_j^{\geq i+1})(t) > \Phi_{\geq i+1}(\beta_{i-1} - H_j^{\geq i})(t). \quad (14)$$

Re-arranging the terms, we equivalently get

$$\phi_i \beta_{i-1}(t) - \Phi_{\geq i}\alpha_i(t) + \phi_i H_i^{\geq i} > \Phi_{\geq i} H_j^{\geq i+1} - \Phi_{\geq i+1} H_j^{\geq i} (\geq 0), \quad (15)$$

which implies $t > \tau_i$. □

Example 2. *Let us continue Example 1. The individual service curve for flow 2 is not continuous. Lemma 6 suggests replacing* β_2' *by* $\beta_2''(t) = (\beta_2'(t) - \frac{1}{2})_+ = 7(t - 11/7)1_{t \geq \frac{13}{6}}$ *and the new service curve obtained is depicted in Figure 1 (right).*

Lemma 6 enables to assert that

$$\frac{\phi_n}{\sum_{i \notin M} \phi_i} \Big(\beta - \sum_{i \in M} (\alpha_i - \frac{\phi_i}{\Phi_{\geq i}} H_i^{\geq i}) - H_n^{\geq k+1}\Big)_+ \quad (16)$$

is a strict service curve for flow n. Taking the maximum for all subsets M not containing n finishes the proof of Theorem 1.

IV. Application to bandwidth-sharing policies

In this section, we specialize the results to two classical bandwidth-sharing policies: we retrieve the state-of-the-art optimal service curve of GPS and improve the state-of-the-art DRR service curve.

A. Generalized processor sharing

A server is a GPS server if for all flows i, all $s \leq t$ such that $(s,t]$ is backlogged period for flow i, for each flow j, $\phi_j D_i(s,t) \geq \phi_i D_j(s,t)$.

Theorem 2 (GPS service curve in [4, Theorem 1])**.** *A strict service curve offered to flow n is*

$$\beta_n = \sup_{M\subseteq\{1,\dots,n-1\}} \frac{\phi_n}{\sum_{j\notin M}\phi_j}\Big(\beta - \sum_{i\in M}\alpha_i\Big)_+. \tag{17}$$

We are here in the particular case where $H_{i,j} = 0$ for all i,j, and then we also have $H_j^{\geq i} = 0$. We then obtain exactly the same residual strict service curve.

The proof presented in this paper corrects the proof of [1, Theorem 7.8], where the good order to remove the flows so that the sequence of τ_i is non-decreasing was not defined.

B. Deficit Round Robin

DRR is an implementation of GPS, where each flow i is assigned a quantum Q_i representing the maximum amount of service that can be provided to flow i at each round. The head-of-line packet can be served in its round provided that its length is at most the amount of data remaining for the round. More details can be found in [3] and [1].

Theorem 3 (DRR service curve in [3])**.** *A strict service curve offered to flow n is*

$$\beta_n = \frac{Q_i}{F}\Big(\beta - (L-\ell_i) - \frac{(F-Q_i)(Q_i+\ell_i)}{Q_i}\Big)_+, \tag{18}$$

where ℓ_i is the maximum packet size of flow i and $F = \sum_{i=1}^n Q_i$ and $L = \sum_{i=1}^n \ell_i$.

For DRR, if $(s,t]$ is a backlogged period for flow i, we have for all $i \neq j$,

$$\frac{D_i(s,t)+\ell_i}{Q_i} \geq \frac{D_j(s,t)-\ell_j-Q_j}{Q_j}, \tag{19}$$

which satisfies Definition 1 with $\phi_i = \frac{1}{Q_i}$, $H_{i,i} = 0$ and $j \neq i$, $H_{i,j} = Q_j + \ell_j + \frac{Q_j}{Q_i}\ell_i$. Straightforward computations show that the service curve of [3] can be retrieved by applying Lemma 1 to $M = \{1,\dots,n\}$ (this corresponds to β_2 in Example 1).

Consider a DRR server with four classes of traffic, described in Table I. The server serves data at the constant rate 5 Gb/s: $\beta(t) = 5\,10^9 t$. The quanta offered to each class of traffic is 2 kB, and we have $Q_i = 16000$ b.

Table II shows the comparison of the delays obtained with the service curve of [3] and with the service curve computed in Theorem 1. Classes are given in the good order (according to the proof). We can observe that the delay of the first class is not improved. Indeed, as it is the first one to become empty, it does not benefit from the improvement. In opposition, the improvement for the other classes grows with the number of the class, up to 50% for the last one.

TABLE I
Characteristics of the four classes of flows

Class	burst	arrival rate	packet size
Electric protection	42.56 kb	8.521 Mb/s	3040 b
Virtual reality game	2.16 Mb	180 Mb/s	12 kb
Video conference	3.24 Mb	162 Mb/s	12 kb
4K video	7.2 Mb	180 Mb/s	12 kb

TABLE II
Delays obtained for the four classes of flows and comparison with the state of the art

Class	Thm. 3, [3]	Thm. 1	improvement
Electric protection	52 μs	52 μs	0 %
Virtual reality game	1.75 ms	1.33 ms	24 %
Video conference	2.61 ms	1.82 ms	30 %
4K video	5.78 ms	2.74 ms	53 %

V. Conclusion

In this paper, we presented a new residual service curve for bandwidth-sharing policies in network calculus, and showed how to apply it in two particular cases: GPS and DRR. This result both generalizes the GPS result of [4] to all bandwidth-sharing policies and improve the residual service curves of the bandwidth-sharing policies that did not take into account the arrival curves of the competing traffic.

If more information is available about the packet lengths (such as the minimum length of a packet), the results can be applied to more round-robin-like service policies: Round Robin, Packet Round Robin or Weighted Round Robin...).

Future work will focus on finding optimal service curves. Indeed, it is very doubtful that the service curve presented here is optimal is general, due to the relaxation of $H_{i,j}$, and it might be interesting to see how to improve this aspect of the proof.

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