

Correlation Clustering and Two-edge-connected Augmentation for Planar Graphs

Philip N. Klein¹, Claire Mathieu^{2,3}, and Hang Zhou³

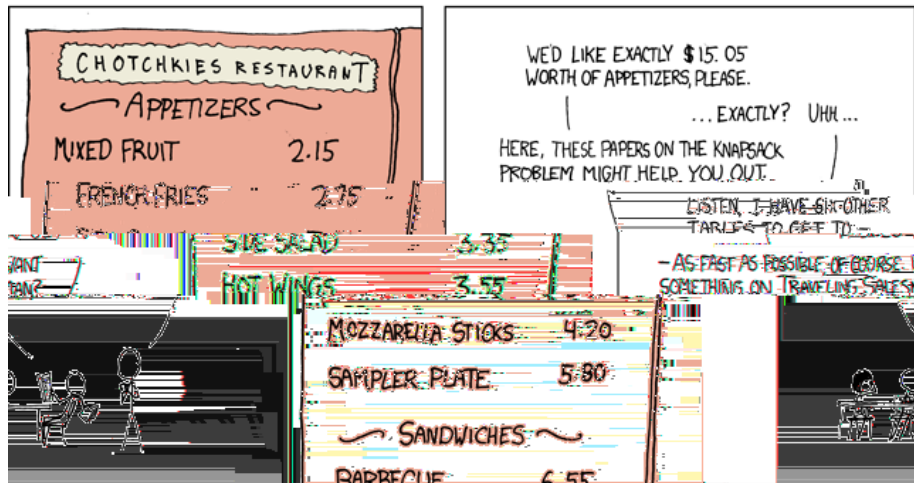
¹Brown University, United States

²CNRS, France

³École Normale Supérieure de Paris, France



MY HOBBY: EMBEDDING NP-COMPLETE PROBLEMS IN RESTAURANT ORDERS



Approximation Schemes for Planar Graphs

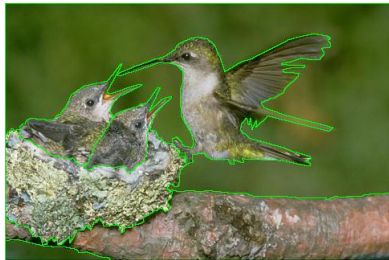
- **Local problems (1977-1983):**
independent set, vertex cover, dominating set, etc.
- **Connectivity problems (2005-2011):**
TSP, Steiner, 2-edge-connected subgraph, etc.
- **Other problems (2012-2014):**
multiway cut, k-center.
- **In our work:**
correlation clustering, 2-edge-connected augmentation.

Correlation Clustering

Input: a graph with edge-weights, where every edge is labelled either $\langle + \rangle$ or $\langle - \rangle$ according to *similarity of its endpoints*

Output: a partition of the vertices that disagrees with the edge labels as little as possible

Motivated by **image segmentation**.

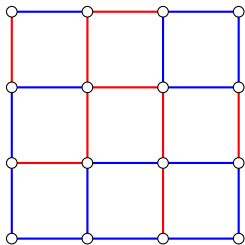


(an example from Berkeley Segmentation Dataset)

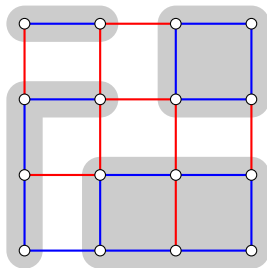
Correlation Clustering

Input: a graph with edge-weights, where every edge is labelled either $\langle + \rangle$ or $\langle - \rangle$ according to *similarity of its endpoints*

Output: a partition of the vertices that disagrees with the edge labels as little as possible



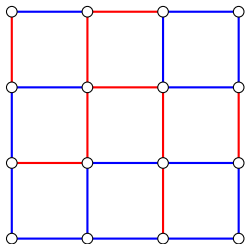

clustering



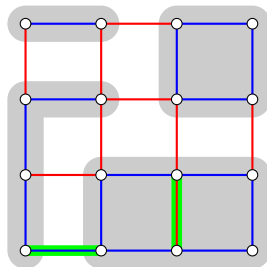
Correlation Clustering

Input: a graph with edge-weights, where every edge is labelled either $\langle + \rangle$ or $\langle - \rangle$ according to *similarity of its endpoints*

Output: a partition of the vertices that disagrees with the edge labels as little as possible




clustering



Correlation Clustering

Input: a graph with edge-weights, where every edge is labelled either $\langle + \rangle$ or $\langle - \rangle$ according to *similarity of its endpoints*

Output: a partition of the vertices that disagrees with the edge labels as little as possible

Our Result

PTAS for correlation clustering in **planar graphs**

Previous results:

- **Constant-factor approximation for minor-excluded graphs** [Demaine, Emanuel, Fiat, Immorlica, 2006]
- **NP-hardness for planar graphs** [Bachrach, Kohli, Kolmogorov, Zadimoghaddam, 2013]
- **APX-hardness for general graphs** [Bansal, Blum, Chawla, 2004]

Two-Edge-Connected Augmentation

Goal: Augment input subset R of edges of a graph so that the result is a collection of 2-edge-connected components.

Our Result

PTAS for two-edge-connected augmentation in **planar graphs**

Previous results for general graphs:

- **APX-hardness** [Kortsarz, Krauthgamer, Lee, 2004]
- **2-approximation** [Jain, 2001]

Reduction from correlation clustering to 2-edge-connected augmentation

- planar duality

PTAS for 2-edge-connected augmentation

- prize-collecting clustering
- brick decomposition
- sphere-cut decomposition
- dynamic programming

Reduction from correlation clustering to 2-edge-connected augmentation:

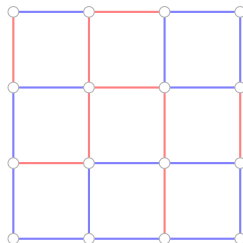
Start from an instance of correlation clustering

Construct an instance of 2-edge-connected augmentation:

- the graph is the dual of the correlation clustering graph
- the set R contains the duals of the $\langle - \rangle$ edges

Solve the 2-edge-connected augmentation instance

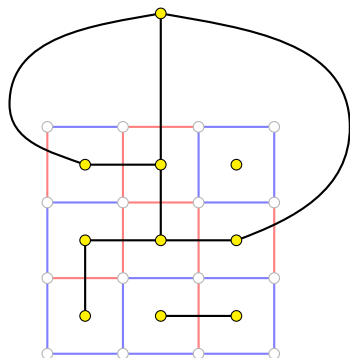
Switch the labels of the dual edges of the augmentation to obtain a partition



Reduction

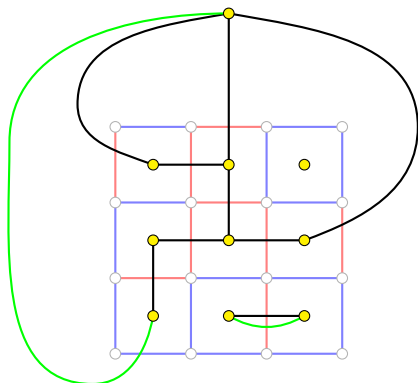
Reduction from correlation clustering to 2-edge-connected augmentation:

- 1 Start from an instance of correlation clustering
- 2 Construct an instance of 2-edge-connected augmentation:
 - the graph is the dual of the correlation clustering graph
 - the set R contains the duals of the $\langle - \rangle$ edges
- 3 Solve the 2-edge-connected augmentation instance
- 4 Switch the labels of the dual edges of the augmentation to obtain a partition



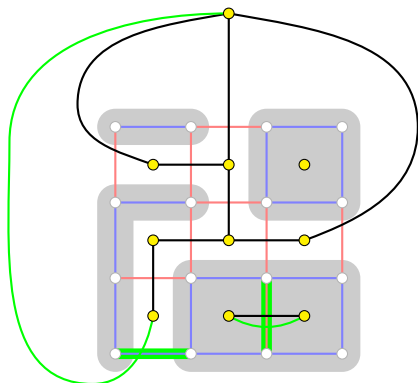
Reduction from correlation clustering to 2-edge-connected augmentation:

- 1 Start from an instance of correlation clustering
- 2 Construct an instance of 2-edge-connected augmentation:
 - the graph is the dual of the correlation clustering graph
 - the set R contains the duals of the $\langle - \rangle$ edges
- 3 Solve the 2-edge-connected augmentation instance
- 4 Switch the labels of the dual edges of the augmentation to obtain a partition



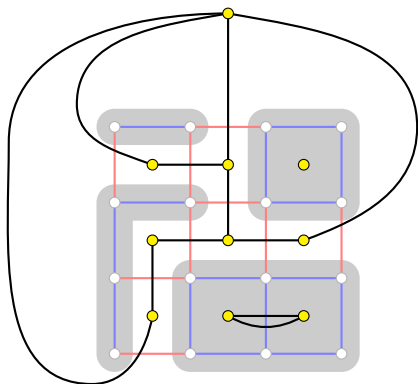
Reduction from correlation clustering to 2-edge-connected augmentation:

- 1 Start from an instance of correlation clustering
- 2 Construct an instance of 2-edge-connected augmentation:
 - the graph is the dual of the correlation clustering graph
 - the set R contains the duals of the $\langle - \rangle$ edges
- 3 Solve the 2-edge-connected augmentation instance
- 4 Switch the labels of the dual edges of the augmentation to obtain a partition



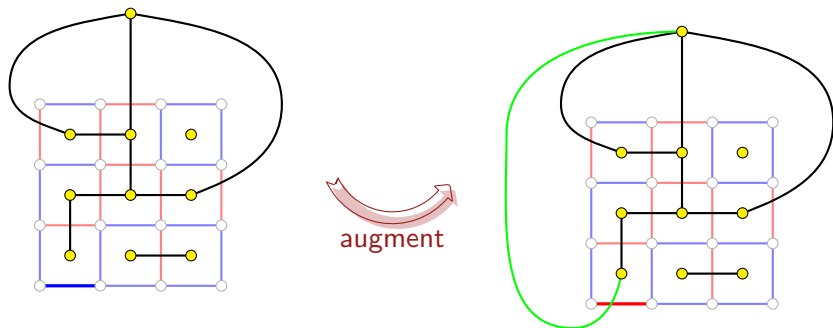
Reduction

Observation: A partition of $V[G]$ has a 1-to-1 correspondence to a collection of 2-edge-connected components of G^* .



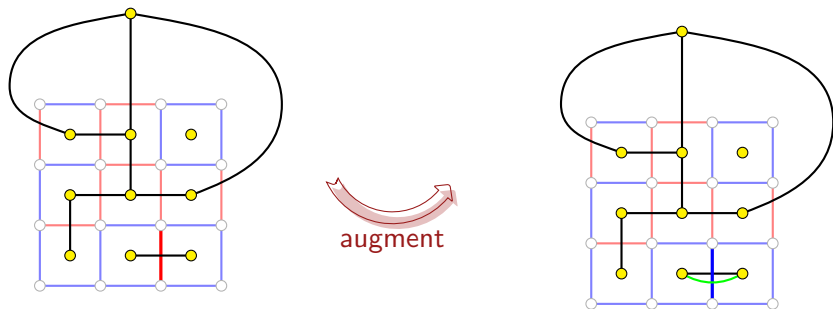
Reduction

- Switch a $\langle + \rangle$ edge to a $\langle - \rangle$ edge \Leftrightarrow add an edge in the dual
- Switch a $\langle - \rangle$ edge to a $\langle + \rangle$ edge \Leftrightarrow remove an edge in the dual \Leftrightarrow double that edge in the dual
- Only need to find an augmentation in the dual so that every component is 2-edge-connected.



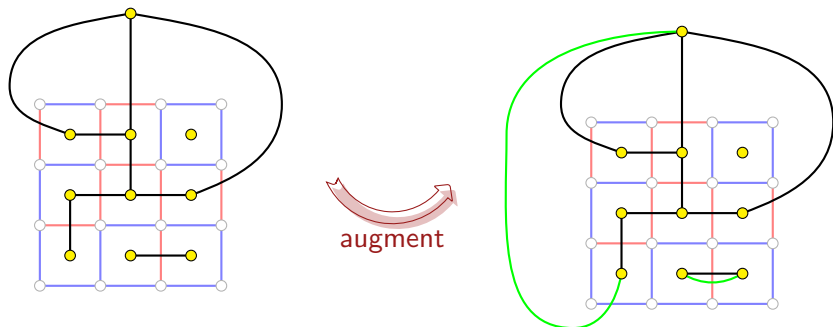
Reduction

- Switch a $\langle + \rangle$ edge to a $\langle - \rangle$ edge \Leftrightarrow add an edge in the dual
- Switch a $\langle - \rangle$ edge to a $\langle + \rangle$ edge \Leftrightarrow remove an edge in the dual \Leftrightarrow double that edge in the dual
- Only need to find an augmentation in the dual so that every component is 2-edge-connected.



Reduction

- Switch a $\langle + \rangle$ edge to a $\langle - \rangle$ edge \Leftrightarrow add an edge in the dual
- Switch a $\langle - \rangle$ edge to a $\langle + \rangle$ edge \Leftrightarrow remove an edge in the dual \Leftrightarrow double that edge in the dual
- Only need to find an augmentation in the dual so that every component is 2-edge-connected.



Reduction

- Switch a $\langle + \rangle$ edge to a $\langle - \rangle$ edge \Leftrightarrow add an edge in the dual
- Switch a $\langle - \rangle$ edge to a $\langle + \rangle$ edge \Leftrightarrow remove an edge in the dual \Leftrightarrow double that edge in the dual
- Only need to find an augmentation in the dual so that every component is 2-edge-connected.

Theorem

There is an approximation-preserving reduction from correlation clustering to 2-edge-connected augmentation.

Approximation Scheme

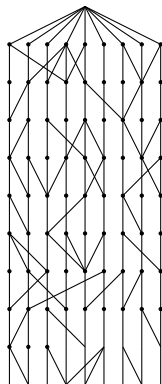
Universal paradigm

- 1 Reduce to bounded treewidth
- 2 Solve the problem by dynamic programming

Q: How do we reduce to bounded treewidth?

A: For local problems, Baker's basic framework:

- 1 Breadth First Search
- 2 Delete edges on one level every $1/\epsilon$ levels



Approximation Scheme

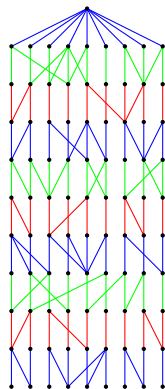
Universal paradigm

- 1 Reduce to bounded treewidth
- 2 Solve the problem by dynamic programming

Q: How do we reduce to bounded treewidth?

A: For local problems, Baker's basic framework:

- 1 Breadth First Search
- 2 Delete edges on one level every $1/\epsilon$ levels



Approximation Scheme

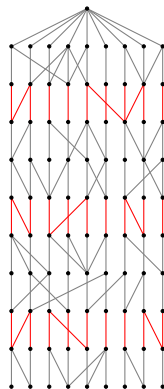
Universal paradigm

- 1 Reduce to bounded treewidth
- 2 Solve the problem by dynamic programming

Q: How do we reduce to bounded treewidth?

A: For local problems, Baker's basic framework:

- 1 Breadth First Search
- 2 Delete edges on one level every $1/\epsilon$ levels



Approximation Scheme

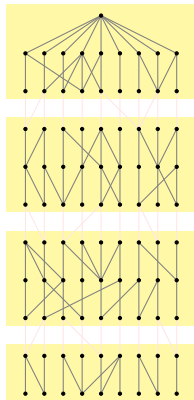
Universal paradigm

- 1 Reduce to bounded treewidth
- 2 Solve the problem by dynamic programming

Q: How do we reduce to bounded treewidth?

A: For local problems, Baker's basic framework:

- 1 Breadth First Search
- 2 Delete edges on one level every $1/\epsilon$ levels



Klein's Dual Framework for Non-Local Problems

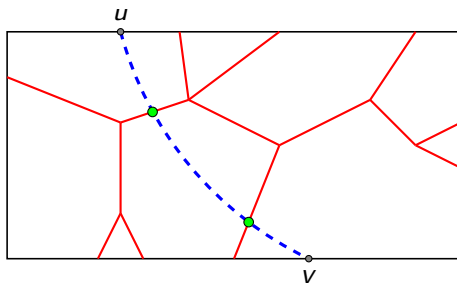
- 1 Compute a brick decomposition such that
 - The total weight of brick boundaries is $O(OPT)$;
 - **Structure Property:** A near-optimal solution inside each brick is simple.
- 2 Breadth First Search on the dual graph of the brick graph
- 3 Delete or contract edges on one level every $1/\epsilon$ levels

Ensure that the brick graph has bounded treewidth.

Our Contribution: A New Structural Property on Bricks

Structure Property

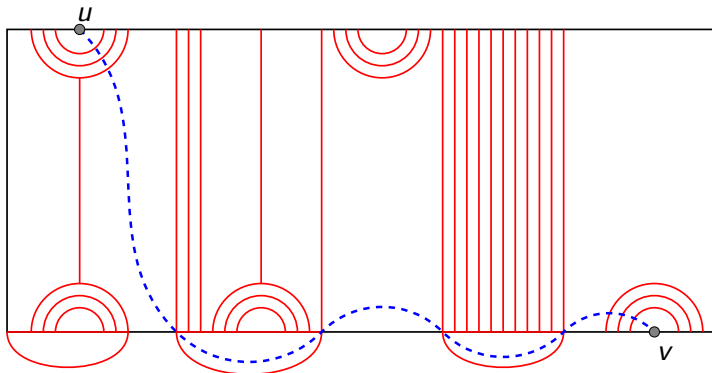
For any two vertices u, v on the boundary of a brick, there exists a u -to- v Jordan curve inside the brick that intersects the near-optimal solution at only a constant number of points.



Our Contribution: A New Structural Property on Bricks

Structure Property

For any two vertices u, v on the boundary of a brick, there exists a u -to- v Jordan curve inside the brick that intersects the near-optimal solution at only a constant number of points.



Our Contribution: A New Structural Property on Bricks

Structure Property

For any two vertices u, v on the boundary of a brick, there exists a u -to- v Jordan curve inside the brick that intersects the near-optimal solution at only a constant number of points.

Theorem

We have a PTAS for 2-edge-connected augmentation in planar graphs.

Conclusion

- **Local problems (1977-1983):**
independent set, vertex cover, dominating set, etc.
- **Connectivity problems (2005-2011):**
TSP, Steiner, 2-edge-connected subgraph, etc.
- **Other problems (2012-2014):**
multiway cut, k-center.
- **In our work:**
correlation clustering, 2-edge-connected augmentation.
- **Open problem:**
Steiner version of the 2-edge-connected subgraph problem.

Thank you!