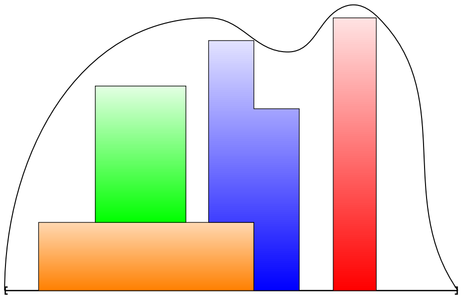
To Augment or Not to Augment: Solving Unsplittable Flow on a Path by Creating Slack

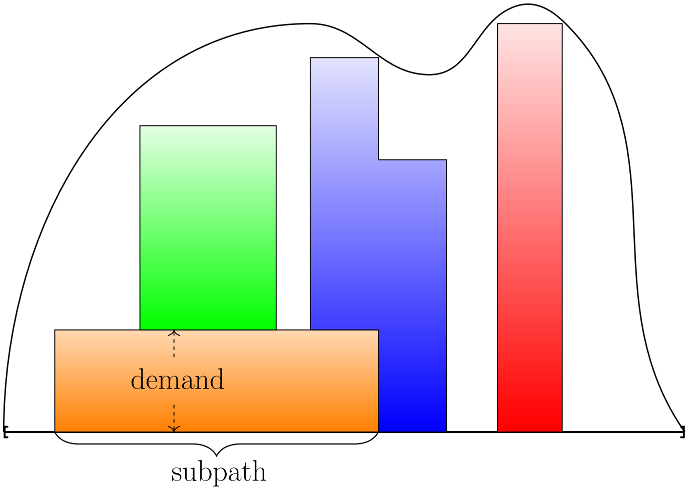
Fabrizio Grandoni | Tobias Mömke | Andreas Wiese | Hang Zhou

IDSIA, Switzerland | Saarland University | University of Chile | Max Planck Institute



Unsplittable Flow on a Path (UFP)

A task: subpath, demand, weight



Polynomial time:

- $O(\log n)$ [Bansal, Friggstad, Khandekar, Salavatipour, SODA 2009]
- $7 + \epsilon$ [Bonsma, Schulz, Wiese, FOCS 2011]
- $2 + \epsilon$ [Anagnostopoulos, Grandoni, Leonardi, Wiese, SODA 2014]
- $1 + \epsilon$ when weight/demand is bounded
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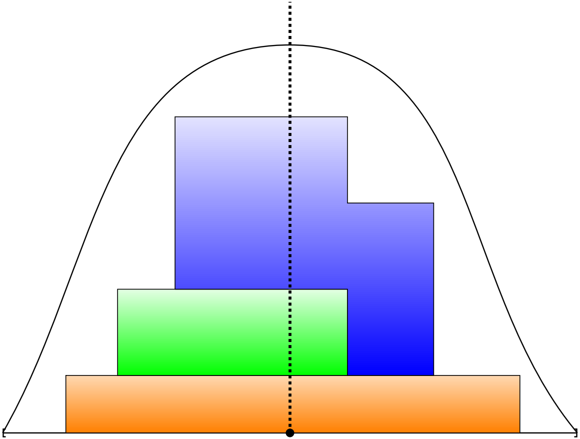
Open Question

Is there a PTAS for UFP?

Our Results

PTASes for three special cases:

- all tasks share a common edge (called *rooted UFP*)
- the weight of each task is propositional to its area
- a task can be included in the solution several times



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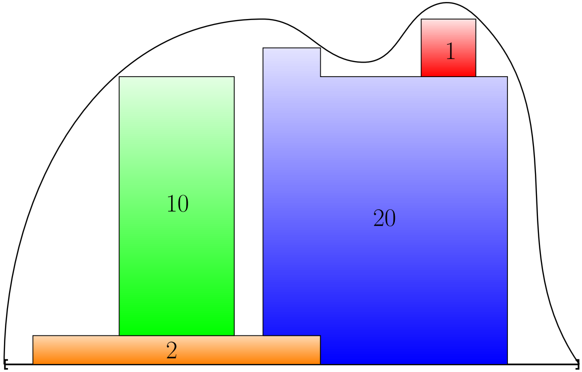

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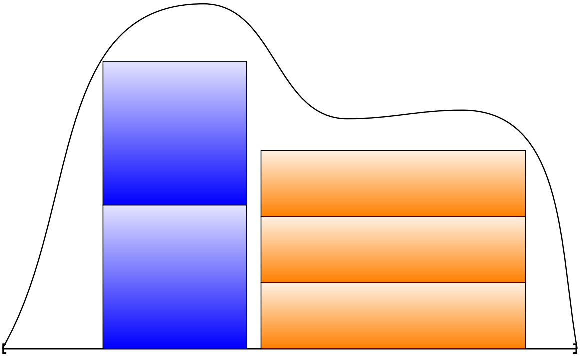
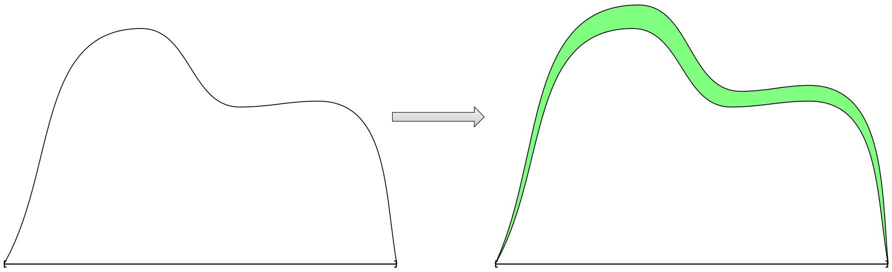


Resource Augmentation

Edge capacities can be violated by an ϵ-fraction.



Our framework:

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PTAS
with *resource augmentation* → creating slack → PTAS
in general

In this talk:

PTAS for **rooted UFP**
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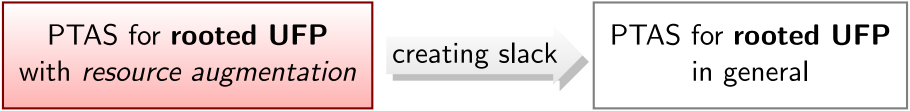
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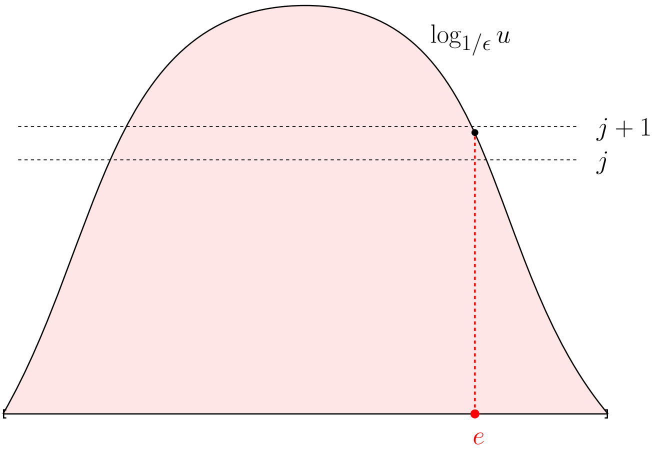
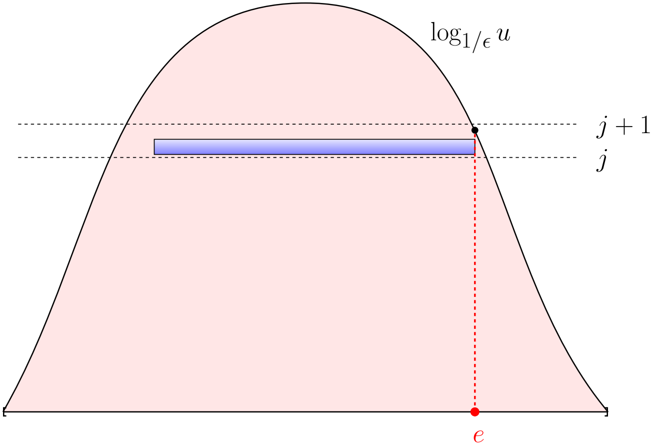


- *type* of an edge
- *type* of a task



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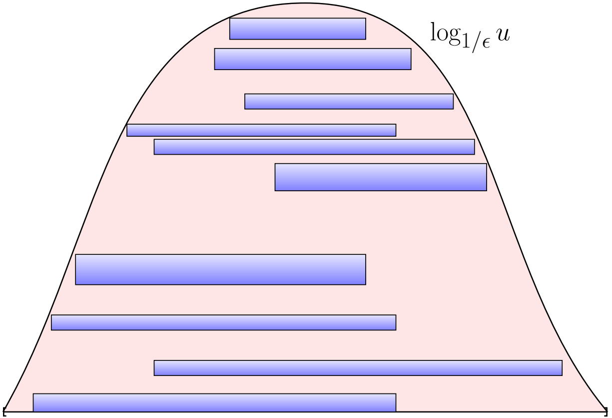
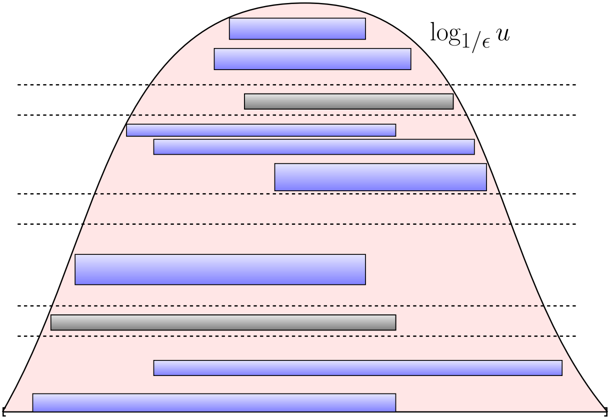


Algorithm

1. Remove the tasks of one type every $1/\epsilon$ types (shifting technique)
2. Solve each subinstance with bounded range of edge capacities
3. Output the union of the solutions to subinstances



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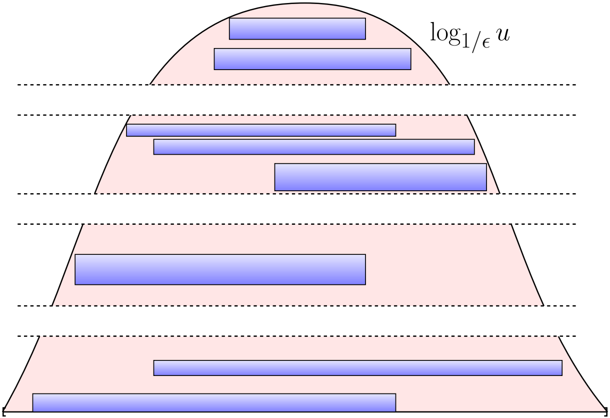

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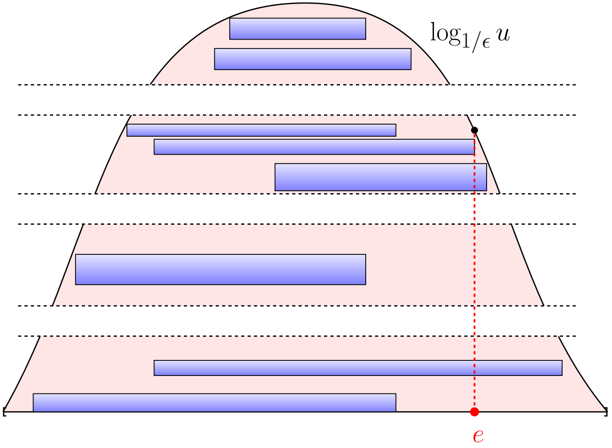

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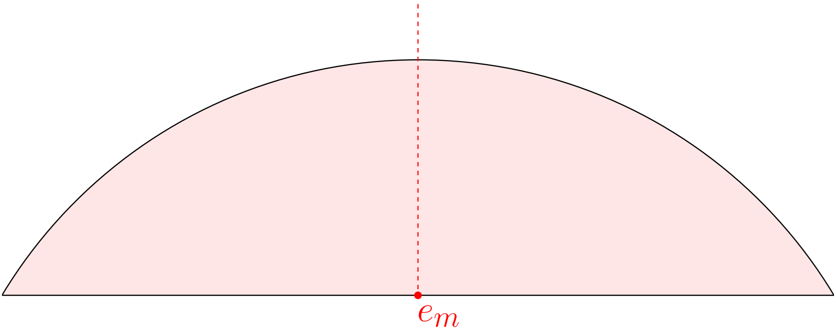
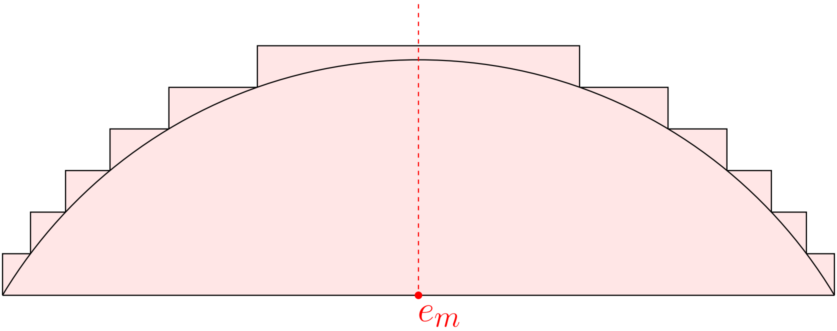


How to solve an instance with a bounded range of edge capacities?

1. observe monotonicity
2. round up edge capacities to powers of $1+\epsilon \implies O_\epsilon(1)$ steps
3. apply PTAS for constant-dimensional knapsack [Frieze, Clarke, 1984]



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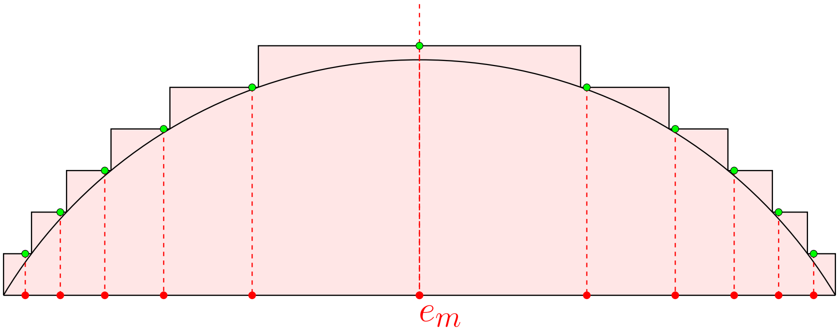

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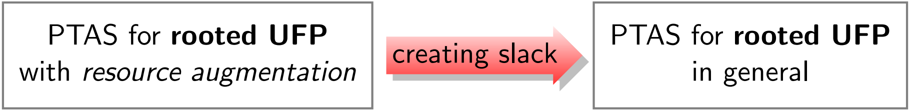
PTAS for **rooted UFP**
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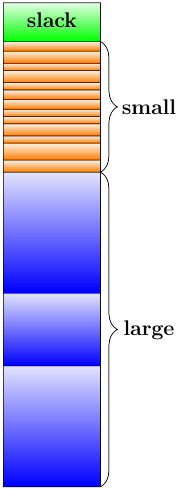
New Slack Lemma

A near-optimal solution where on each edge there is some slack s.t.

1. the number of large tasks is bounded;
2. the slack is at least ϵ fraction of the total demand of small tasks.



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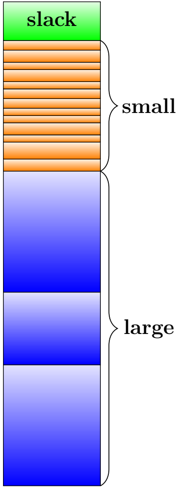

Question: How to obtain Property 2?

Answer: Shrink ϵ fraction of small tasks.
Small integrality gap
[Chekuri, Mydlarz, Shepherd, 2007]

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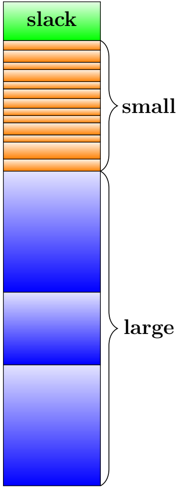

Question: Why does Property 2 help?

Answer: Borrow techniques from the resource augmentation setting.

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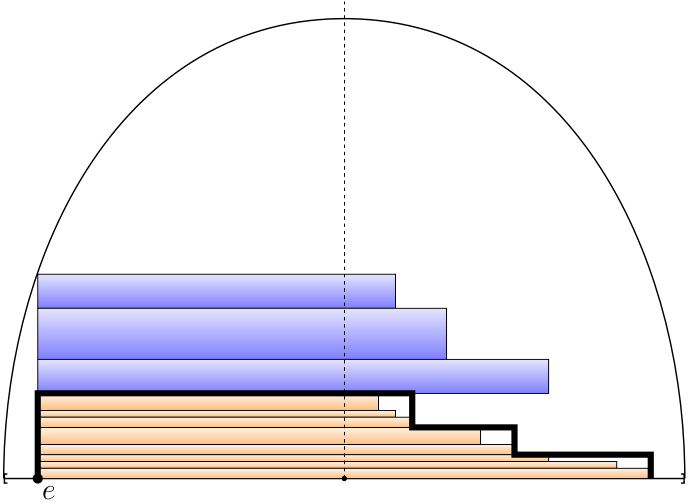
Our dynamic program proceeds in increasing order of types.
For each type, it guesses:

- large tasks
- capacity profile for small tasks

$\left.\right\}$ constant complexity



Difficulty: Not able to remember previously guessed information.

Solution: Remember information only from the last type.

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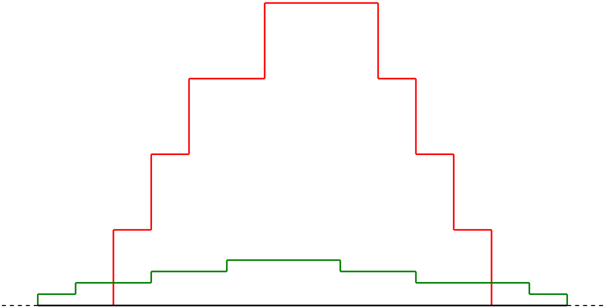

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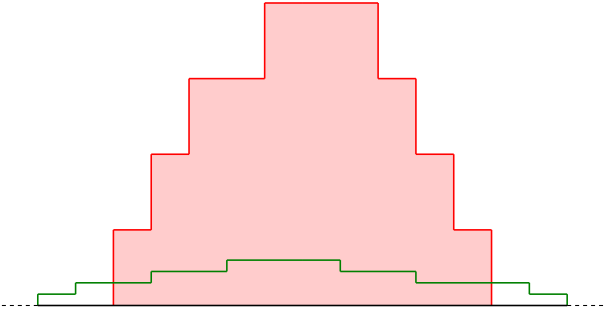

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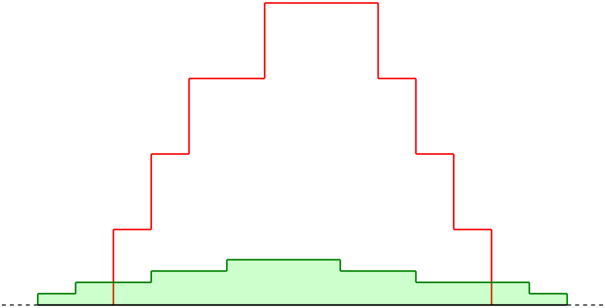

small tasks of type j

small tasks of type $j-2$

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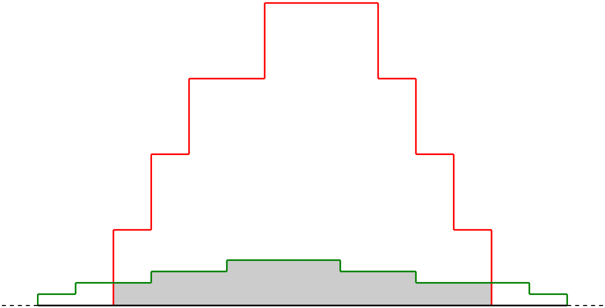

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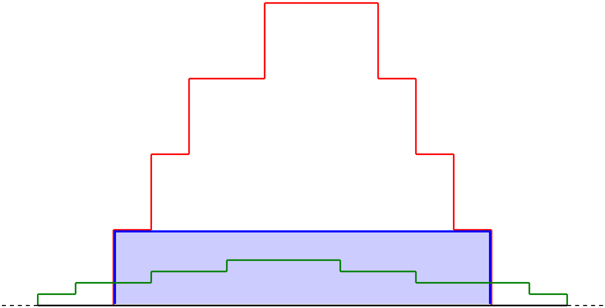

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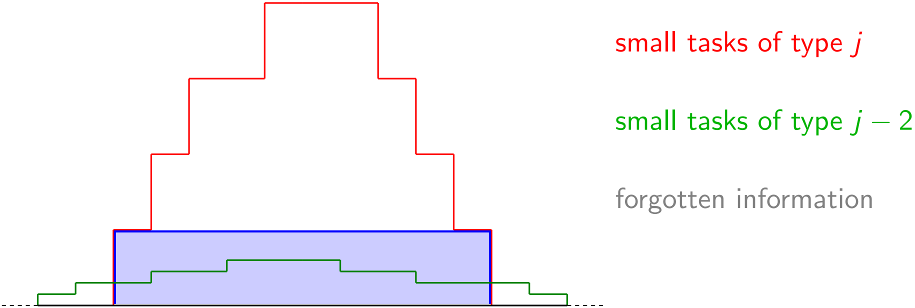

slack at type j accommodates the forgotten information

Polynomial-time dynamic program

Conclusion

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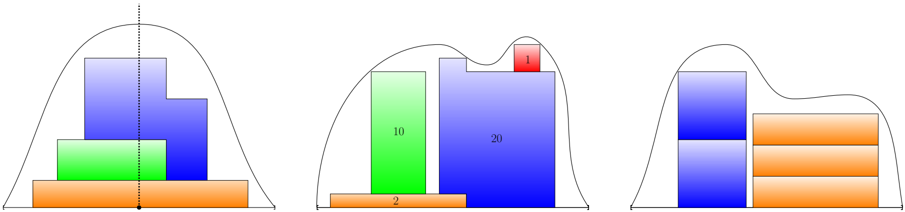
Our framework:

PTAS with *resource augmentation*	creating slack →	PTAS in general



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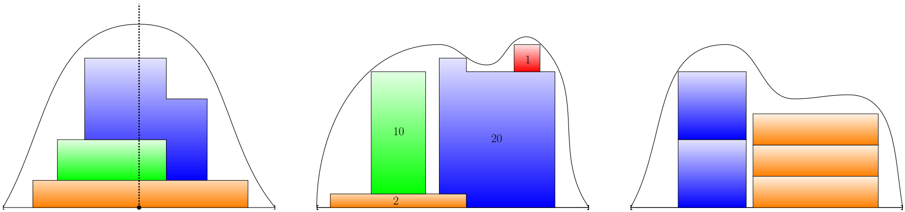

Open Question

Is there a PTAS for general UFP (even with resource augmentation)?

Thank you!