To Augment or Not to Augment: Solving Unsplittable Flow on a Path by Creating Slack

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Unsplittable Flow on a Path (UFP)

A task: subpath, demand, weight
Polynomial time:
- $O(\log n)$ [Bansal, Friggstad, Khandekar, Salavatipour, SODA 2009]
- $7 + \epsilon$ [Bonsma, Schulz, Wiese, FOCS 2011]
- $2 + \epsilon$ [Anagnostopoulos, Grandoni, Leonardi, Wiese, SODA 2014]
- $1 + \epsilon$ when weight/demand is bounded
  [Batra, Garg, Kumar, Mömke, Wiese, SODA 2015]
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Quasi-polynomial time:
- \( 1 + \epsilon \) (*) [Bansal, Chakrabarti, Epstein, Schieber, STOC 2006]
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Open Question
Is there a PTAS for UFP?
Our Results

PTASes for three special cases:
- all tasks share a common edge (called rooted UFP)
- the weight of each task is proportional to its area
- a task can be included in the solution several times
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- all tasks share a common edge (called \textit{rooted UFP})
- the weight of each task is propositional to its area
- a task can be included in the solution several times
Resource Augmentation

Edge capacities can be violated by an $\epsilon$-fraction.
Our framework:

- PTAS with resource augmentation
  Creating slack
- PTAS in general
Our framework:

- PTAS with resource augmentation

Creating slack

PTAS in general

In this talk:

- PTAS for rooted UFP with resource augmentation

Creating slack

PTAS for rooted UFP in general
PTAS for rooted UFP with resource augmentation

creating slack

PTAS for rooted UFP in general
\[ \log_{1/\epsilon} u \]

- **type of an edge**
- **type of a task**
Algorithm

1. Remove the tasks of one type every \(1/\epsilon\) types (shifting technique)
2. Solve each subinstance with bounded range of edge capacities
3. Output the union of the solutions to subinstances
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How to solve an instance with a bounded range of edge capacities?

1. observe monotonicity
2. round up edge capacities to powers of $1 + \epsilon \Rightarrow O(1)_{\epsilon}$ steps
3. apply PTAS for constant-dimensional knapsack [Frieze, Clarke, 1984]
How to solve an instance with a bounded range of edge capacities?

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PTAS for rooted UFP with resource augmentation

creating slack

PTAS for rooted UFP in general
New Slack Lemma

A near-optimal solution where on each edge there is some slack s.t.

1. the number of large tasks is bounded;
2. the slack is at least $\epsilon$ fraction of the total demand of small tasks.
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1. the number of large tasks is bounded;
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Question: How to obtain Property 2?

Answer: Shrink $\epsilon$ fraction of small tasks.

Small integrality gap

[Chekuri, Mydlarz, Shepherd, 2007]
New Slack Lemma

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Question: Why does Property 2 help?

Answer: Borrow techniques from the resource augmentation setting.
PTAS for **rooted UFP** with resource augmentation

creating slack

PTAS for **rooted UFP** in general
Our dynamic program proceeds in increasing order of types. For each type, it guesses:

- large tasks
- capacity profile for small tasks

\[ e \]
Difficulty: Not able to remember previously guessed information.
Solution: Remember information only from the last type.
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small tasks of type $j$
small tasks of type $j - 2$
forgotten information

slack at type $j$ accommodates the forgotten information
Difficulty: Not able to remember previously guessed information.
Solution: Remember information only from the last type.

Polynomial-time dynamic program
Conclusion

In this talk:

PTAS for \textit{rooted UFP} with \textit{resource augmentation}

creating slack

PTAS for \textit{rooted UFP} in general
Conclusion

Our framework:

PTAS with resource augmentation → creating slack → PTAS in general
Conclusion

Our framework:

PTAS with resource augmentation → creating slack → PTAS in general

Open Question

Is there a PTAS for general UFP (even with resource augmentation)?
Thank you!