

Gilbert Levitt

Joint w. Guirardel

Take  $F_n$  a free group,  $Out(F_n) = Aut(F_n)/Inn(F_n)$

$\mathcal{C}$  a finite set of classes in  $F_n$

$Out_{\mathcal{C}}(F_n)$  pointwise stabilizer of  $\mathcal{C}$

$\mathcal{C} = \{[x_1, x_2], [x_3, x_4]\}$  in  $F_4$

$Out_{\mathcal{C}}(F_n) =$  Mapping class group of  $C$

(McCool 1955)  $Out_{\mathcal{C}}(F_n)$  is finitely presented

(Guirardel Levitt)  $Out_{\mathcal{C}}(F_n)$  is

VLF (virtually libre finie) :  $\exists$  finite index subgroup with finite  $K(T, 1)$  should be

is valid if  $\mathcal{C}$  infinite or replace  $F_n$  by torsion-free

hyperbolic  $G$   $Out_{\mathcal{C}}(G)$

Motivation : Thurston The stabilizer of a point in the boundary of outer space is built out of McCool groups. So it's VFL.

Two steps in understanding  $Out_{\mathcal{C}}(F_n)$  and  $Out_{\mathcal{C}}(G)$

(1) Use outer space (Culler - Vogtmann 1986) to reduce to one ended case

(2) Use  $\mathbb{R}$ -trees (Rips - Sela) to do one ended case.

~~Out(F\_n) ...~~

Other v

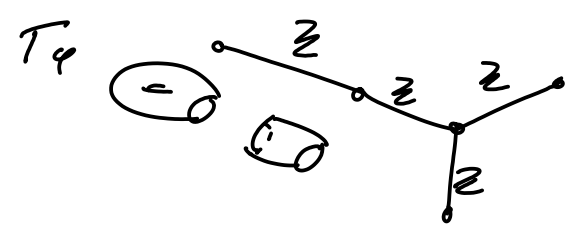
Example  $\mathcal{C} = \{[c]\}$  If  $T$  is a tree in  $\mathcal{C}V_c(\mathcal{C})$  only are non-trivial vertex stabilizers (up to conjugacy) smaller for factor centers

# Structure of $Out(G)$

$$1 \rightarrow \mathcal{Q} \rightarrow Out^0(G) \rightarrow \pi_1 MCG(\Sigma_g) \rightarrow 1$$

frank index subgroup: disregard permutations, reflexions  
 $\rightarrow$  Dehn twists in yellow annuli  $\cong \mathbb{Z}^2$

Example:  $\mathcal{Q} = \{\text{purple curve}\} = C$



Is  $Out_{\mathcal{Q}}(G)$  invariant  
- every splitting over  $\mathbb{Z}$  rel  $\mathcal{Q}$  is visible in  $T_{\mathcal{Q}}$ .  
-  $T_{\mathcal{Q}}$  is  $Out_{\mathcal{Q}}(G)$ -invariant  
get similar exact sequence for  $Out_{\mathcal{Q}}(G)$ .

## In GENERAL:

Have to use relative JSJ relative to  $\mathcal{Q}$ .