Dahmani JW Guirardel

The isom pb & pinched neg curvature

Equation problem G: given a system of equations how to know whether there is a solution.

$x a x^{-1} = b$ $\qquad$ a, b, c, d parameters

$x y a x^{-1} b x^{3} c = y d y$ $\qquad$ x, y unknown

Isomorphism problem Given two (f.p.) groups how to know whether they are isom.

E.P

Abelian groups OK ; Nilpotent groups : NO in general
Romankov

Free groups OK ; Hyperbolic groups OK ;
Makanin $\qquad$ Rips – Sela
Dahmani – Guirardel

Rel hyperbolic groups OK
toral
Dahmani

Abelian OK ; Nilpotent OK ;
Gruenwald-Segal

IP:

Solvable groups: NO

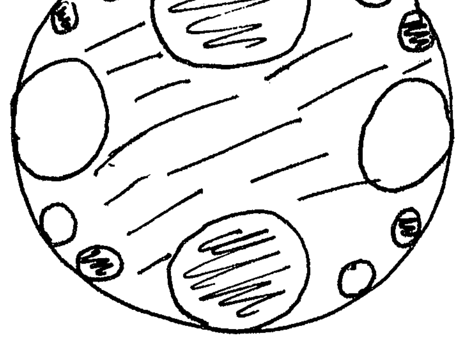
Free groups OK ; Hyperbolic groups OK
Sela, Dahmani Groves
Guirardel ~~Guirard~~

toral rel hyp groups OK
Dahmani - Groves

Relatively hyperbolic groups

G is hyperbolic if it acts properly discontinuously cocompacty on a geodesic hyperbolic space X.

G is relatively hyperbolic if G acts p.d. on X hyperbolic such that: there exists a separated invariant family of horoballs in X s.t. G acts cocompactly on the complement.



parabolic subgroups : stabilizers of horoballs

toral : t f with abelian parabolic subgroups

Examples : • fdmt groups of finite volume hyp manifolds

• fdmt groups of finite vol mfds with pinched neg curvature $-b^2 < \varkappa < -a^2 < 0$

⚠ parabolic subgroups are v. nilpotent

How EP and IP are related ?

G, H , understand $\mathrm{Hom}_{inj}(G,H)/H$

they are solution of equations in H.

Prop (Bestvina - Paulin - Rips)

If G, H are hyperbolic and if $\mathrm{Hom}_{inj}(G,H)/H$ is infinite, then G admits a splitting (which is interesting).

Let $\varphi_n : G \hookrightarrow H$ non conjugated

Let S a generating set of G.

Fact: $\inf\limits_{x\in H} \max\limits_{s\in S} d(x, \varphi_n(s)x) =: \alpha_n \longrightarrow \infty$

(using that the φ_n's are not conj)

Rescale the word metric on H (divide by α_n)

Bestvina - Paulin : one can extract a limit action on a limit space, which is an $\mathbb{R}$-tree.

By Rips theory we can promote this action to a splitting of G. $G \simeq A *_C B$ or $G \simeq A *_C$

~~New~~ <u>Problematic</u> : rel hyp groups with nilpotent <u>parabolic</u> subgroups

<u>Idea</u> : use residual finiteness to solve IP (when available)

⚠ $\exists\, G, H$ f.p. groups, $G \xhookrightarrow{i} H$, $G \not\cong H$ and i induces an isom on profinite completion. (Bridson - Grunewald)

<u>Thm</u> (Osin, Groves-Manning) (Dehn filling thm)

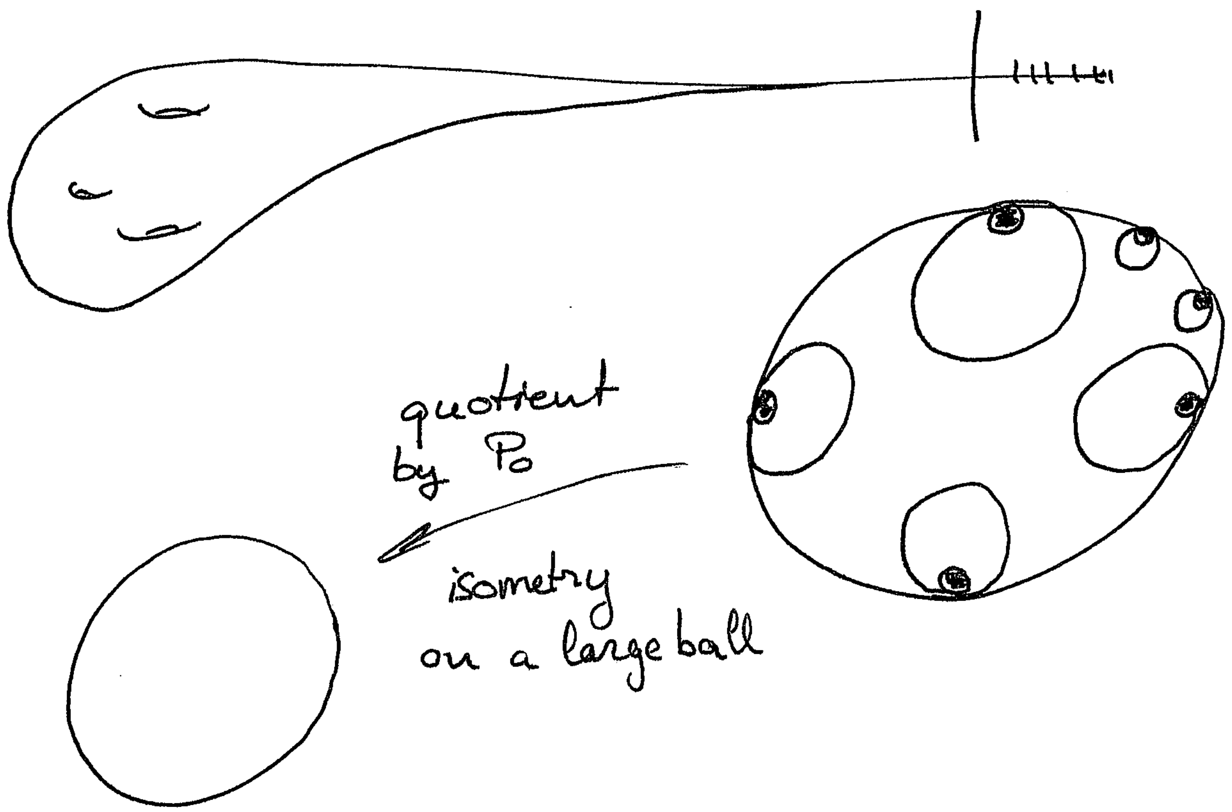
Let G be a rel hyperbolic group, with one parabolic subgroup P

$\forall S \subset G$ finite, $\exists F_0 \subset P$ finite

s.t. $\forall P_0 \lhd P$ of finite index with $\cancel{P_0 \cap F_0 = \emptyset}$

$P_0 \cap F_0 = \emptyset$, $G / \langle\langle P_0 \rangle\rangle$ is hyperbolic and

S embeds.



<u>Isomorphism theorem</u> (G, P_1), (H, P_2) two relatively hyp groups f.p. Assume P_1, P_2 resid finite and G has no splitting over virt cyclic or parabolic subgroups.

Assume that the sequence of Characteristic

Dehn filling of (G, P_0) and (H, P_2) are isomorphic.

Then $G \hookrightarrow H$.

Seq of charac Dehn fillings of G

$$G_n = G / \langle\langle P_n \rangle\rangle \qquad P_n = \bigcap_{[P' : P] \leq n} P'$$

~~Corollary~~

$$\begin{tikzcd}
G \arrow[d] \arrow[drr, "\varphi_n"] & & H \arrow[d] \\
G_n \arrow[rr] & & H_n
\end{tikzcd}$$

Corollary IP is solvable for $\pi_1(M)$, M finite vol mfds of pinched negative curvature.