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The isom pb & pinched neg curvature

Equation problem G: given a system of equations how to know whether there is a solution.

$x a x^{-1} = b$ $\qquad$ a, b, c, d parameters

$x y a x^{-1} b x^3 c = y d y$ $\qquad$ x, y unknown

Isomorphism problem Given two (f.p.) groups how to know whether they are isom.

E.P

Abelian groups OK; Nilpotent groups: NO in general
Romankov

Free groups OK; Hyperbolic groups OK;
Makanin
Rips – Sela
Dahmani – Guirardel

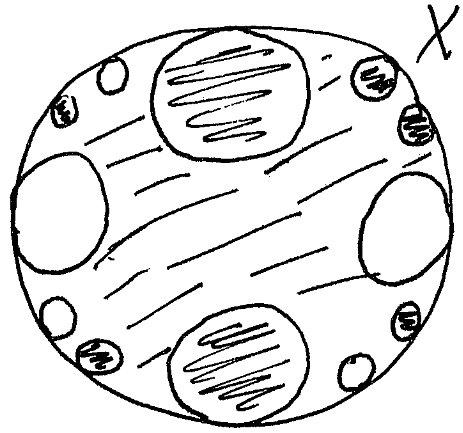
Rel hyperbolic groups OK
toral
Dahmani

IP:
- Abelian OK ; Nilpotent OK ;
  Gruenwald - Segal
- Solvable groups : NO
- Free groups OK ; Hyperbolic groups OK
  Sela, Dahmani Groves
  Guirardel ~~[illegible]~~
- toral rel hyp groups OK
  Dahmani - Groves

Relatively hyperbolic groups

G is hyperbolic if it acts properly discontinuously cocompacty on a geodesic hyperbolic space X.

G is relatively hyperbolic if G acts p.d. on X hyperbolic such that : there exists a separated invariant family of horoballs in X s.t. G acts cocompactly on the complement.



X

parabolic subgroups : stabilizers of horoballs

toral : t f with abelian parabolic subgroups

Examples : • fdmt groups of finite volume hyp manifolds

• fdmt groups of finite vol mfds with pinched neg curvature $-b^2 < \varkappa < -a^2 < 0$

⚠ parabolic subgroups are v. nilpotent

How EP and IP are related ?

G, H, understand $\mathrm{Hom}_{inj}(G,H)/H$

they are solution of equations in H.

Prop (Bestvina - Paulin - Rips)

If G, H are hyperbolic and if $\mathrm{Hom}_{inj}(G,H)/H$ is infinite, then G admits a splitting (which is interesting).

Let $\varphi_n : G \hookrightarrow H$ non conjugated

Let S a generating set of G.

Fact: $\inf_{x \in H} \max_{s \in S} d(x, \varphi_n(s)x) =: \alpha_n \longrightarrow \infty$

(using that the φ_n's are not conj)

Rescale the word metric on H (divide by α_n)

Bestvina - Paulin: one can extract a limit action on a limit space, which is an $\mathbb{R}$-tree.

By Rips theory we can promote this action to a splitting of G. $G \simeq A *_C B$ or $G \simeq A *_C$

~~New~~ Problematic: rel hyp groups with nilpotent parabolic subgroups

Idea: use residual finiteness to solve IP (when available)

⚠ $\exists\, G, H$ f.p. groups, $G \xhookrightarrow{i} H$, $G \not\simeq H$ and i induces an isom on profinite completion. (Bridson - Grunewald)

<u>Thm</u> (Osin, Groves-Manning) (Dehn filling thm)

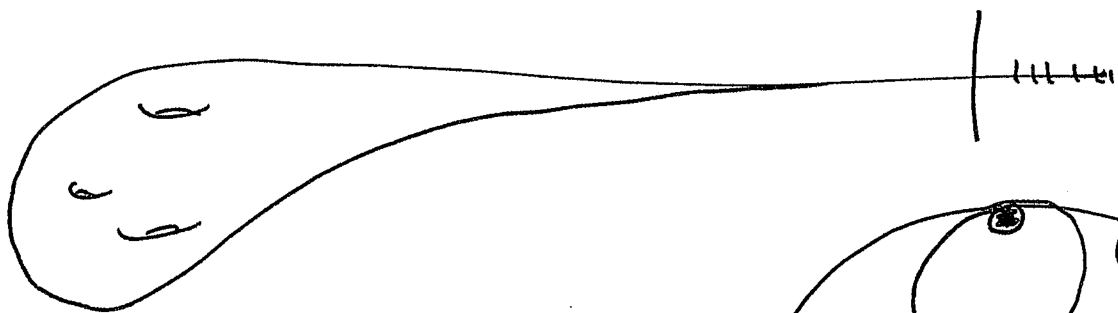
Let G be a rel hyperbolic group, with one parabolic subgroup P

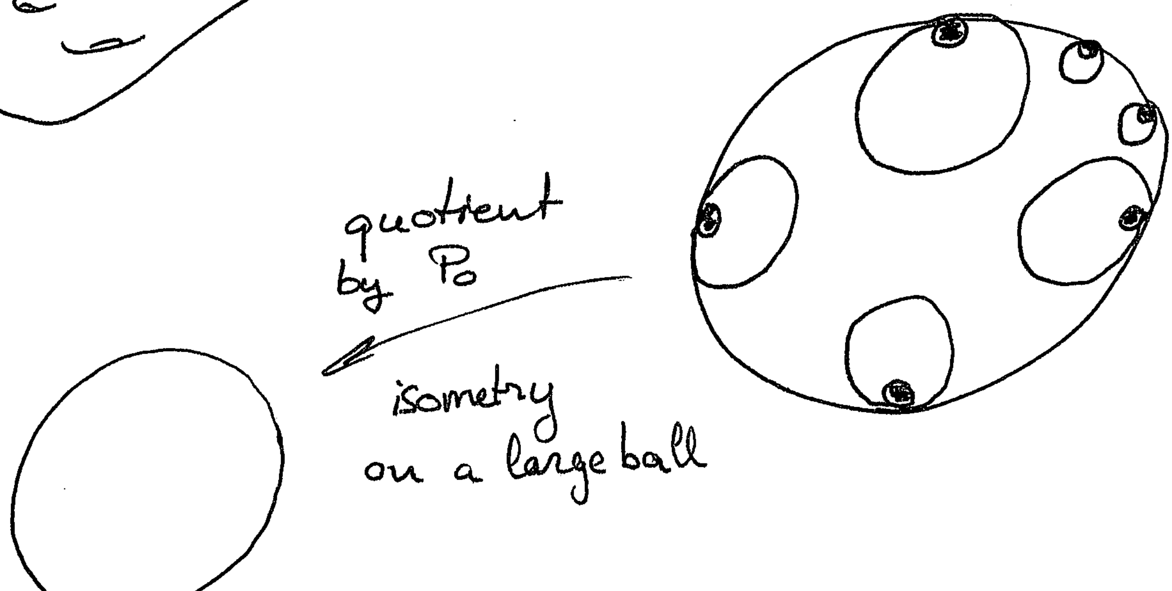
$\forall S \subset G$ finite, $\exists F_0 \subset P$ finite

s.t. $\forall P_0 \triangleleft P$ of finite index with ~~$P_0 \cap F_0 = \emptyset$~~

$P_0 \cap F_0 = \emptyset$, $G/\langle\langle P_0 \rangle\rangle$ is hyperbolic and

S embeds.





<u>Isomorphism theorem</u> (G, P_1), (H, P_2) two relatively hyp groups f.p. Assume P_1, P_2 resid finite and G has no splitting over virt cyclic or parabolic subgroups.

Assume that the sequence of Characteristic

Dehn filling of (G, P_0) and (H, P_2) are isomorphic.
Then $G \hookrightarrow H$.

Seq of charac Dehn fillings of G

$$G_n = G / \langle\langle P_n \rangle\rangle \qquad P_n = \bigcap_{[P':P] \leq n} P'$$

~~Corollary~~

$$\begin{tikzcd}
G \arrow[d] \arrow[dr, "\varphi_n"] & H \arrow[d] \\
G_n \arrow[r] & H_n
\end{tikzcd}$$

Corollary IP is solvable for $\pi_1(M)$, M finite vol mfds of pinched negative curvature.