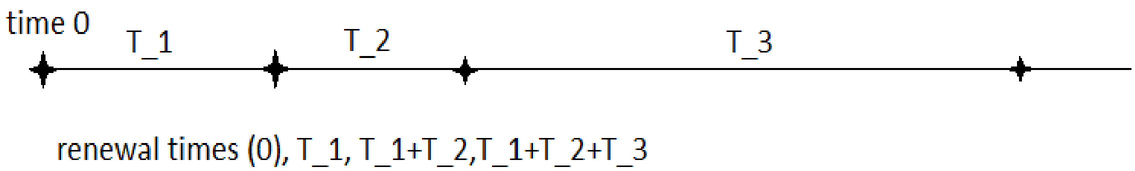
Summary of renewal theory notation

Non-delayed renewal processes

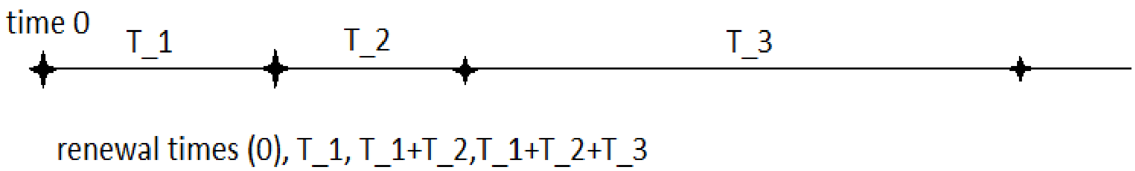


- Random variable T_i time between renewals $i-1$ and i. May have positive probability of being infinite (transient case).
- $f_n = P(T_i = n)$, $f_0 = 0$, generating function $F(s) = \sum_{n=1}^{\infty} f_n s^n$.
- Behaves as if there is a renewal at time 0 so f_n is the probability that the **first** renewal is at time n.
- $f = \sum_{n=1}^{\infty} f_n = F(1)$.
- E_n event that there is a renewal at time n.
- $u_n = P(E_n) = P(E_{t+n}|E_t)$, $u_0 = 1$, generating function $U(s) = \sum_{n=0}^{\infty} u_n s^n$.

Terminology

- Recurrent if $f = 1$ (so there will always be a next renewal), transient otherwise.
- Positive recurrent if $E(T_i)$ finite; null recurrent if recurrent but not positive recurrent.
- Period is highest common factor of $\{n : f_n > 0\}$, e.g. if f_n is non-zero if and only if n is even then period is 2. Aperiodic if period is 1.

Delayed renewal processes



- Random variable D time until the first renewal. May have positive probability of being zero or infinite.
- $b_n = P(D = n)$, generating function $B(s) = \sum_{n=0}^{\infty} b_n s^n$.
- Once first renewal occurs behaves exactly like the delayed case, f_n defined as $P(T_i = n)$ as before.
- $v_n = P(E_n)$: probability there is a renewal at time n, generating function $V(s)$.
- $u_n = P(E_{t+n}|E_t)$, probability there is a renewal in n steps time given there is one now, generating function $U(s)$. NB $u_0 = 1$, but $v_0 = b_0$.