MAS275 Probability Modelling Example 21: Monopoly

Dimitrios Kiagias

School of Mathematics and Statistics, University of Sheffield

Spring Semester, 2018

Monopoly simplified

Board game with 40 "squares", which we will number 0 to 39 (or 1 to 40). Most are labelled with a street or other location.

Monopoly simplified

Board game with 40 “squares”, which we will number 0 to 39 (or 1 to 40). Most are labelled with a street or other location.

In the US edition locations are in Atlantic City, New Jersey (e.g. square 39 is “Boardwalk”); in the standard UK edition they are in London (e.g. square 39 is “Mayfair”). Most locations are colour-coded into groups (e.g. 37 and 39 are dark blue; 16, 18 and 19 are orange). Others are railways and utilities (electricity and water).

Monopoly simplified

Board game with 40 "squares", which we will number 0 to 39 (or 1 to 40). Most are labelled with a street or other location.

In the US edition locations are in Atlantic City, New Jersey (e.g. square 39 is "Boardwalk"); in the standard UK edition they are in London (e.g. square 39 is "Mayfair"). Most locations are colour-coded into groups (e.g. 37 and 39 are dark blue; 16, 18 and 19 are orange). Others are railways and utilities (electricity and water).

Each player has a token which moves round the board in a circuit. On each turn the initial movement is determined by moving two dice.

Monopoly simplified

Some squares are labelled “Chance” or “Community Chest”; the player draws a card which may lead to a further move.

Monopoly simplified

Some squares are labelled “Chance” or “Community Chest”; the player draws a card which may lead to a further move.

One corner (square 10) is labelled “Jail” and the opposite corner (square 30) “Go to Jail”.

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain.

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain. We start with a very simplified version.

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain. We start with a very simplified version.

Let X_n be the nth square visited by the player. The player starts on square 0 (or 40; labelled "Go"), so $X_0 = 0$.

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain. We start with a very simplified version.

Let X_n be the nth square visited by the player. The player starts on square 0 (or 40; labelled "Go"), so $X_0 = 0$.

In most cases, if on square j, the player moves to square $j + k \pmod{40}$ with probability given by d_k, where d_k is the probability of k being the sum of the numbers on the two dice.

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain. We start with a very simplified version.

Let X_n be the nth square visited by the player. The player starts on square 0 (or 40; labelled "Go"), so $X_0 = 0$.

In most cases, if on square j, the player moves to square $j + k(\mod 40)$ with probability given by d_k, where d_k is the probability of k being the sum of the numbers on the two dice. (E.g. $p_{5,11} = d_6 = 5/36$, $p_{37,8} = d_{11} = 2/36$.)

Markov Monopoly - very simple version

We will model the sequence of squares the player visits as a Markov chain. We start with a very simplified version.

Let X_n be the nth square visited by the player. The player starts on square 0 (or 40; labelled "Go"), so $X_0 = 0$.

In most cases, if on square j, the player moves to square $j + k(\mod 40)$ with probability given by d_k, where d_k is the probability of k being the sum of the numbers on the two dice. (E.g. $p_{5,11} = d_6 = 5/36$, $p_{37,8} = d_{11} = 2/36$.)

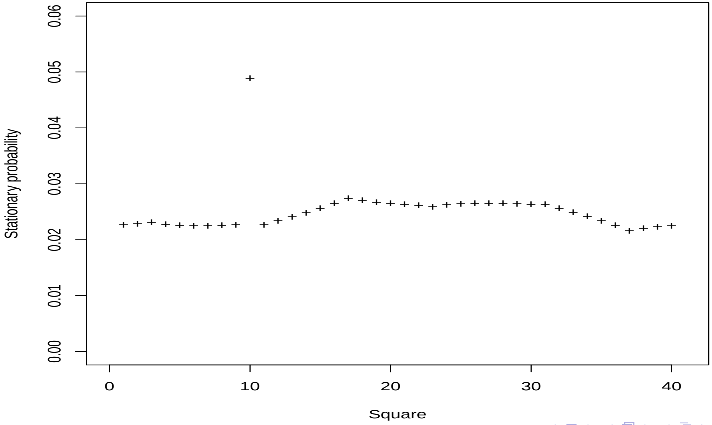
If the player lands on square 30 ("Go to Jail") the token moves to square 10 ("Jail"). So we set $p_{30,10} = 1$ (and $p_{30,i} = 0$ for $i \neq 10$).

First stationary distribution

With the Markov model on the previous slide, we get the following stationary distribution by finding eigenvectors in R:

First stationary distribution

With the Markov model on the previous slide, we get the following stationary distribution by finding eigenvectors in R:



First stationary distribution

Jail high probability; squares between about 15 and 31 (including orange, red, yellow sets) have higher probabilities than others.

Adding Chance and Community Chest

(Based on US edition circa 1983)

Adding Chance and Community Chest

If the player lands on squares 7, 22 or 36, they draw a "Chance" card. There are 16 cards, 10 of which cause a further move.

Adding Chance and Community Chest

If the player lands on squares 7, 22 or 36, they draw a "Chance" card. There are 16 cards, 10 of which cause a further move.

Assume that these are shuffled before one is drawn (otherwise non-Markov).

Adding Chance and Community Chest

If the player lands on squares 7, 22 or 36, they draw a "Chance" card. There are 16 cards, 10 of which cause a further move.

Assume that these are shuffled before one is drawn (otherwise non-Markov).

Then $p_{7,i}$ is $\frac{6}{16}d_{i-7}$ plus the probability of being sent to i by the Chance card, which is $1/16$ for squares 0 (Go), 4 (back 3 spaces), 5, 10 (Jail), 11, 12 (nearest utility, Electric Company), 24 and 39, and $1/8$ for square 15 (nearest railway, because there are two of these cards).

Adding Chance and Community Chest

If the player lands on squares 7, 22 or 36, they draw a "Chance" card. There are 16 cards, 10 of which cause a further move.

Assume that these are shuffled before one is drawn (otherwise non-Markov).

Then $p_{7,i}$ is $\frac{6}{16}d_{i-7}$ plus the probability of being sent to i by the Chance card, which is $1/16$ for squares 0 (Go), 4 (back 3 spaces), 5, 10 (Jail), 11, 12 (nearest utility, Electric Company), 24 and 39, and $1/8$ for square 15 (nearest railway, because there are two of these cards).

Similar changes to rows 22 and 36

Adding Chance and Community Chest

Landing on squares 2, 17 and 33 gives “Community Chest”. Similar analysis to Chance, but only two cause a further move, one to Jail and one to Go.

Adding Chance and Community Chest

Landing on squares 2, 17 and 33 gives "Community Chest". Similar analysis to Chance, but only two cause a further move, one to Jail and one to Go.

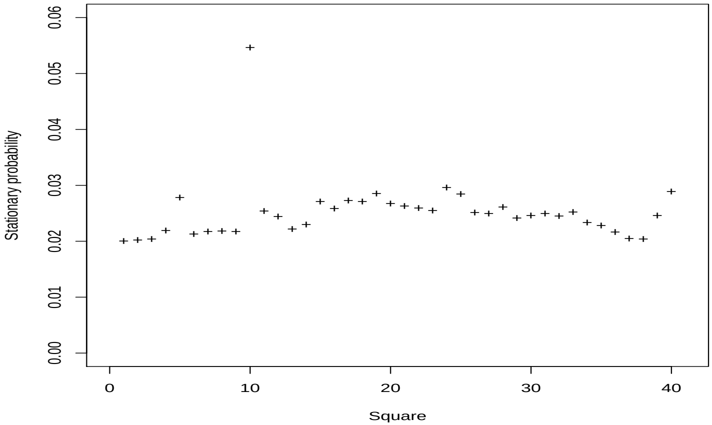
Then $p_{2,i}$ is $\frac{14}{16}d_{i-2}$ plus the probability of being sent to i by the Chance card, which is $1/16$ for squares 0 (Go) and 10 (Jail). Similar changes to rows 17 and 33.

New stationary distribution

With the new Markov model, we get the following stationary distribution:

New stationary distribution

With the new Markov model, we get the following stationary distribution:



New stationary distribution

Square 10 (jail) still has the highest probability. Square 0 or 40 (Go) is second highest.

New stationary distribution

Square 10 (jail) still has the highest probability. Square 0 or 40 (Go) is second highest.

Property with highest probability is square 24, Illinois Avenue (US edition), one of the red group. The other properties in this area, the orange and red groups, also have relatively high probabilities.

New stationary distribution

Square 10 (jail) still has the highest probability. Square 0 or 40 (Go) is second highest.

Property with highest probability is square 24, Illinois Avenue (US edition), one of the red group. The other properties in this area, the orange and red groups, also have relatively high probabilities.

Properties on the side of the board after Go (squares 1-9) have relatively low probabilities, except square 5, which is a railway and benefits from Chance cards sending tokens there.

NB

This is still somewhat simplified from the real game, e.g.:

NB

This is still somewhat simplified from the real game, e.g.:

Jail rules are more complicated, and not entirely Markov (e.g. "Get Out of Jail Free" cards).

NB

This is still somewhat simplified from the real game, e.g.:

Jail rules are more complicated, and not entirely Markov (e.g. "Get Out of Jail Free" cards).

Chance and Community Chest cards are used in order, not shuffled. (This is non-Markov.)