On the spectrum of (some) quantum integrable models

Sébastien Leurent, Institut de Mathématiques de Bourgogne

May 27, 2015
Conférence "Cordes en France"

Outline

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$ $\quad \mathcal{H} = (\mathbb{C}^2)^{\otimes L}; \quad \vec{\sigma}_{L+1} = \vec{\sigma}_1$

$= L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{P}_{1,2}\,|{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$

$\mathcal{P}_{1,2}\,|{\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$

$|\{3\}\rangle = |{\downarrow\downarrow\uparrow\downarrow\downarrow}\dots\rangle$

$|\{1,4\}\rangle = |{\uparrow\downarrow\downarrow\uparrow\downarrow}\dots\rangle$

- "Vacuum": $|{\downarrow\downarrow}\cdots{\downarrow}\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k} (e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)})\,|\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}; \qquad \vec{\sigma}_{L+1} = \vec{\sigma}_1$$

$$\mathcal{P}_{1,2}\,|\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$$

$$\mathcal{P}_{1,2}\,|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$$

$$|\{3\}\rangle = |\downarrow\downarrow\uparrow\downarrow\downarrow\ldots\rangle$$

$$|\{1,4\}\rangle = |\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\rangle$$

- "Vacuum": $|\downarrow\downarrow\cdots\downarrow\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j + p_2 k)} + S\,e^{i(p_1 k + p_2 j)}\right)|\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma_i} \cdot \vec{\sigma_{i+1}}$
$= L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}$; $\quad \vec{\sigma}_{L+1} = \vec{\sigma}_1$

$\mathcal{P}_{1,2}\,|\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$\mathcal{P}_{1,2}\,|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$|\{3\}\rangle = |\downarrow\downarrow\uparrow\downarrow\downarrow\ldots\rangle$

$|\{1,4\}\rangle = |\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\rangle$

- "Vacuum": $|\downarrow\downarrow\cdots\downarrow\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j + p_2 k)} + S\,e^{i(p_1 k + p_2 j)}\right)|\{j,k\}\rangle$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}; \qquad \vec{\sigma}_{L+1} = \vec{\sigma}_1$$

$$\mathcal{P}_{1,2}\,|{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$$

$$\mathcal{P}_{1,2}\,|{\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$$

$$|\{3\}\rangle = |{\downarrow\downarrow\uparrow\downarrow\downarrow}\dots\rangle$$

$$|\{1,4\}\rangle = |{\uparrow\downarrow\downarrow\uparrow\downarrow}\dots\rangle$$

- "Vacuum": $|{\downarrow\downarrow}\cdots{\downarrow}\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j + p_2 k)} + S\,e^{i(p_1 k + p_2 j)}\right)|\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$
$= L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}$; $\vec{\sigma}_{L+1} = \vec{\sigma}_1$

$\mathcal{P}_{1,2}\,|\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$\mathcal{P}_{1,2}\,|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$|\{3\}\rangle = |\downarrow\downarrow\uparrow\downarrow\downarrow\ldots\rangle$

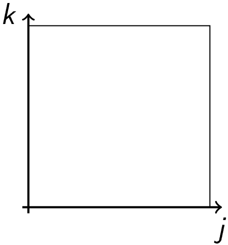
$|\{1,4\}\rangle = |\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\rangle$

- "Vacuum": $|\downarrow\downarrow\cdots\downarrow\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j + p_2 k)} + S\,e^{i(p_1 k + p_2 j)}\right)|\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$



"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}; \qquad \vec{\sigma}_{L+1} = \vec{\sigma}_1$

$\mathcal{P}_{1,2}\,|\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$\mathcal{P}_{1,2}\,|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$|\{3\}\rangle = |\downarrow\downarrow\uparrow\downarrow\downarrow\ldots\rangle$

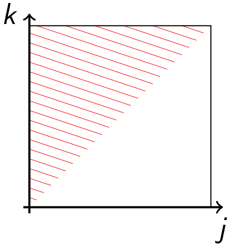
$|\{1,4\}\rangle = |\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\rangle$

- "Vacuum": $|\downarrow\downarrow\cdots\downarrow\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k} \left(e^{i(p_1 j + p_2 k)} + S\,e^{i(p_1 k + p_2 j)}\right) |\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$



"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma_i} \cdot \vec{\sigma_{i+1}}$
$= L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}$; $\vec{\sigma}_{L+1} = \vec{\sigma}_1$

$\mathcal{P}_{1,2}\,|{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$

$\mathcal{P}_{1,2}\,|{\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$

$|\{3\}\rangle = |{\downarrow\downarrow\uparrow\downarrow\downarrow}\ldots\rangle$

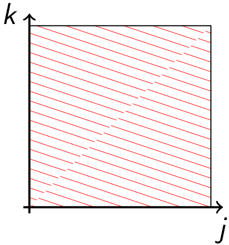
$|\{1,4\}\rangle = |{\uparrow\downarrow\downarrow\uparrow\downarrow}\ldots\rangle$

- "Vacuum": $|{\downarrow\downarrow}\cdots{\downarrow}\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$|\psi\rangle \propto \sum_{j<k}(e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)})\,|\{j,k\}\rangle$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$



"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}; \qquad \vec{\sigma}_{L+1} = \vec{\sigma}_1$$

$$\mathcal{P}_{1,2}\,|{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$$

$$\mathcal{P}_{1,2}\,|{\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle = |{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow}\rangle$$

$$|\{3\}\rangle = |{\downarrow\downarrow\uparrow\downarrow\downarrow}\ldots\rangle$$

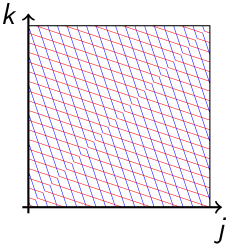
$$|\{1,4\}\rangle = |{\uparrow\downarrow\downarrow\uparrow\downarrow}\ldots\rangle$$

- "Vacuum": $|{\downarrow\downarrow} \cdots {\downarrow}\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations: $|\psi\rangle = \sum_{j,k} \Psi(j,k)\,|\{j,k\}\rangle$

$$|\psi\rangle \propto \sum_{j<k} \left(e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)}\right)|\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)} - 2e^{ip_2}}{1+e^{i(p_1+p_2)} - 2e^{ip_1}}$$



"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

Eigenstates of $H = -\sum_{i=1}^{L} \vec{\sigma_i} \cdot \vec{\sigma_{i+1}}$
$= L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1}$

$\mathcal{H} = (\mathbb{C}^2)^{\otimes L}$; $\vec{\sigma}_{L+1} = \vec{\sigma}_1$

$\mathcal{P}_{1,2}\, |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$\mathcal{P}_{1,2}\, |\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle$

$|\{3\}\rangle = |\downarrow\downarrow\uparrow\downarrow\downarrow\ldots\rangle$

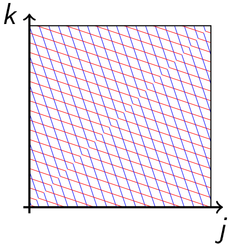
$|\{1,4\}\rangle = |\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\rangle$

- "Vacuum": $|\downarrow\downarrow\cdots\downarrow\rangle$
- Single excitation: $|\psi\rangle \propto \sum_k e^{i\,k\,p}\,|\{k\}\rangle$ where $e^{2i\,p\,L} = 1$
- Two excitations:

$$|\psi\rangle \propto \sum_{j<k} (e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)})\, |\{j,k\}\rangle$$

where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with

$$S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$$



"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

- Two excitations: $|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j+p_2 k)} + S\, e^{i(p_1 k+p_2 j)}\right)|\{j,k\}\rangle$

  where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with $S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$

- n excitations:

$$|\Psi\rangle = \sum_{1\leqslant j_1<j_2<\cdots<j_n\leqslant L}\ \sum_{\sigma\in\mathfrak{S}_n} \mathcal{A}_\sigma e^{i\sum_k p_{\sigma(k)} j_k}\,|\{j_1,j_2,\ldots,j_n\}\rangle$$

is an eigenstate if

  - $\mathcal{A}_\sigma \propto (-1)^\sigma \prod_{j<k}\left(1+e^{i(p_{\sigma(j)}+p_{\sigma(k)})}-2e^{ip_{\sigma(k)}}\right)$
  - $\forall j,\ e^{i\,L\,p_j} = \prod_{k\neq j} S(p_j,p_k)$ where $S(p,p') = -\frac{1+e^{i(p+p')}-2e^{i\,p}}{1+e^{i(p+p')}-2e^{i\,p'}}$

The corresponding eigenvalue is

$$E = -L + \sum_k (4-4\cos p_k)$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

- Two excitations: $|\psi\rangle \propto \sum_{j<k} (e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)}) \,|\{j,k\}\rangle$
  where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with $S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$
- n excitations:

$$|\Psi\rangle = \sum_{1\leqslant j_1<j_2<\cdots<j_n\leqslant L} \sum_{\sigma\in\mathfrak{S}_n} \mathcal{A}_\sigma e^{i\sum_k p_{\sigma(k)} j_k} \,|\{j_1, j_2, \ldots, j_n\}\rangle$$

is an eigenstate if

- $\mathcal{A}_\sigma \propto (-1)^\sigma \prod_{j<k} \left(1 + e^{i(p_{\sigma(j)}+p_{\sigma(k)})} - 2e^{ip_{\sigma(k)}}\right)$
- $\forall j, e^{i\,L\,p_j} = \prod_{k\neq j} S(p_j, p_k)$ where $S(p,p') \equiv -\frac{1+e^{i(p+p')}-2e^{i\,p}}{1+e^{i(p+p')}-2e^{i\,p'}}$

The corresponding eigenvalue is

$$E = -L + \sum_k (4 - 4\cos p_k)$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

- Two excitations: $|\psi\rangle \propto \sum_{j<k} \left(e^{i(p_1 j + p_2 k)} + S\, e^{i(p_1 k + p_2 j)}\right) |\{j,k\}\rangle$
  where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with $S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$
- n excitations:

$$|\Psi\rangle = \sum_{1\leqslant j_1<j_2<\cdots<j_n\leqslant L} \sum_{\sigma\in\mathfrak{S}_n} \mathcal{A}_\sigma e^{i\sum_k p_{\sigma(k)} j_k} |\{j_1, j_2, \ldots, j_n\}\rangle$$

  is an eigenstate if
  - $\mathcal{A}_\sigma \propto (-1)^\sigma \prod_{j<k} \left(1 + e^{i(p_{\sigma(j)}+p_{\sigma(k)})} - 2e^{ip_{\sigma(k)}}\right)$
  - $\forall j, e^{i\,L\,p_j} = \prod_{k\neq j} S(p_j, p_k)$ where $S(p,p') \equiv -\frac{1+e^{i(p+p')}-2e^{i\,p}}{1+e^{i(p+p')}-2e^{i\,p'}}$

The corresponding eigenvalue is

$$E = -L + \sum_k (4 - 4\cos p_k)$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

- Two excitations: $|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j+p_2 k)} + S\, e^{i(p_1 k+p_2 j)}\right)|\{j,k\}\rangle$
  where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with $S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$
- n excitations:

$$|\Psi\rangle = \sum_{1\leqslant j_1<j_2<\cdots<j_n\leqslant L}\ \sum_{\sigma\in\mathfrak{S}_n} \mathcal{A}_\sigma e^{i\sum_k p_{\sigma(k)} j_k}\,|\{j_1,j_2,\ldots,j_n\}\rangle$$

  is an eigenstate if
  - $\mathcal{A}_\sigma \propto (-1)^\sigma \prod_{j<k}\left(1+e^{i(p_{\sigma(j)}+p_{\sigma(k)})}-2e^{ip_{\sigma(k)}}\right)$
  - $\forall j,\, e^{i\,L\,p_j} = \prod_{k\neq j} S(p_j,p_k)$ where $S(p,p') \equiv -\frac{1+e^{i(p+p')}-2e^{i\,p}}{1+e^{i(p+p')}-2e^{i\,p'}}$

The corresponding eigenvalue is

$$E = -L + \sum_k (4-4\cos p_k)$$

"Coordinate Bethe Ansatz" for $XXX_{1/2}$ Heisenberg spin chain

- Two excitations: $|\psi\rangle \propto \sum_{j<k}\left(e^{i(p_1 j+p_2 k)} + S\, e^{i(p_1 k+p_2 j)}\right)|\{j,k\}\rangle$
  where $e^{i\,L\,p_2} = S = e^{-i\,L\,p_1}$, with $S = -\frac{1+e^{i(p_1+p_2)}-2e^{ip_2}}{1+e^{i(p_1+p_2)}-2e^{ip_1}}$
- n excitations:

$$|\Psi\rangle = \sum_{1\leqslant j_1<j_2<\cdots<j_n\leqslant L} \sum_{\sigma\in\mathfrak{S}_n} \mathcal{A}_\sigma e^{i\sum_k p_{\sigma(k)} j_k} |\{j_1,j_2,\ldots,j_n\}\rangle$$

is an eigenstate if

- $\mathcal{A}_\sigma \propto (-1)^\sigma \prod_{j<k}\left(1+e^{i(p_{\sigma(j)}+p_{\sigma(k)})} - 2e^{ip_{\sigma(k)}}\right)$
- $\forall j,\ e^{i\,L\,p_j} = \prod_{k\neq j} S(p_j,p_k)$ where $S(p,p') \equiv -\frac{1+e^{i(p+p')}-2e^{i\,p}}{1+e^{i(p+p')}-2e^{i\,p'}}$

The corresponding eigenvalue is

$$E = -L + \sum_k (4 - 4\cos p_k)$$

spectrum

Plan

1. Rational spin chains
    - Coordinate Bethe Ansatz
    - Transfer matrices
    - Q-system and Bethe Ansatz
2. Finite size spectrum of sigma models
    - Thermodynamic Bethe Ansatz
    - "Quantum Spectral Curve" for AdS/CFT

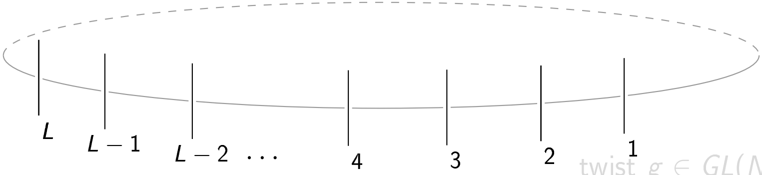
Transfer matrices

example of *XXX*-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T(u)\Big|_{u=0}$$

$$T(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a}) \cdot (u\,\mathbb{I} + \mathcal{P}_{L-1,a}) \cdots (u\,\mathbb{I} + \mathcal{P}_{1,a})\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



twist $g \in GL(N)$

- $[T(u), T(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)T^{a,s}(u) = T^{a+1,s}(u+1)T^{a-1,s}(u) + T^{a,s-1}(u+1)T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

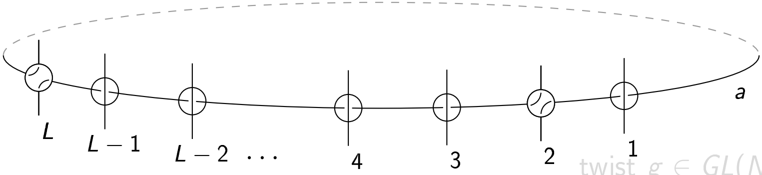
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L}\mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}}\log T(u)\Big|_{u=0}$$

$$T(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



twist $g \in GL(N)$

- $[T(u), T(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)T^{a,s}(u) = T^{a+1,s}(u+1)T^{a-1,s}(u) + T^{a,s-1}(u+1)T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

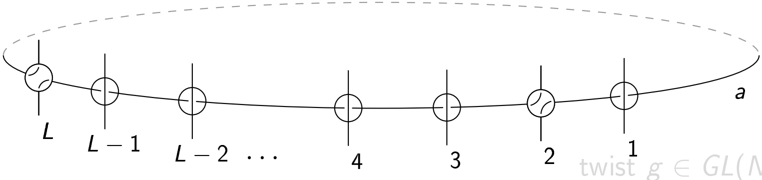
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{d}u}\log T(u)\Big|_{u=0}$$

$$T(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T(u), T(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)T^{a,s}(u) = T^{a+1,s}(u+1)T^{a-1,s}(u) + T^{a,s-1}(u+1)T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T(u)\Big|_{u=0}$$

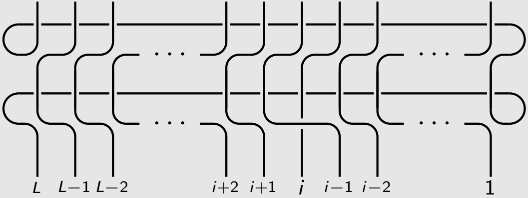
$$T(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a}) \cdot (u\,\mathbb{I} + \mathcal{P}_{L-1,a}) \cdots (u\,\mathbb{I} + \mathcal{P}_{1,a})\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$

Proof

$$\frac{\mathrm{d}}{\mathrm{du}} \log T(u)\Big|_{u=0} = \Big(T(0)\Big)^{-1} \cdot \partial_u T(u)|_{u=0}$$

$$= \sum_i$$



$$= \sum_i \mathcal{P}_{i,i+1}$$

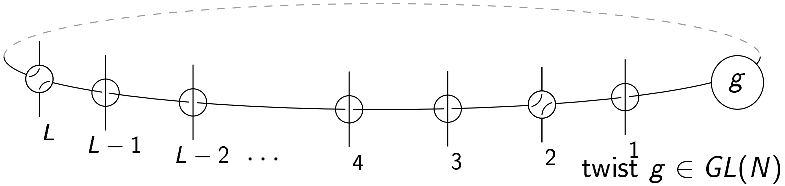
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T(u)\Big|_{u=0}$$

$$T(u) = \mathrm{tr}_a \left((u\,\mathbb{I} + \mathcal{P}_{L,a}) \cdot (u\,\mathbb{I} + \mathcal{P}_{L-1,a}) \cdots (u\,\mathbb{I} + \mathcal{P}_{1,a})\, g \right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T(u), T(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)T^{a,s}(u) = T^{a+1,s}(u+1)T^{a-1,s}(u) + T^{a,s-1}(u+1)T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

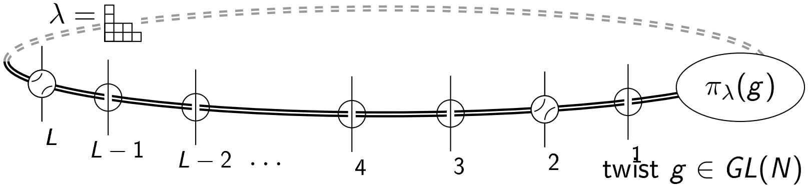
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{d}u} \log T^{\square}(u)\Big|_{u=0}$$

$$T^{\lambda}(u) = \mathrm{tr}_a\left(\left(u\,\mathbb{I} + \mathcal{P}_{L,a}\right)\cdot\left(u\,\mathbb{I} + \mathcal{P}_{L-1,a}\right)\cdots\left(u\,\mathbb{I} + \mathcal{P}_{1,a}\right)\pi_\lambda(g)\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T^{\lambda}(u), T^{\mu}(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)T^{a,s}(u) = T^{a+1,s}(u+1)T^{a-1,s}(u) + T^{a,s-1}(u+1)T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

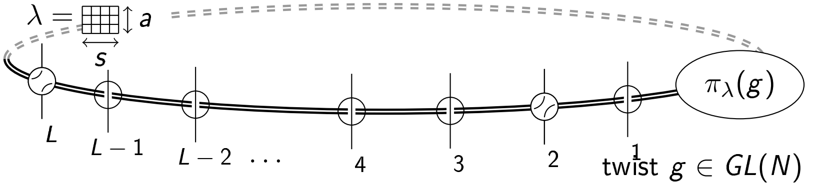
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T^{\square}(u)\Big|_{u=0}$$

$$T^{\lambda}(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\,\pi_\lambda(g)\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T^{\lambda}(u), T^{\mu}(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

$$T^{a,s}(u+1)\,T^{a,s}(u) = T^{a+1,s}(u+1)\,T^{a-1,s}(u) + T^{a,s-1}(u+1)\,T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

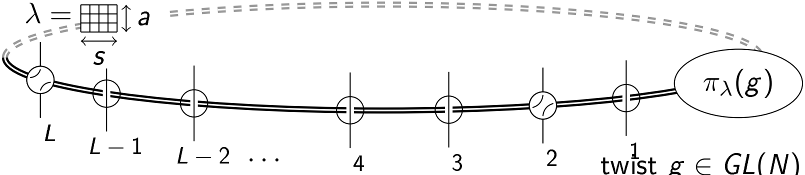
Transfer matrices

example of *XXX*-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{d}u}\log T^{\square}(u)\Big|_{u=0}$$

$$T^{\lambda}(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\,\pi_\lambda(g)\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



twist $g \in GL(N)$

- $[T^{\lambda}(u), T^{\mu}(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation: **classical integrability, mKP hierarchy**

$$T^{a,s}(u+1)\,T^{a,s}(u) = T^{a+1,s}(u+1)\,T^{a-1,s}(u) + T^{a,s-1}(u+1)\,T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

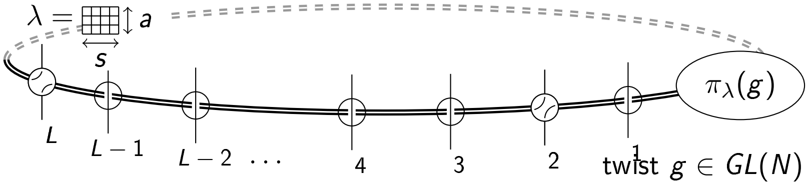
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T^{\square}(u)\Big|_{u=0}$$

$$T^{\lambda}(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\,\pi_\lambda(g)\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T^{\lambda}(u), T^{\mu}(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation:

classical integrability, mKP hierarchy
[Alexandrov Kazakov SL Tsuboi Zabrodin 13]

$$T^{a,s}(u+1)\,T^{a,s}(u) = T^{a+1,s}(u+1)\,T^{a-1,s}(u) + T^{a,s-1}(u+1)\,T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most ... rows

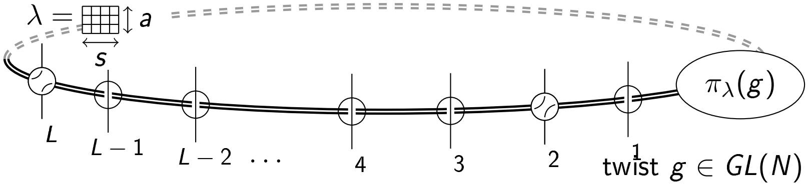
Transfer matrices

example of XXX-type spin chains

$$H = L - 2\sum_{i=1}^{L} \mathcal{P}_{i,i+1} = L - 2\,\frac{\mathrm{d}}{\mathrm{d}u}\log T^{\square}(u)\Big|_{u=0}$$

$$T^{\lambda}(u) = \mathrm{tr}_a\left((u\,\mathbb{I} + \mathcal{P}_{L,a})\cdot(u\,\mathbb{I} + \mathcal{P}_{L-1,a})\cdots(u\,\mathbb{I} + \mathcal{P}_{1,a})\,\pi_\lambda(g)\right)$$

mutually commuting operators on the Hilbert space $\left(\mathbb{C}^N\right)^{\otimes L}$



- $[T^{\lambda}(u), T^{\mu}(v)] = 0$ (due to relations like $\mathcal{P}_{i,j}\mathcal{P}_{j,k} = \mathcal{P}_{j,k}\mathcal{P}_{i,k}$)
- Hirota equation: **classical integrability, mKP hierarchy**

$$T^{a,s}(u+1)\,T^{a,s}(u) = T^{a+1,s}(u+1)\,T^{a-1,s}(u) + T^{a,s-1}(u+1)\,T^{a,s+1}(u)$$

- For $g \in GL(N)$, Young tableaux have at most a rows

Plan

1. Rational spin chains
   - Coordinate Bethe Ansatz
   - Transfer matrices
   - Q-system and Bethe Ansatz
2. Finite size spectrum of sigma models
   - Thermodynamic Bethe Ansatz
   - "Quantum Spectral Curve" for AdS/CFT

Q-system and Nested Bethe Ansatz

Generic solution of Hirota equation

[Krichever Lipan Wiegmann Zabrodin 97]

$$T^{\lambda}(u) = \frac{\det\left(x_j^{1-k+\lambda_k} Q_j(u-k+1+\lambda_k)\right)_{1\le j,k\le N}}{\Delta(x_1,\cdots,x_N)}$$

where $g = \mathrm{diag}(x_1,\cdots x_N)$; $\quad \Delta(x_1,\cdots,x_N) = \det\left(x_j^{1-k}\right)_{1\le j,k\le N}$

- Example of construction

$\exists \hat{D} : T^{\lambda}(u) = (u+\hat{D})^{\otimes L}\chi_{\lambda}(g)$ [Kazakov Vieira 08]

$Q_i = (u+N-1+\hat{D})^{\otimes L} \prod_{k\neq i} \det \frac{1}{1-g\,t_k}\Big|_{t_k\to 1/x_k}$ [Kazakov SL Tsuboi 12]

- Other construction: specific representation in auxiliary space

Q-system and Nested Bethe Ansatz

Generic solution of Hirota equation

[Krichever Lipan Wiegmann Zabrodin 97]

$$T^\lambda(u) = \frac{\det\left(x_j^{1-k+\lambda_k} Q_j(u-k+1+\lambda_k)\right)_{1\le j,k\le N}}{\Delta(x_1,\cdots,x_N)}$$

where $g = \mathrm{diag}(x_1,\cdots x_N)$; $\quad \Delta(x_1,\cdots,x_N) = \det\left(x_j^{1-k}\right)_{1\le j,k\le N}$

- Example of construction

$\exists \hat{D} : T^\lambda(u) = (u+\hat{D})^{\otimes L}\chi_\lambda(g)$ [Kazakov Vieira 08]

$Q_i = (u+N-1+\hat{D})^{\otimes L}\prod_{k\neq i}\det\frac{1}{1-g\,t_k}\Big|_{t_k\to 1/x_k}$ [Kazakov SL Tsuboi 12]

- Other construction: specific representation in auxiliary space

Q-system and Nested Bethe Ansatz

Generic solution of Hirota equation
[Krichever Lipan Wiegmann Zabrodin 97]

$$T^{\lambda}(u) = \frac{\det\left(x_j^{1-k+\lambda_k} Q_j(u-k+1+\lambda_k)\right)_{1\leq j,k\leq N}}{\Delta(x_1,\cdots,x_N)}$$

where $g = \mathrm{diag}(x_1,\cdots x_N)$; $\quad \Delta(x_1,\cdots,x_N) = \det\left(x_j^{1-k}\right)_{1\leq j,k\leq N}$

- Example of construction

$\exists \hat{D} : T^{\lambda}(u) = (u+\hat{D})^{\otimes L}\chi_\lambda(g)$ [Kazakov Vieira 08]

$Q_i = (u+N-1+\hat{D})^{\otimes L}\prod_{k\neq i} \det \frac{1}{1-g\,t_k}\Big|_{t_k\to 1/x_k}$ [Kazakov SL Tsuboi 12]

- Other construction: specific representation in auxiliary space

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2\, \frac{\mathrm{d}}{\mathrm{du}} \log T^{\Box}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$u^L = Q_1(u)\, Q_2(u-1) - Q_2(u) Q_1(u-1)$$

$$(u+1)^L = Q_1(u+1) Q_2(u) - Q_2(u+1)\, Q_1(u)$$

$$\frac{\prod_k(\theta_j-\theta_k+1)}{\prod_k(\theta_j-\theta_k-1)} = -\left(\frac{\theta_j+1}{\theta_j}\right)^L \qquad \xleftrightarrow{\theta=\frac{e^{ip}}{1-e^{ip}}} \qquad \forall j,\ e^{iLp_j} = \prod_{k\neq j} S(p_j,p_k)$$

$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j+1)} \qquad\qquad E = -L + \sum_k (4 - 4\cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T^{\square}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\qquad Q_1(u) = \prod_j (u - \theta_j)$

$$u^L = Q_1(u)\,Q_2(u-1) - Q_2(u)Q_1(u-1)$$

$$(u+1)^L = Q_1(u+1)Q_2(u) - Q_2(u+1)\,Q_1(u)$$

$$\frac{\prod_k(\theta_j-\theta_k+1)}{\prod_k(\theta_j-\theta_k-1)} = -\left(\frac{\theta_j+1}{\theta_j}\right)^L$$
$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j+1)}$$

$$\xleftrightarrow{\theta=\frac{e^{ip}}{1-e^{ip}}}$$

$$\forall j, e^{iLp_j} = \prod_{k\neq j} S(p_j, p_k)$$
$$E = -L + \sum_k (4 - 4\cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2 \frac{\mathrm{d}}{\mathrm{du}} \log T^{\Box}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$u^L = Q_1(u)\, Q_2(u-1) - Q_2(u) Q_1(u-1)$$

$$(u+1)^L = Q_1(u+1) Q_2(u) - Q_2(u+1)\, Q_1(u)$$

$$\frac{\prod_k (\theta_j - \theta_k + 1)}{\prod_k (\theta_j - \theta_k - 1)} = -\left(\frac{\theta_j + 1}{\theta_j}\right)^L \qquad \xLeftrightarrow{\theta = \frac{e^{i\,p}}{1 - e^{i\,p}}} \qquad \forall j, e^{i\,L\,p_j} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = -L - 2 \sum_j \frac{1}{\theta_j(\theta_j + 1)} \qquad E = -L + \sum_k (4 - 4 \cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2 \, \frac{\mathrm{d}}{\mathrm{du}} \log T^{\Box}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$u^L = Q_1(u)\, Q_2(u-1) - Q_2(u) Q_1(u-1)$$

$$(u+1)^L = Q_1(u+1) Q_2(u) - Q_2(u+1)\, Q_1(u)$$

$$\frac{\prod_k(\theta_j-\theta_k+1)}{\prod_k(\theta_j-\theta_k-1)} = -\left(\frac{\theta_j+1}{\theta_j}\right)^L$$

$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j+1)}$$

$$\theta = \frac{e^{ip}}{1-e^{ip}} \quad \longleftrightarrow$$

$$\forall j,\ e^{iLp_j} = \prod_{k\neq j} S(p_j, p_k)$$

$$E = -L + \sum_k (4 - 4\cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2 \frac{\mathrm{d}}{\mathrm{du}} \log T^{\Box}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$\theta_j^L = \cancel{Q_1(\theta_j)}\, Q_2(\theta_j - 1) - Q_2(\theta_j) Q_1(\theta_j - 1)$$

$$(\theta_j + 1)^L = Q_1(\theta_j + 1) Q_2(\theta_j) - Q_2(\theta_j + 1)\, \cancel{Q_1(\theta_j)}$$

$$\frac{\prod_k(\theta_j - \theta_k + 1)}{\prod_k(\theta_j - \theta_k - 1)} = -\left(\frac{\theta_j + 1}{\theta_j}\right)^L$$

$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j + 1)}$$

$$\xleftrightarrow{\theta = \frac{e^{ip}}{1 - e^{ip}}}$$

$$\forall j, e^{iLp_j} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = -L + \sum_k (4 - 4\cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2 \, \frac{\mathrm{d}}{\mathrm{du}} \log T^{\square}(u)\Big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$\theta_j^L = \cancel{Q_1(\theta_j)} \, Q_2(\theta_j - 1) - Q_2(\theta_j) Q_1(\theta_j - 1)$$

$$(\theta_j + 1)^L = Q_1(\theta_j + 1) Q_2(\theta_j) - Q_2(\theta_j + 1) \, \cancel{Q_1(\theta_j)}$$

$$\frac{\prod_k(\theta_j - \theta_k + 1)}{\prod_k(\theta_j - \theta_k - 1)} = -\left(\frac{\theta_j + 1}{\theta_j}\right)^L$$

$$E = -L - 2 \sum_j \frac{1}{\theta_j(\theta_j + 1)}$$

$$\xleftrightarrow{\theta = \frac{e^{i p}}{1 - e^{i p}}}$$

$$\forall j, e^{i L p_j} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = -L + \sum_k (4 - 4 \cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \leq j \leq N$)
- $T^{\emptyset} = u^L$
- $H = L - 2\,\frac{\mathrm{d}}{\mathrm{du}} \log T^{\Box}(u)\Big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $\quad Q_1(u) = \prod_j (u - \theta_j)$

$$\theta_j^L = \cancel{Q_1(\theta_j)}\, Q_2(\theta_j - 1) - Q_2(\theta_j) Q_1(\theta_j - 1)$$

$$(\theta_j + 1)^L = Q_1(\theta_j + 1) Q_2(\theta_j) - Q_2(\theta_j + 1)\, \cancel{Q_1(\theta_j)}$$

$$\frac{\prod_k(\theta_j - \theta_k + 1)}{\prod_k(\theta_j - \theta_k - 1)} = -\left(\frac{\theta_j + 1}{\theta_j}\right)^L$$

$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j + 1)}$$

$$\theta = \frac{e^{ip}}{1 - e^{ip}} \quad \Longleftrightarrow$$

$$\forall j,\ e^{i L p_j} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = -L + \sum_k (4 - 4\cos p_k)$$

Spectrum

Three conditions fix the spectrum

- Q_j is polynomial in u (for $1 \le j \le N$)
- $T^{\emptyset} = u^L$
- $H = L - 2\, \frac{\mathrm{d}}{\mathrm{du}} \log T^{\square}(u)\big|_{u=0}$

- Example for periodic $SU(2)$ spin chain $Q_1(u) = \prod_j (u - \theta_j)$

$$\theta_j^L = \cancel{Q_1(\theta_j)}\, Q_2(\theta_j - 1) - Q_2(\theta_j) Q_1(\theta_j - 1)$$
$$(\theta_j + 1)^L = Q_1(\theta_j + 1) Q_2(\theta_j) - Q_2(\theta_j + 1)\, \cancel{Q_1(\theta_j)}$$

$$\frac{\prod_k(\theta_j-\theta_k+1)}{\prod_k(\theta_j-\theta_k-1)} = -\left(\frac{\theta_j+1}{\theta_j}\right)^L \qquad \forall j,\, e^{i L p_j} = \prod_{k\neq j} S(p_j, p_k)$$
$$E = -L - 2\sum_j \frac{1}{\theta_j(\theta_j+1)} \quad \overset{\theta=\frac{e^{i p}}{1-e^{i p}}}{\Longleftrightarrow} \quad E = -L + \sum_k (4 - 4\cos p_k)$$

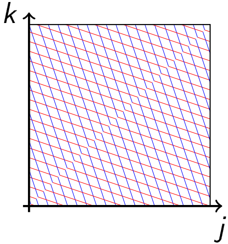
Plan

Integrable field theories

Bethe Ansatz of the form $\psi(n_1, n_2, \cdots, n_M) \equiv \sum_{\sigma \in \mathcal{S}^M} \mathcal{A}_\sigma e^{i \sum_k p_{\sigma(k)} n_k}$

$\rightsquigarrow$ wave-function of the eigenstates of several theories such that

- The space is one-dimensional and there are periodic boundary conditions.
- The interactions are local.
- A factorization formula holds



One can argue that it is sufficient to have infinitely many conserved charges

[Zamolodchikov Zamolodchikov 79]

- "Locality" requires a large spatial period

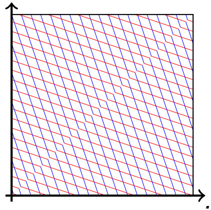
$\rightsquigarrow$ Question of the finite size effects

Integrable field theories

Bethe Ansatz of the form $\psi(n_1, n_2, \cdots, n_M) \equiv \sum_{\sigma \in \mathcal{S}^M} \mathcal{A}_\sigma e^{i \sum_k p_{\sigma(k)} n_k}$

$\rightsquigarrow$ wave-function of the eigenstates of several theories such that

- The space is one-dimensional and there are periodic boundary conditions.
- The interactions are local.
- A factorization formula holds



One can argue that it is sufficient to have infinitely many conserved charges

[Zamolodchikov Zamolodchikov 79]

- "Locality" requires a large spatial period

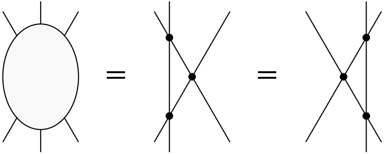
$\rightsquigarrow$ Question of the finite size effects

Integrable field theories

Bethe Ansatz of the form $\psi(n_1, n_2, \cdots, n_M) \equiv \sum_{\sigma \in \mathcal{S}^M} \mathcal{A}_\sigma e^{i \sum_k p_{\sigma(k)} n_k}$

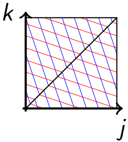
$\rightsquigarrow$ wave-function of the eigenstates of several theories such that

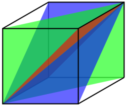
- The space is one-dimensional and there are periodic boundary conditions.
- The interactions are local.
- A factorization formula holds:



One can argue that it is sufficient to have infinitely many conserved charges

[Zamolodchikov Zamolodchikov 79]





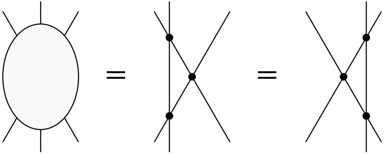
- "Locality" requires a large spatial period

$\rightsquigarrow$ Question of the finite size effects

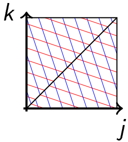
Integrable field theories

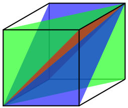
Bethe Ansatz of the form $\psi(n_1, n_2, \cdots, n_M) \equiv \sum_{\sigma \in \mathcal{S}^M} \mathcal{A}_\sigma e^{i \sum_k p_{\sigma(k)} n_k}$

$\rightsquigarrow$ wave-function of the eigenstates of several theories such that



- The space is one-dimensional and there are periodic boundary conditions.
- The interactions are local.
- A factorization formula holds





One can argue that it is sufficient to have infinitely many conserved charges

[Zamolodchikov Zamolodchikov 79]

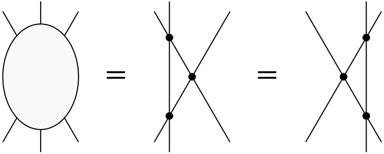
- "Locality" requires a large spatial period

$\rightsquigarrow$ Question of the finite size effects

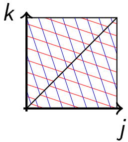
Integrable field theories

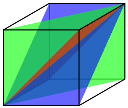
Bethe Ansatz of the form $\psi(n_1, n_2, \cdots, n_M) \equiv \sum_{\sigma \in \mathcal{S}^M} \mathcal{A}_\sigma e^{i \sum_k p_{\sigma(k)} n_k}$

$\rightsquigarrow$ wave-function of the eigenstates of several theories such that



- The space is one-dimensional and there are periodic boundary conditions.
- The interactions are local.
- A factorization formula holds





One can argue that it is sufficient to have infinitely many conserved charges

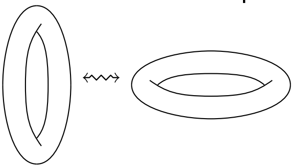
[Zamolodchikov Zamolodchikov 79]

- "Locality" requires a large spatial period

$\rightsquigarrow$ Question of the finite size effects

Thermodynamic Bethe Ansatz

- *Matsubara Trick*: "double Wick Rotation"
  finite size ⇜⇝ finite temperature



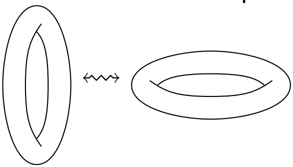
- At finite temperature, the Bethe equations give rise to several different types of bound states
  ⇝ introduce one density of excitations (as a function of the rapidity) for each type of bound state.

- For vacuum, densities given by T-functions $T_{a,s}(u)$ obeying the Hirota equation
  $T_{a,s}(u+i/2)T_{a,s}(u-i/2) = T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$

Thermodynamic Bethe Ansatz

- *Matsubara Trick*: "double Wick Rotation"

  finite size $\leftrightsquigarrow$ finite temperature

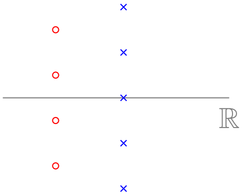


- At finite temperature, the Bethe equations give rise to several different types of bound states

  $\rightsquigarrow$ introduce one density of excitations (as a function of the rapidity) for each type of bound state.

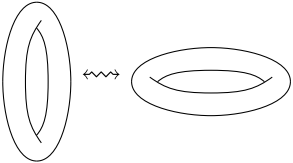
- For vacuum, densities given by T-functions $T_{a,s}(u)$ obeying the Hirota equation

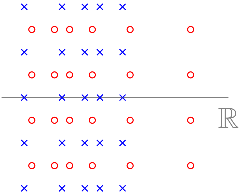
  $$T_{a,s}(u+i/2)T_{a,s}(u-i/2) = T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$$



Thermodynamic Bethe Ansatz

- *Matsubara Trick*: "double Wick Rotation"
  finite size $\leftrightsquigarrow$ finite temperature

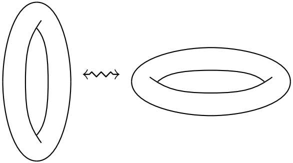


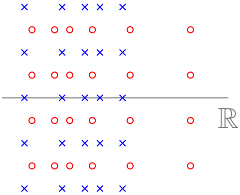


- At finite temperature, the Bethe equations give rise to several different types of bound states $\rightsquigarrow$ introduce one density of excitations (as a function of the rapidity) for each type of bound state.
- For vacuum, densities given by T-functions $T_{a,s}(u)$ obeying the Hirota equation
  $T_{a,s}(u+i/2)T_{a,s}(u-i/2) = T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$

Thermodynamic Bethe Ansatz

- *Matsubara Trick*: "double Wick Rotation"
  finite size ⇜⇝ finite temperature

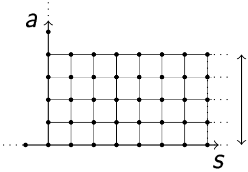




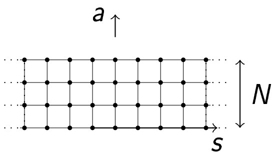
- At finite temperature, the Bethe equations give rise to several different types of bound states ⇝ introduce one density of excitations (as a function of the rapidity) for each type of bound state.
- For vacuum, densities given by T-functions $T_{a,s}(u)$ obeying the Hirota equation
  $T_{a,s}(u+i/2)\,T_{a,s}(u-i/2) = T_{a+1,s}(u)\,T_{a-1,s}(u) + T_{a,s+1}(u)\,T_{a,s-1}(u)$

Boundary conditions for Hirota equation

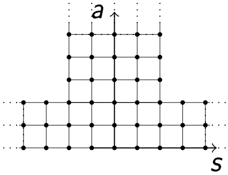
- **Symmetry Group ⇝ boundary condition**



$SU(N)$ spin chain



$SU(N) \times SU(N)$
principal chiral model



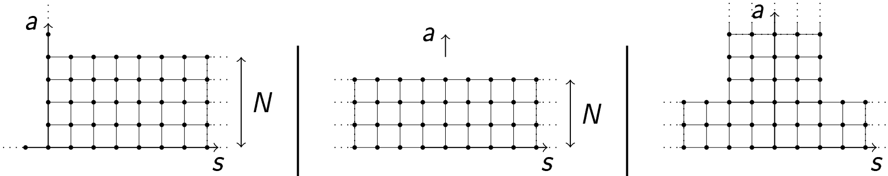
AdS_5/CFT_4
$PSU(2,2|4)$

⇝ $T^{a,s}(u) = \det(Q_j(u + \dots))_{1 \le j,k \le \dots}$

- Additional information: analyticity properties

Boundary conditions for Hirota equation

- **Symmetry Group ⇝ boundary condition**



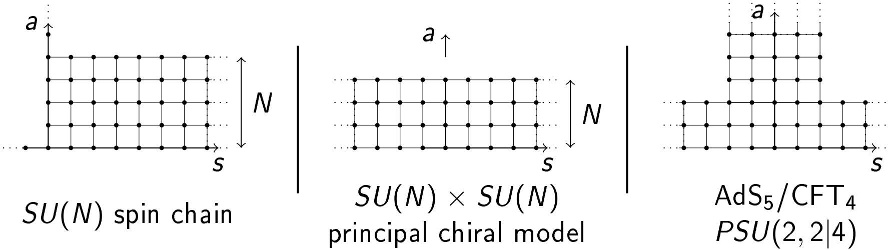
$SU(N)$ spin chain | $SU(N) \times SU(N)$ principal chiral model | $\mathrm{AdS}_5/\mathrm{CFT}_4$ $PSU(2,2|4)$

⇝ $T^{a,s}(u) = \det(Q_j(u + \dots))_{1 \le j,k \le \dots}$

- Additional information: analyticity properties
  - Singularities (poles, branch points, ...)
  - Large u behaviour (power-like, exponentially small, ...)
  - Analytic continuation around branch points.

Boundary conditions for Hirota equation

- **Symmetry Group $\leftrightsquigarrow$ boundary condition**

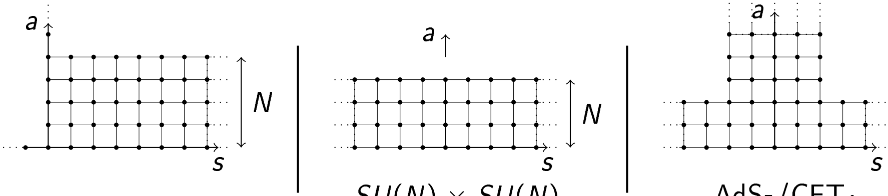


$\rightsquigarrow$ $T^{a,s}(u) = \det(Q_j(u + \dots))_{1 \leq j,k \leq \dots}$

- Additional information: analyticity properties
  - Singularities (poles, branch points, ...)
  - Large u behaviour (power-like, exponentially small, ...)
  - Analytic continuation around branch points.

Boundary conditions for Hirota equation

- **Symmetry Group ⇜⇝ boundary condition**



$SU(N)$ spin chain | $SU(N) \times SU(N)$ principal chiral model | AdS_5/CFT_4 $PSU(2,2|4)$

$\rightsquigarrow$ $T^{a,s}(u) = \det(Q_j(u + \dots))_{1 \le j,k \le \dots}$

- Additional information: analyticity properties
  - Singularities (poles, branch points, . . .)
  - Large u behaviour (power-like, exponentioally small, . . .)
  - Analytic continuation around branch points.

Plan

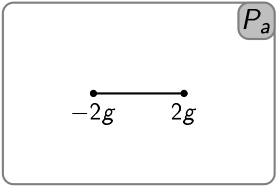
1. Rational spin chains
   - Coordinate Bethe Ansatz
   - Transfer matrices
   - Q-system and Bethe Ansatz

2. Finite size spectrum of sigma models
   - Thermodynamic Bethe Ansatz
   - "Quantum Spectral Curve" for AdS/CFT

Analyticity requirements

Quantum Spectral Curve

- Functions P_a and P^a holomorphic on $\mathbb{C} \setminus [-2g, 2g]$ $(1 \leq a \leq 4)$ where $g = \frac{\sqrt{\lambda}}{4\pi}$, $\lambda = g_{YM} N_c^2$.



- Denote by tilde the analytic continuation around $\pm 2g$.

- Functions μ_{ab} i-periodic and holomorphic on $\mathbb{C} \setminus ((]-\infty, -2g] \cup [2g, \infty[) + i\mathbb{Z})$

$\mu_{ab} = -\mu_{ba}$ $1 \leq a, b \leq 4$

$\mu_{12}\,\mu_{34} - \mu_{13}\,\mu_{24} + \mu_{14}\,\mu_{23} = 1$

- Power-like asymptotics

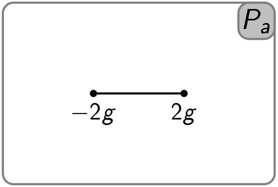
$$\tilde{\mu}_{ab} - \mu_{ab} = P_a \tilde{P}_b - P_b \tilde{P}_a \qquad \tilde{P}_a = \mu_{ab} P^b \qquad P_a P^a = 0$$

[Gromov Kazakov SL Volin 14]

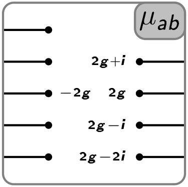
Analyticity requirements

Quantum Spectral Curve

- Functions P_a and P^a holomorphic on $\mathbb{C} \setminus [-2g, 2g]$ $(1 \leq a \leq 4)$
  where $g = \frac{\sqrt{\lambda}}{4\pi}$, $\lambda = g_{YM} N_c^2$.



- Functions μ_{ab} i-periodic and holomorphic on
  $\mathbb{C} \setminus ((]-\infty, -2g] \cup [2g, \infty[) + i\mathbb{Z})$
  $\mu_{ab} = -\mu_{ba}$ $1 \leq a, b \leq 4$
  $\mu_{12}\,\mu_{34} - \mu_{13}\,\mu_{24} + \mu_{14}\,\mu_{23} = 1$



- Denote by tilde the analytic continuation around $\pm 2g$.
- Power-like asymptotics

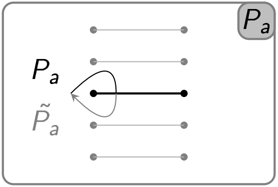
$$\tilde{\mu}_{ab} - \mu_{ab} = P_a \tilde{P}_b - P_b \tilde{P}_a \qquad \tilde{P}_a = \mu_{ab} P^b \qquad P_a P^a = 0$$

[Gromov Kazakov SL Volin 14]

Analyticity requirements

Quantum Spectral Curve

- Functions P_a and P^a holomorphic on $\mathbb{C} \setminus [-2g, 2g]$ $(1 \leq a \leq 4)$ where $g = \frac{\sqrt{\lambda}}{4\pi}$, $\lambda = g_{YM} N_c^2$.

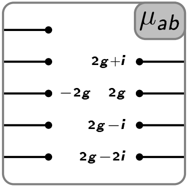


- Denote by tilde the analytic continuation around $\pm 2g$.

- Functions μ_{ab} i-periodic and holomorphic on $\mathbb{C} \setminus ((]-\infty, -2g] \cup [2g, \infty[) + i\mathbb{Z})$

$\mu_{ab} = -\mu_{ba}$ $\quad 1 \leq a, b \leq 4$

$\mu_{12}\,\mu_{34} - \mu_{13}\,\mu_{24} + \mu_{14}\,\mu_{23} = 1$



- Power-like asymptotics

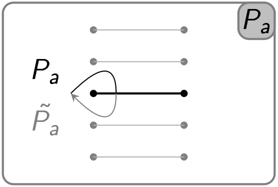
$$\tilde{\mu}_{ab} - \mu_{ab} = P_a \tilde{P}_b - P_b \tilde{P}_a \qquad \tilde{P}_a = \mu_{ab} P^b \qquad P_a P^a = 0$$

[Gromov Kazakov SL Volin 14]

Analyticity requirements

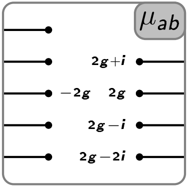
Quantum Spectral Curve

- Functions P_a and P^a holomorphic on $\mathbb{C} \setminus [-2g, 2g]$ $(1 \leq a \leq 4)$ where $g = \frac{\sqrt{\lambda}}{4\pi}$, $\lambda = g_{YM} N_c^2$.



- Denote by tilde the analytic continuation around $\pm 2g$.

- Functions μ_{ab} i-periodic and holomorphic on $\mathbb{C} \setminus ((]-\infty, -2g] \cup [2g, \infty[) + i\mathbb{Z})$
$\mu_{ab} = -\mu_{ba}$ $1 \leq a, b \leq 4$
$\mu_{12}\, \mu_{34} - \mu_{13}\, \mu_{24} + \mu_{14}\, \mu_{23} = 1$



- Power-like asymptotics

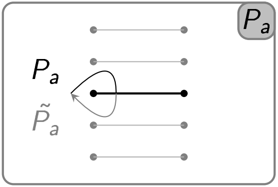
$$\tilde{\mu}_{ab} - \mu_{ab} = P_a \tilde{P}_b - P_b \tilde{P}_a \qquad \tilde{P}_a = \mu_{ab} P^b \qquad P_a P^a = 0$$

[Gromov Kazakov SL Volin 14]

Analyticity requirements

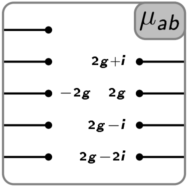
Quantum Spectral Curve

- Functions P_a and P^a holomorphic on $\mathbb{C} \setminus [-2g, 2g]$ $(1 \le a \le 4)$ where $g = \frac{\sqrt{\lambda}}{4\pi}$, $\lambda = g_{YM} N_c^2$.



- Denote by tilde the analytic continuation around $\pm 2g$.

- Functions μ_{ab} i-periodic and holomorphic on $\mathbb{C} \setminus ((]-\infty, -2g] \cup [2g, \infty[) + i\mathbb{Z})$
$\mu_{ab} = -\mu_{ba}$ $1 \le a, b \le 4$
$\mu_{12}\,\mu_{34} - \mu_{13}\,\mu_{24} + \mu_{14}\,\mu_{23} = 1$



- Power-like asymptotics

$$\tilde{\mu}_{ab} - \mu_{ab} = P_a \tilde{P}_b - P_b \tilde{P}_a \qquad \tilde{P}_a = \mu_{ab} P^b \qquad P_a P^a = 0$$

[Gromov Kazakov SL Volin 14]

Tests and generalizations

Non-exhaustive list

Tests and applications of the QSC

- Weak coupling expansion [Marboe Velizhanin Volin 14]
- Numeric resolution [Gromov Levkovich-Maslyuk Sizov 15]
- BFKL regime [Alfimov Gromov Kazakov 15]

Generalisations of the QSC

- Deformations of AdS_5/CFT_4 [Kazakov SL Volin *in preparation*]
- ABJM [Cavaglià Fioravanti Gromov Tateo 14]

Tests and generalizations

Non-exhaustive list

Tests and applications of the QSC

- Weak coupling expansion [Marboe Velizhanin Volin 14]
- Numeric resolution [Gromov Levkovich-Maslyuk Sizov 15]
- BFKL regime [Alfimov Gromov Kazakov 15]

Generalisations of the QSC

- Deformations of AdS_5/CFT_4 [Kazakov SL Volin *in preparation*]
- ABJM [Cavaglià Fioravanti Gromov Tateo 14]

Tests and generalizations

Non-exhaustive list

Tests and applications of the QSC

- Weak coupling expansion [Marboe Velizhanin Volin 14]
- Numeric resolution [Gromov Levkovich-Maslyuk Sizov 15]
- BFKL regime [Alfimov Gromov Kazakov 15]

Generalisations of the QSC

- Deformations of AdS_5/CFT_4 [Kazakov SL Volin *in preparation*]
- ABJM [Cavaglià Fioravanti Gromov Tateo 14]

Tests and generalizations

Non-exhaustive list

Tests and applications of the QSC

- Weak coupling expansion [Marboe Velizhanin Volin 14]
- Numeric resolution [Gromov Levkovich-Maslyuk Sizov 15]
- BFKL regime [Alfimov Gromov Kazakov 15]

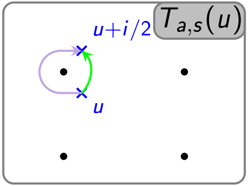
Generalisations of the QSC

- Deformations of AdS_5/CFT_4 [Kazakov SL Volin *in preparation*]
- ABJM [Cavaglià Fioravanti Gromov Tateo 14]

Interpretation of the QSC

Ambiguity in shifts

$$T_{a,s}(u+i/2)T_{a,s}(u-i/2) =$$
$$T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$$



Symmetries

$$T^{a,s}(u) = \det(Q_j(u+\dots))_{1\leq j,k\leq\dots}$$

Invariant under

- rotation $Q_j \leftrightsquigarrow H_j{}^k Q_k$
- hodge duality:
  $Q_j \leftrightsquigarrow Q^j \equiv \det(Q_k(u+\dots))_{k\neq j}$

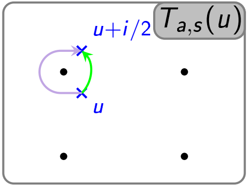
Quantum spectral curve

$$\tilde{P}_a = \mu_{ab}P^b$$

Interpretation of the QSC

Ambiguity in shifts

$$T_{a,s}(u+i/2)T_{a,s}(u-i/2) =$$
$$T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$$



Symmetries

$$T^{a,s}(u) = \det(Q_j(u+\dots))_{1\leq j,k\leq\dots}$$

Invariant under

- rotation $Q_j \leftrightsquigarrow {H_j}^k Q_k$
- hodge duality:
  $Q_j \leftrightsquigarrow Q^j \equiv \det(Q_k(u+\dots))_{k\neq j}$

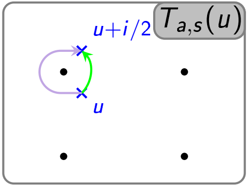
Quantum spectral curve

$$\tilde{P}_a = \mu_{ab} P^b$$

Interpretation of the QSC

Ambiguity in shifts

$T_{a,s}(u+i/2)\,T_{a,s}(u-i/2) =$
$T_{a+1,s}(u)\,T_{a-1,s}(u) + T_{a,s+1}(u)\,T_{a,s-1}(u)$



Symmetries

$$T^{a,s}(u) = \det(Q_j(u+\dots))_{1\le j,k\le\dots}$$

Invariant under

- rotation $Q_j \leftrightsquigarrow {H_j}^k Q_k$
- hodge duality:
  $Q_j \leftrightsquigarrow Q^j \equiv \det(Q_k(u+\dots))_{k\neq j}$

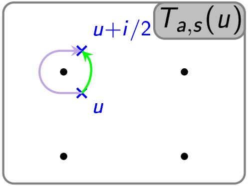
Quantum spectral curve

$$\tilde{P}_a = \mu_{ab} P^b$$

Interpretation of the QSC

Ambiguity in shifts

$T_{a,s}(u+i/2)T_{a,s}(u-i/2) =$
$T_{a+1,s}(u)T_{a-1,s}(u) + T_{a,s+1}(u)T_{a,s-1}(u)$



Symmetries

$$T^{a,s}(u) = \det(Q_j(u+\dots))_{1\leq j,k\leq\dots}$$

Invariant under

- rotation $Q_j \leftrightsquigarrow H_j{}^k Q_k$
- hodge duality:
  $Q_j \leftrightsquigarrow Q^j \equiv \det(Q_k(u+\dots))_{k\neq j}$

Quantum spectral curve

$$\tilde{P}_a = \mu_{ab}P^b$$

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for TBA?
    - Similar procedure for N-point interactions ?
    - Other symmetry algebras?

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for *TBA*?
    - Similar procedure for N-point interactions?
    - Other symmetry algebras?

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for TBA?
    - Similar procedure for N-point interactions ?
    - Other symmetry algebras?

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for TBA?
    - Similar procedure for N-point interactions ?
    - Other symmetry algebras?

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for *TBA*?
    - Similar procedure for N-point interactions ?
    - Other symmetry algebras?

Outlook

- Rational spin chains (very well understood)
  - Expression of the Hamiltonian from T and Q-functions
  - Bethe equations $\leftrightarrow$ polynomiality of Q-operators
- For finite-size effects in integrable field theories, gives a guideline to write FiNLIE
  - Parameterization in terms of Q-functions
  - Q-functions turn out to have quite simple analyticity properties
- Open questions:
  - Proof these analyticity properties, understanding of the new symmetries, from eg a lattice regularisation ?
  - How general is the Q-function procedure for TBA?
    - Similar procedure for N-point interactions ?
    - Other symmetry algebras?

Thank you for your attention