D Unfolding thorems

Theorem 7. *If* - $\mu X_v.\ P$ *is v-closed* - $[\![\mu X_v.\ P]\!]$ *exists* *then* $\mu X_v.\ P \equiv P\{(\mu X_v.\ P)/X_v\}$.

Proof (Th. 7).

$$
\begin{array}{ll}
\ \ [\![\mu X_v.P]\!]_\rho & \\
= \mathrm{lfp}_\emptyset^\subseteq \lambda X.\ [\![P]\!]_{[\rho|X_v \to X]} & \text{def} \\
= [\![P]\!]_{[\rho|X_v \to ([\![\mu X_v.P]\!]_\rho)]} & \text{(since it's a fix point)} \\
= [\![P(\mu X_v.P)/X_v]\!]_\rho & \text{(from the substitution theorem for } BI^{\mu\nu} \text{ general Th. 10)}
\end{array}
$$

Theorem 8. *If* - $\nu X_v.\ P$ *is v-closed* - $[\![\nu X_v.\ P]\!]$ *exists* *then* $\nu X_v.\ P \equiv P\{(\nu X_v.\ P)/X_v\}$.

Proof (Th. 8).

$$
\begin{array}{ll}
\ \ [\![\nu X_v.P]\!]_\rho & \\
= \mathrm{gfp}_\emptyset^\subseteq \lambda X.\ [\![P]\!]_{[\rho|X_v \to X]} & \text{def} \\
= [\![P]\!]_{[\rho|X_v \to ([\![\nu X_v.P]\!]_\rho)]} & \text{(since it's a fix point)} \\
= [\![P(\nu X_v.P)/X_v]\!]_\rho & \text{(from the substitution theorem for } BI^{\mu\nu} \text{ general Th. 10)}
\end{array}
$$