A Definitions

We recall the definitions of Var (Def. 7), FV (Def. 8), FV_v (Def. 9), v-closed (Def. 10), $\{[\ /\ ]\}$ (Def. 11):

Definition 7.

$$
\begin{array}{ll}
Var(x) & = \{x\} \\
Var(42) & = \emptyset \\
Var(\texttt{nil}) & = \emptyset \\
Var(True) & = \emptyset \\
Var(False) & = \emptyset \\
Var(E_1 op E_2) & = Var(E_1) \cup Var(E_2) \\
\ldots & \\
Var(E_1 = E_2) & = Var(E_1) \cup Var(E_2) \\
Var(E \mapsto E_1, E_2) & = Var(E) \cup Var(E_1) \cup Var(E_2) \\
Var(\texttt{false}) & = \emptyset \\
Var(P \Rightarrow Q) & = Var(P) \cup Var(Q) \\
Var(\exists x.P) & = Var(P) \cup \{x\} \\
Var(\texttt{emp}) & = \emptyset \\
Var(P * Q) & = Var(P) \cup Var(Q) \\
Var(P \mathrel{-\!\!*} Q) & = Var(P) \cup Var(Q) \\
Var(X_v) & = \emptyset \\
Var(\mu X_v.P) & = Var(P) \\
Var(\nu X_v.P) & = Var(P) \\
Var(P[E/x]) & = Var(P) \cup Var(E) \cup \{x\}
\end{array}
$$

Definition 8.

$$
\begin{array}{ll}
FV(E_1 = E_2) & = Var(E_1) \cup Var(E_2) \\
FV(E \mapsto E_1, E_2) & = Var(E) \cup Var(E_1) \cup Var(E_2) \\
FV(\texttt{false}) & = \emptyset \\
FV(P \Rightarrow Q) & = FV(P) \cup FV(Q) \\
FV(\exists x.P) & = FV(P) \setminus \{x\} \\
FV(\texttt{emp}) & = \emptyset \\
FV(P * Q) & = FV(P) \cup FV(Q) \\
FV(P \mathrel{-\!\!*} Q) & = FV(P) \cup FV(Q) \\
FV(X_v) & = \emptyset \\
FV(\mu X_v.P) & = FV(P) \\
FV(\nu X_v.P) & = FV(P) \\
FV(P[E/x]) & = FV(P) \cup FV(E) \cup \{x\}
\end{array}
$$

Definition 9.

$$
\begin{array}{ll}
FV_v(E_1 = E_2) & = \emptyset \\
FV_v(E \mapsto E_1, E_2) & = \emptyset \\
FV_v(\texttt{false}) & = \emptyset \\
FV_v(P \Rightarrow Q) & = FV_v(P) \cup FV_v(Q) \\
FV_v(\exists x.P) & = FV_v(P) \\
FV_v(\texttt{emp}) & = \emptyset \\
FV_v(P * Q) & = FV_v(P) \cup FV_v(Q) \\
FV_v(P -\!\!* Q) & = FV_v(P) \cup FV_v(Q) \\
FV_v(X_v) & = \{X_v\} \\
FV_v(\mu X_v.P) & = FV_v(P) \setminus \{X_v\} \\
FV_v(\nu X_v.P) & = FV_v(P) \setminus \{X_v\} \\
FV_v(P[E/x]) & = FV_v(P)
\end{array}
$$

Definition 10. *P is v-closed iff $FV_v(P) = \emptyset$.*

Definition 11.

$$
\begin{array}{ll}
x\{z/y\} & = x \ \mathit{if}\ y \neq x \\
y\{z/y\} & = z \\
42\{z/y\} & = 42 \\
\texttt{nil}\{z/y\} & = \texttt{nil} \\
True\{z/y\} & = True \\
False\{z/y\} & = False \\
(E_1 op E_2)\{z/y\} & = E_1\{z/y\} op E_2\{z/y\} \\
... & \\
(E_1 = E_2)\{[z/y]\} & = E_1\{z/y\} = E_2\{z/y\} \\
(E \mapsto E_1, E_2)\{[z/y]\} & = E\{z/y\} \mapsto E_1\{z/y\}, E_2\{z/y\} \\
(\texttt{false})\{[z/y]\} & = \texttt{false} \\
(P \Rightarrow Q)\{[z/y]\} & = (P\{[z/y]\}) \Rightarrow (Q\{[z/y]\}) \\
(\exists x.P)\{[z/y]\} & = \exists (x\{z/y\}).(P\{[z/y]\}) \\
(\texttt{emp})\{[z/y]\} & = \texttt{emp} \\
(P * Q)\{[z/y]\} & = (P\{[z/y]\}) * (Q\{[z/y]\}) \\
(P -\!\!* Q)\{[z/y]\} & = (P\{[z/y]\}) -\!\!* (Q\{[z/y]\}) \\
(X_v)\{[z/y]\} & = X_v \\
(\mu X_v.P)\{[z/y]\} & = \mu X_v.(P\{[z/y]\}) \\
(\nu X_v.P)\{[z/y]\} & = \nu X_v.(P\{[z/y]\}) \\
(P[E/x])\{[z/y]\} & = (P\{[z/y]\})[E\{z/y\}/x\{z/y\}]
\end{array}
$$