An abstract domain for separation logic formulae

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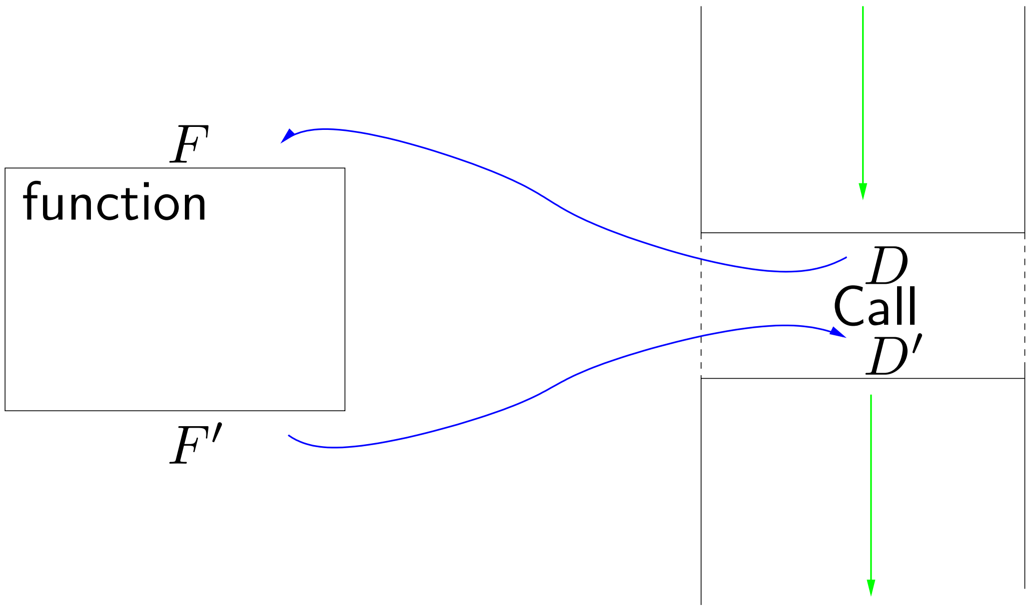
Situation

- Goal: pointer analysis: check dereferencing errors, aliases, ...
- $BI^{\mu\nu}$ a separation logic which permit easy descriptions of the memory, e.g.
  - x points to a list of [1;2;3]
    $\exists x_2, x_3.\ (x \hookrightarrow 1, x_2) * (x_2 \hookrightarrow 2, x_3) * (x_3 \hookrightarrow 3, \texttt{nil})$
  - x and y are aliased pointers
    $x = y \wedge \exists x_1, x_2.\ (x \hookrightarrow x_1, x_2)$
  - Partitionning: x and y belong to two disjoints parts of the heap which have no pointers from one to the other...

➤ We want to use this logic as an interface language for modular analysis



Analysis 1 $\longrightarrow BI^{\mu\nu} \longrightarrow$ Analysis 2
Program $\longrightarrow BI^{\mu\nu} \longrightarrow$ Analysis 3

We have build an intermediate domain such that:

- it is similar to the existing shape/alias analysis domains to allow translations from/to those domains
- it comes with a concrete semantics in term of sets of states which is the same domain as for the formulae's semantics
- we can translate the formulas into our domain
- it is a cartesian product of different subdomains so that we can cheaply tune the precision depending on the needs (for example, the domain is parametrised by a numerical domain which can be forgotten if we do not care about numericals)

The concrete domain: $State$

We have a set of variables Var.

$$\begin{array}{rcll} Val & = & Int \cup Bool \cup Atoms \cup Loc & Values \\ S & = & Var \rightharpoonup Val & Stacks \\ H & = & Loc \rightharpoonup Val \times Val & Heaps \\ State & = & S \times H & \end{array}$$

Rq: stacks can be partial functions

The logic: $BI^{\mu\nu}$

Classical	*connectives*				
	$E = E'$		|	`false`	
|	$P \Rightarrow Q$		|	$\exists x.P$	
	Spatial	*connectives*			
|	`emp`	Empty heap	|	$E \mapsto E_1, E_2$	Points to
|	$P * Q$	Spatial conj.	|	$P \mathrel{-\!\!*} Q$	Spatial imp.
	Fixpoints	*connectives*			
|	X_v	Variable for formulae	|	$P[E/x]$	Posponned substitution
|	$\nu X_v.P$	Greatest fixpoint	|	$\mu X_v.P$	Least fixpoint

$Var_v = \{X_v, Y_v, ...\}$ infinite set of variables of formulae

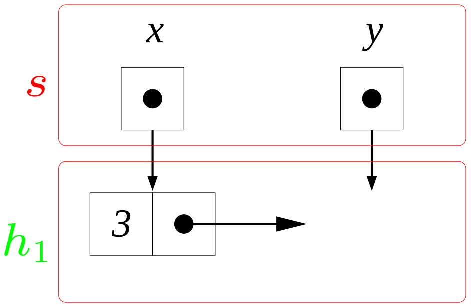
Semantic of $*$

$$[\![P * Q]\!]_\rho = \left\{ s, h_0 \cdot h_1 \;\middle|\; \begin{array}{ll} \bullet & \boldsymbol{dom(h_0) \cap dom(h_1) = \emptyset} \\ \bullet & s, h_0 \in [\![P]\!]_\rho \\ \bullet & s, h_1 \in [\![Q]\!]_\rho \end{array} \right\}$$

Examples of formulae

Ex. 1

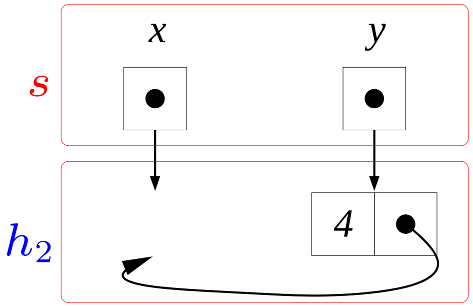
$$s = [x \to l_1, y \to l_2]$$
$$h_1 = [l_1 \to \langle 3, l_2 \rangle]$$



$$\models \quad (x \mapsto 3, y)$$

Ex. 2

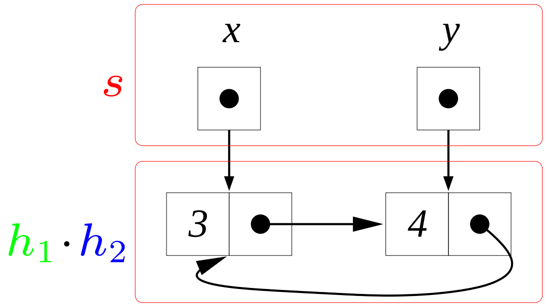
$$s = [x \to l_1, y \to l_2]$$
$$h_2 = [l_2 \to \langle 4, l_1 \rangle]$$



$$\models (y \mapsto 4, x)$$

Ex. 3

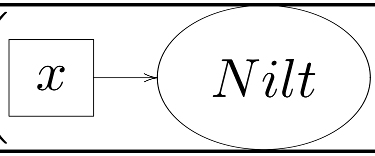
$$s = [x \to l_1, y \to l_2]$$
$$h_1 \cdot h_2 = \left[\begin{array}{l} l_1 \to \langle 3, l_2 \rangle, \\ l_2 \to \langle 4, l_1 \rangle \end{array} \right]$$

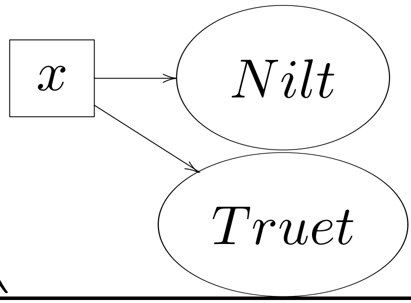


$$\models (x \mapsto 3, y) * (y \mapsto 4, x)$$
$$\not\models (x \mapsto 3, y) \wedge (y \mapsto 4, x)$$

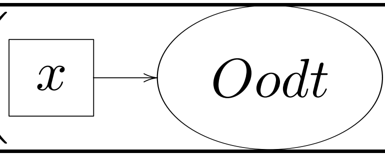
Presentation of the choices for the domain by examples of translation of formulae

Ex1

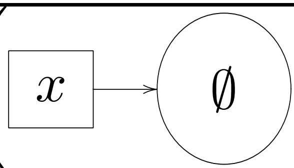
Formulae	$x = \texttt{nil}$
Semantics	$\{s, h \mid s(x) = \texttt{nil}\}$,...
Translation	$\Big($  $, \_, \_, \_, \_, \_, \_ \Big)$

Formulae	$(x = \texttt{nil} \vee x = \texttt{true})$
Translation	$\Big($  $, \_, \_, \_, \_, \_, \_ \Big)$

Ex2

Formula	$\neg(x = x)$
Semantic	$\{s, h \mid x \notin dom(s)\}$
Translation	( , _, _, _, _, _, _)

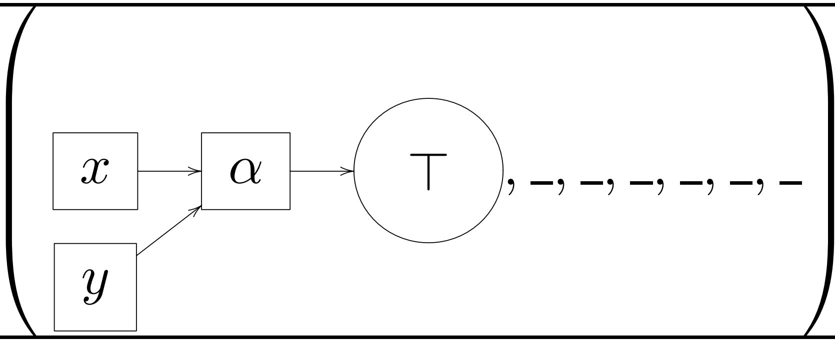
Do not confuse with:

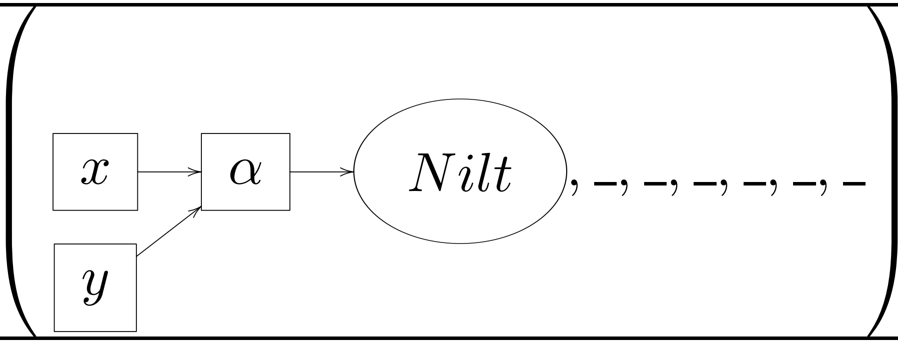
Formula	`false`
Semantic	$\emptyset$
Translation (for ex.)	( , _, _, _, _, _, _)

Ex3

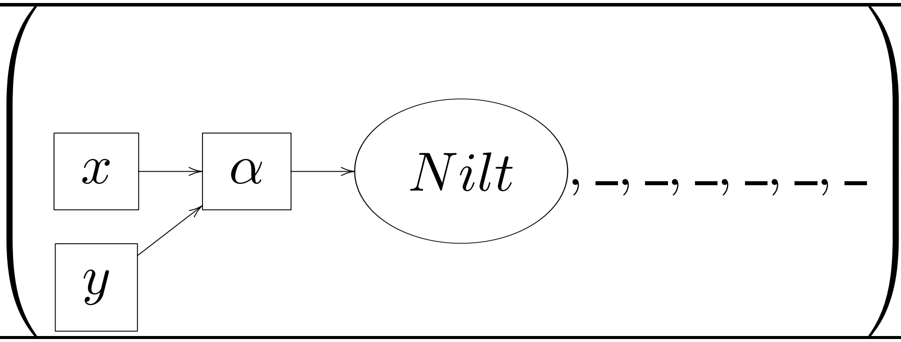
Formula	$A \wedge B$
Constraints	cheap translation of $\wedge$
Translation	$T(A \wedge B) \triangleq T'(T'(\top, A), B)$

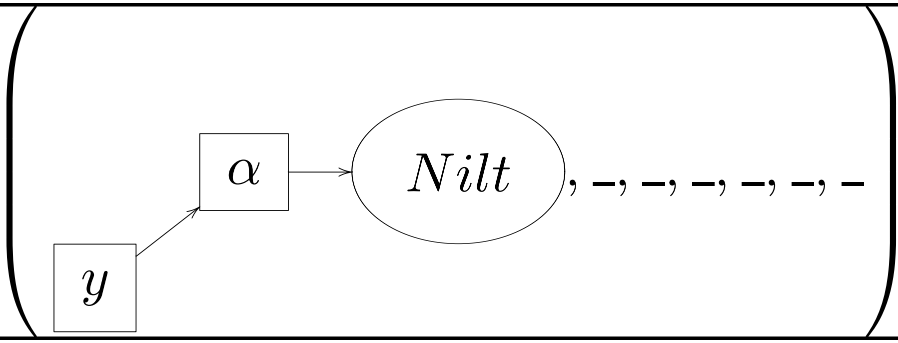
Ex4,   Ex5

Formula	$x = y$
Constraints	refine the information for one variables while also refining the information of the second one in a cheap way
Adds	infinite set of auxiliary variables $TVar$ $VAR \triangleq Var \uplus TVar$
Translation	

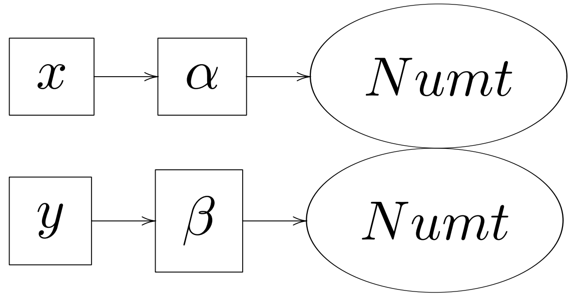
Formula	$x = y \wedge x = \mathtt{nil}$
Translation	

Ex6

Formula	$x = y \wedge x = \texttt{nil}$
Translation	

Formula	$(\exists x.\ x = y \wedge x = \texttt{nil}) \equiv (y = \texttt{nil})$
Translation	

Ex7

<table>
<tr><td>Formula</td><td>$(x < y + 3)$</td></tr>
<tr><td>Translation</td><td>$\Bigg($ 

$, \_, \_, \_, \_, \_, d \Bigg)$
$d \in \mathcal{D}$ encodes that $\alpha < \beta + 3$</td></tr>
</table>

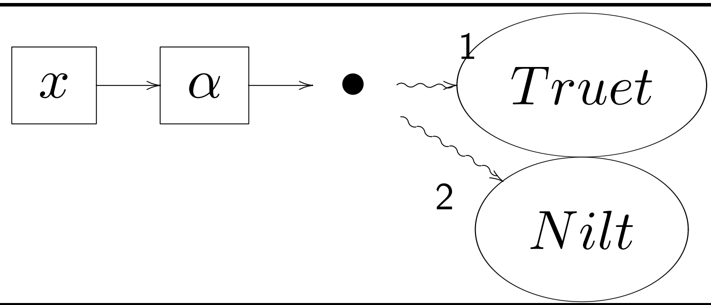
Ex8

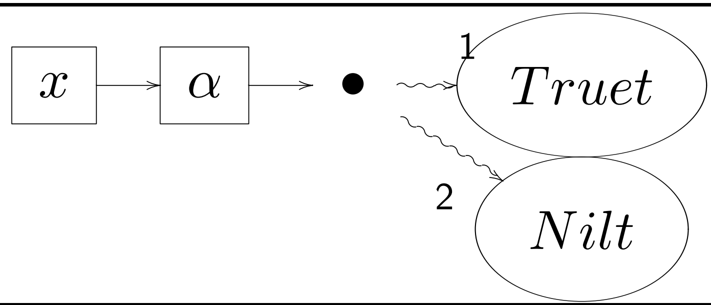
Formula	"x is a location not allocated"
Semantic	$\{s, h \mid s(x) \in Loc \wedge s(x) \notin dom(h)\}$
Translation	( , _, _, _, _, _)

Ex9

Formula	$\texttt{emp}$
Semantic	$\{s, h \mid dom(h) = \emptyset\}$
Adds	$HU \triangleq \mathcal{P}(TVar)$ $HO \triangleq \mathcal{P}(TVar) \uplus \texttt{full}$
Translation	$(\_, \_, \emptyset, \_, \_, \_, \_)$

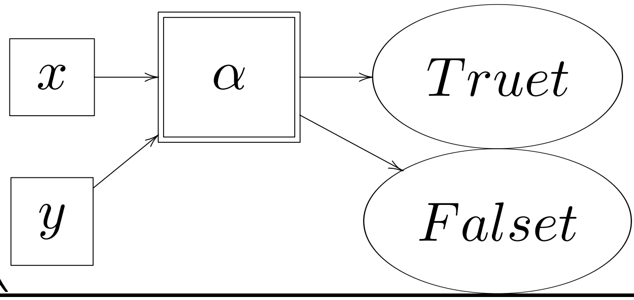
Ex10

Formula	$(x \mapsto \texttt{true}, \texttt{nil})$
Semantic	$\{s, h \| [s(x) \to \langle True, \texttt{nil} \rangle] = h\}$
Translation	$\Bigg($  $, \{\alpha\}, \{\alpha\}, \_, \_, \_, \_ \Bigg)$

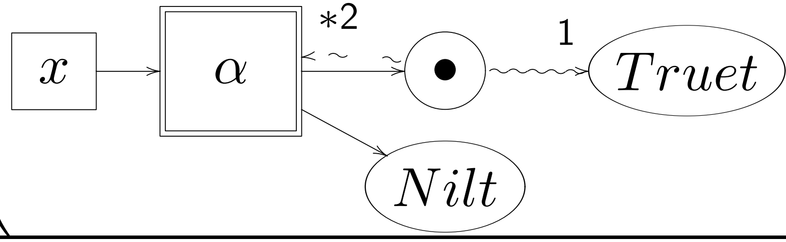
Formula	$(x \hookrightarrow \texttt{true}, \texttt{nil})$
Semantic	$\{s, h \| [s(x) \to \langle True, \texttt{nil} \rangle] \subseteq h\}$
Translation	$\Bigg($  $, \{\alpha\}, \texttt{full}, \_, \_, \_, \_ \Bigg)$

Ex11

Variables represent at most one value. To allow approximation we introduce summary nodes which can represent several values.

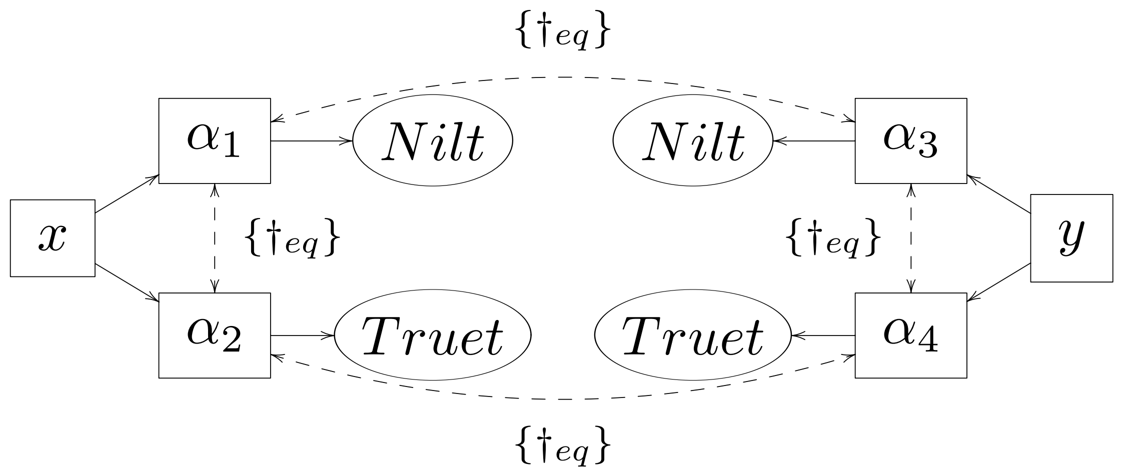
Formula	approx. of $(x = \texttt{true} \wedge y = \texttt{false})$ by $\begin{pmatrix} x = \texttt{true} \\ \vee x = \texttt{false} \end{pmatrix} \wedge \begin{pmatrix} y = \texttt{false} \\ \vee y = \texttt{true} \end{pmatrix}$
Translation	$($  $, \_, \_, \{\alpha\}, \_, \_, \_)$

Ex12: finite acyclic list of $True$ starting from x

Formula	$\mu X_v. \left(\begin{array}{c} (x = \texttt{nil}) \vee \exists x_2. \\ x \hookrightarrow (\texttt{true}, x_2) \ * \ X_v[x_2/x] \end{array} \right)$
Translation	( $, \_, \_, \{\alpha\}, \emptyset, \_, \_$)

$\emptyset$ is the set of infinite summary nodes, for infinite list μ would be replaced by ν and $\emptyset$ by $\{\alpha\}$).

Ex13: increase precision of union

<table>
<tr><td>Formula</td><td>$\begin{pmatrix} x = \texttt{nil} \\ \wedge y = \texttt{true} \end{pmatrix} \vee \begin{pmatrix} x = \texttt{true} \\ \wedge y = \texttt{nil} \end{pmatrix}$</td></tr>
<tr><td>Translation</td><td>
</td></tr>
</table>

Formal definition of the domain

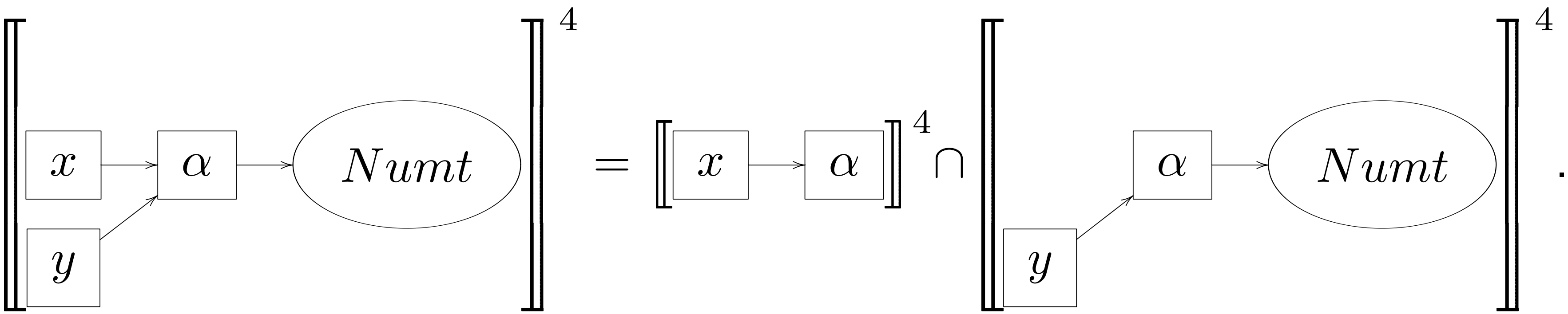
$$
\begin{array}{rcl}
VD1 & ::= & Numt \mid Truet \mid Falset \mid Oodt \mid Nilt \mid Dangling\_Loc \mid TVar \\
VD & ::= & VD1 \mid Loc(\mathcal{P}(\{*1, *2\}) \times VD1 \times VD1) \\
PVD^{+} & ::= & (\mathcal{P}(VD) \uplus \circledast, \sqcup, \sqcap) \\
 & & \\
AD & ::= & VAR \overset{total}{\rightarrow} PVD^{+} \\
 & & \\
CL_{eq} & ::= & \mathcal{P}(\{\ddagger_{eq}, \dagger_{eq}, =_{eq}, \subset_{eq}, \supset_{eq}, \sharp_{eq}, \bigcirc\!\!\!\bigcirc_{eq}\}) \\
TB & ::= & (TVar \times TVar) \overset{total}{\rightarrow} CL_{eq} \\
 & & \\
AR & ::= & AD \times \mathcal{P}(TVar) \times (\mathcal{P}(TVar) \uplus \texttt{full}) \times \mathcal{P}(TVar) \times \mathcal{P}(TVar) \times TB \\
 & & \times (\mathcal{D}, [\![\cdot]\!]^{\mathcal{D}} : \mathcal{D} \rightarrow (TVar \overset{total}{\rightarrow} \mathcal{P}(\mathbb{Z})))
\end{array}
$$

Semantic

$$
\begin{array}{rcl}
Val & \triangleq & \mathbb{Z} \uplus Bool \uplus \mathtt{nil} \uplus Loc \\
S & \triangleq & Var \rightharpoonup Val \\
H & \triangleq & Loc \rightharpoonup (Val \times Val) \\
 & & \\
State & \triangleq & S \times H
\end{array}
\qquad
\begin{array}{rcl}
Val' & \triangleq & Val \cup \{\mathtt{ood}\} \\
S' & \triangleq & Var \overset{total}{\rightarrow} Val' \\
F & \triangleq & TVar \overset{total}{\rightarrow} \mathcal{P}(Val') \\
R & \triangleq & Loc \rightharpoonup \mathcal{P}(Loc) \\
MFR & \triangleq & \mathcal{P}(S' \times H \times F \times R)
\end{array}
$$

Why the F?

We want



$$
\begin{aligned}
[\![\cdot]\!] &\in \boldsymbol{AR} \rightarrow \boldsymbol{\mathcal{P}(State)} \\
[\![ar]\!] &\triangleq \{\bar{s}, h \mid s, h, f, r \in [\![ar]\!]'\}
\end{aligned}
$$

$$
\begin{aligned}
[\![\cdot]\!]' &\in \boldsymbol{AR} \rightarrow \boldsymbol{MFR} \\
[\![(ad, hu, ho, sn, sn^{\infty}, t, d)]\!]' &\triangleq [\![ad]\!]^4 \cap [\![hu]\!]^1 \cap [\![ho]\!]^{1'} \cap [\![sn]\!]^2 \cap [\![sn^{\infty}]\!]^{2'} \\
&\quad \cap [\![t]\!]^3 \cap [\![d]\!]^7 \cap sem*
\end{aligned}
$$

$$
\begin{aligned}
[\![\cdot]\!]^4 &\in \boldsymbol{AD} \rightarrow \boldsymbol{MFR} \\
[\![ad]\!]^4 &\triangleq \bigcap_{v \in VAR} [\![v, ad(v)]\!]^5
\end{aligned}
$$

Operations

– union

– extension (replace $[v \rightarrow S]$ by $[v \rightarrow \{v'\}|v' \rightarrow S]$ with a fresh v')
used to tune the precision of the union

– merging (replace $[v_1 \rightarrow S_1 \mid v_2 \rightarrow S_2]$ by $[v_2 \rightarrow (S_1 \cup S_2)]$)
used with the widening

– translations from formulae to the domain

Comparisons

- the $\bullet \overset{1}{\leadsto} \overset{2}{\leadsto}$ represent locations in the usual shape graphs

- summary nodes as for other shape graphs, seems to give more possibilities than predicate abstraction (with each time a specific predicate for list, etc...) but the technics of predicate and their algorithm/heuristics (like folding/unfolding) could probably also be use on our graphs

- a lonely outgoing edge can be seen as a "must" arrow (or valued 1), several outgoing edges from a variable can be seen as a "may" arrow (or valued 1/2, but it is a bit more precise because we know that one of them should exist), and an edge to $\emptyset$ can be seen as a "must not" arrow (or valued 0)

- we deal with numerical (for what we know, only *Magill & al.* also do)

- we have a formal semantic of our domain, the semantics of auxiliary variables are formally defined and formally used in the proofs

- we don't have to check for equalities of variables

- the domain is a cartesian product, we can add or remove some parts depending on the precision we want

Ende