

Here is a list of corrections to the published version of *A Guide to NIP Theories*. Those have all been incorporated in the online version available on my webpage: <http://www.normalesup.org/~simon/book.html>.

Many thanks to Itay Kaplan, Nadja Hempel, Domenico Zambella, Alessandro Beraducci, Christian d'Elbée and Levon Haykazyan for pointing them out to me.

Last revision: October 16, 2015

- Lemma 2.7

In the proof of left to right:  $\{\phi(x; c) : c \in I_0\} \cup \{\neg\phi(x; c) : c \in I_1\}$  should be  $\{\phi(c; y) : c \in I_0\} \cup \{\neg\phi(c; y) : c \in I_1\}$ .

- Chapter 2, References and related subjects, p.30:

Kaplan, Scanlon and Wagner show that NIP fields are Artin-Schreier closed, along with results about valued fields.

- Observation 3.2

If  $\pi(x)$  is a definable set with at least two elements and is stably embedded, then one can choose the formula  $\psi(x_1, \dots, x_n; z)$  in a way that it depends only on  $\phi(x_1, \dots, x_n; y)$  and not on the parameters  $b$ .

- Remark 3.34

It follows from Proposition 3.32 that if  $I$  is ordered by a complete order and if there is a formula  $\theta(x, y) \in L(I)$  which orders  $I$ , then  $I$  is stably embedded.

- Lemma 5.17

The end of the proof should read:

By Ramsey, we may find an  $Aa'$ -indiscernible sequence  $(b'_i : i < \omega)$  realizing the EM-type of  $(b_i : i < \omega)$  over  $Aa'$ . Then  $a' \models \pi(x; b'_i)$  for every  $i < \omega$ . Let  $f \in \text{Aut}(\mathcal{U}/A)$  send  $(b'_i : i < \omega)$  to  $(b_i : i < \omega)$  and set  $a = f(a')$ . Then  $a \models \pi$  and the sequence  $I$  is indiscernible over  $Aa$ .

- Definition 6.8

The set  $X_0$  should be a multiset, i.e. we allow repetitions.

- Corollary 6.13

The centered equation should read

$$\left| \mu(S) - \frac{|\{i : x_i \in S\}|}{q} \right| \leq \epsilon.$$

- Section 7.1, Borel measures.

They are some details missing in the proof of construction of the regular Borel measure extending a Keisler measure. Here is a more complete argument.

Let  $\mu \in \mathfrak{M}_x(A)$  be a Keisler measure. It assigns a measure to every clopen set of the space  $S_x(A)$ . We show how to extend that measure to a  $\sigma$ -additive Borel probability measure. First, if  $O \subseteq S_x(A)$  is open, we define  $\mu(O) = \sup\{\mu(D) : D \subseteq O, D \text{ clopen}\}$ . Similarly, the measure of a closed set  $F$  is the infimum of the measures of clopen sets which contain it. If  $F \subseteq O$  are respectively closed and open, then there is a definable set between them. This implies that if  $X$  is either closed or open, we have

$$(Reg) \quad \sup\{\mu(F) : F \subseteq X, F \text{ closed}\} = \inf\{\mu(O) : X \subseteq O, O \text{ open}\}.$$

It is not hard to see that that  $\mu$  is subadditive on open sets and that  $\mu(O \setminus F) = \mu(O) - \mu(F)$  for  $F$  closed inside the open set  $O$ .

The next step is to show that the set of subsets  $X \subseteq S_x(A)$  satisfying (Reg) is closed under complement and countable union. Complement is clear. For countable union: let  $X = \bigcup_{i < \omega} X_i$  and fix  $\epsilon > 0$ . For each  $i < \omega$ , take  $F_i \subseteq X_i \subseteq O_i$  with  $\mu(O_i) - \mu(F_i) \leq \epsilon 2^{-i}$ . Let  $O = \bigcup_{i < \omega} O_i$ . Note that  $\mu(O) = \lim_n \mu(\bigcup_{i < n} O_i)$ , because by compactness any clopen set inside  $O$  is already inside some  $\bigcup_{i < n} O_i$ . Then we can find some finite  $N$  such that  $\mu(O) - \mu(\bigcup_{i < N} O_i) \leq \epsilon$ . Let  $F = \bigcup_{i < N} F_i$ . Then we have  $F \subseteq X \subseteq O$  and  $\mu(O) - \mu(F) = \mu(\bigcup_{i < \omega} O_i \setminus F) \leq \mu(\bigcup_{i < N} O_i \setminus F) + \epsilon \leq \epsilon + \sum_{i < N} \mu(O_i) - \mu(F_i) \leq 3\epsilon$ .

It follows that every Borel subset of  $S_x(A)$  satisfies (Reg). We can therefore define  $\mu$  on all such sets by  $\mu(X) = \sup\{\mu(F) : F \subseteq X, F \text{ closed}\} = \inf\{\mu(O) : X \subseteq O, O \text{ open}\}$ . It is easy to check that this defines a  $\sigma$ -additive measure on  $S_x(A)$ . Property (Reg) is referred to as *regularity* of the measure  $\mu$ .

- Proposition 7.10, last paragraph of the proof.

Now take points  $(a_i : i < n)$  in  $\mathcal{U}$  such that  $a_i \models p_i$ . Set  $\lambda' = \frac{1}{n} \sum_{i < n} \text{tp}(a_i/\mathcal{U})$ . Let  $b \in \mathcal{U}$  and let  $i < n$  be such that  $\models \psi_i(b)$ . Then we have  $\models \theta_i^0(x) \rightarrow \phi(x; b) \rightarrow \theta_i^1(x)$  and  $\mu(\theta_i^1(x)) - \mu(\theta_i^0(x)) \leq \epsilon$ . Thus  $|\mu(\phi(x; b) \cap X) - \mu(\theta_i^0(x) \cap X)| \leq \epsilon$  and similarly  $|\lambda'(\phi(x; b) \cap X) - \lambda'(\theta_i^0(x) \cap X)| \leq 3\epsilon$ . Finally, since  $\lambda'(\theta_i^0(x) \cap X)$  is within  $\epsilon$  of  $\mu(\theta_i^0(x) \cap X)$ , we have that  $\lambda'(\phi(x; b) \cap X)$  is within  $5\epsilon$  of  $\mu(\phi(x; b) \cap X)$ .

- Definition 7.23

Let  $\mu(x)$  be a global  $M$ -invariant measure. We say that  $\mu$  is *fim* (frequency interpretation measure) if for any formula  $\phi(x; y) \in L$ , there is a family  $(\theta_n(x_1, \dots, x_n) : n < \omega)$  of formulas in  $L(M)$  such that:

- $\lim \mu^{(n)}(\theta_n(x_1, \dots, x_n)) = 1$ ;
- for any  $\epsilon > 0$ , for  $n$  big enough, for any  $(a_1, \dots, a_n) \in \theta_n(\mathcal{U})$ , and any  $b \in \mathcal{U}$ ,  $\text{Av}(a_1, \dots, a_n; \phi(x; b))$  is within  $\epsilon$  of  $\mu(\phi(x; b))$ .

- Theorem 7.29

(ii) for any formula  $\phi(x; y) \in L$  and  $\epsilon > 0$ , there are  $a_1, \dots, a_n \in M$  such that for any  $b \in \mathcal{U}$ ,  $\text{Av}(a_1, \dots, a_n; \phi(x; b))$  is within  $\epsilon$  of  $\mu(\phi(x; b))$ .

- Proposition 7.30, end of the proof.

Fix  $\epsilon > 0$ . By Proposition 7.27 there are  $a_1, \dots, a_n \in N$  such that for all  $b' \in \mathcal{U}$ ,  $\text{Av}(a_1, \dots, a_n; \phi(x; b'))$  is within  $\epsilon$  of  $\mu(\phi(x; b'))$  and also  $\text{Av}(a_1, \dots, a_n; X)$  is within  $\epsilon$  of  $\mu(X)$ . Then  $q_y \otimes \mu_x(\phi(x; y))$  is within  $\epsilon$  of  $\text{Av}(a_1, \dots, a_n; X)$  which by definition of  $X$  is equal to  $\text{Av}(a_1, \dots, a_n; \phi(x; b))$ , which is within  $\epsilon$  of  $\mu_x \otimes q_y(\phi(x; y))$ .

As this holds for all  $\epsilon > 0$ , the result follows.

- Proposition 8.21

The reduction to countable  $L$  is not so clear. One can argue using facts from the paper “Definably amenable NIP groups” with A. Chernikov: if  $p$  is  $f$ -generic in some language  $L$ , then its reduct to any sublanguage is also  $f$ -generic because it has bounded orbit. The reader may simply prefer to assume that  $L$  is countable in this proposition.

- Section 8.4 Compact domination. The paragraph before Lemma 8.39 should be:

Fix a countable elementary submodel  $\mathcal{U}$  of the set theoretic universe containing  $L, T, M, G, \mu$  etc. If  $a \in \mathcal{U}$  is a finite tuple, a point  $b \in G(\mathcal{U})$  is said to be *random* over  $Ma$  if there does not exist some Borel set  $B \subseteq S_{xy}(M)$  coded in  $\mathcal{U}$  such that  $B(a, b)$  holds and  $\mu(B(a, y)) = 0$ . Note that such a  $b$  always exists because we have to avoid countably many Borel sets of measure 0.