Focalization in Linear Logic

Eighth Lecture

7th September 2004

LL Sequent Calculus

Identity Rules:

$$\frac{}{\vdash A^{\perp}, A}\ ax \qquad \frac{\vdash A, \Gamma \quad \vdash A^{\perp}, \Delta}{\vdash \Gamma, \Delta}\ cut$$

Structural Rule:

$$\frac{\vdash \Gamma, B, A, \Delta}{\vdash \Gamma, A, B, \Delta}\ Ex$$

Logical Rules:

Multiplicatives:

$$\frac{\vdash F, G, \Gamma}{\vdash F \parr G, \Gamma}\ \parr \qquad \frac{\vdash F, \Gamma \quad \vdash G, \Delta}{\vdash F \otimes G, \Gamma, \Delta}\ \otimes$$

Additives:

$$\frac{\vdash F, \Gamma \quad \vdash G, \Gamma}{\vdash F \& G, \Gamma}\ \& \qquad \frac{\vdash F, \Gamma}{\vdash F \oplus G, \Gamma}\ \oplus 1 \qquad \frac{\vdash G, \Gamma}{\vdash F \oplus G, \Gamma}\ \oplus 2$$

Constants:

$$\frac{}{\vdash 1}\ 1 \qquad \frac{\vdash \Gamma}{\vdash \bot, \Gamma}\ \bot \qquad \frac{}{\vdash \top, \Gamma}\ \top \qquad \textit{no rule for } 0$$

Quantifiers:

$$\frac{\vdash F, \Gamma}{\vdash \forall x F, \Gamma}\ \forall \quad (\star) \qquad \frac{\vdash F[t/x], \Gamma}{\vdash \exists x F, \Gamma}\ \exists$$

Exponentials:

$$\frac{\vdash F, \Gamma}{\vdash ?F, \Gamma}\ ? \qquad \frac{\vdash F, ?\Gamma}{\vdash !F, ?\Gamma}\ !$$

$$\frac{\vdash \Gamma}{\vdash ?F, \Gamma}\ ?W \qquad \frac{\vdash ?F, ?F, \Gamma}{\vdash ?F, \Gamma}\ ?C$$

Commutation Properties

Commutation $\otimes/\oplus$:

$$\dfrac{\dfrac{\vdash A, C, \Gamma \quad \vdash D, \Delta}{\vdash A, C \otimes D, \Gamma, \Delta}\otimes}{\vdash A \oplus B, C \otimes D, \Gamma, \Delta}\oplus$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, C, \Gamma}{\vdash A \oplus B, C, \Gamma}\oplus \quad \vdash D, \Delta}{\vdash A \oplus B, C \otimes D, \Gamma, \Delta}\otimes$$

Commutation Properties

Commutation $\oplus/\oplus$:

$$\dfrac{\dfrac{\vdash A, C, \Gamma}{\vdash A, C \oplus D, \Gamma}\oplus}{\vdash A \oplus B, C \oplus D, \Gamma}\oplus$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, C, \Gamma}{\vdash A \oplus B, C, \Gamma}\oplus}{\vdash A \oplus B, C \oplus D, \Gamma}\oplus$$

Commutation Properties

Commutation $\otimes/\otimes$:

$$\dfrac{\dfrac{\vdash A, C, \Gamma \quad \vdash D, \Delta}{\vdash A, C \otimes D, \Gamma, \Delta}\otimes \qquad \vdash B, \Sigma}{\vdash A \otimes B, C \otimes D, \Gamma, \Delta, \Sigma}\otimes$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, C, \Gamma \quad \vdash B, \Sigma}{\vdash A \otimes B, C, \Gamma, \Sigma}\otimes \qquad \vdash D, \Delta}{\vdash A \otimes B, C \otimes D, \Gamma, \Delta, \Sigma}\otimes$$

$$\dfrac{\dfrac{\dfrac{}{\vdash p, p^{\perp}}\,ax \qquad \dfrac{}{\vdash q, q^{\perp}}\,ax}{\vdash p, q, p^{\perp} \otimes q^{\perp}}\,\otimes}{\vdash p \parr q, p^{\perp} \otimes q^{\perp}}\,\parr$$

Commutation $\parr/\parr$:

$$\dfrac{\dfrac{\vdash A, B, C, D, \Gamma}{\vdash A, B, C \parr D, \Gamma}\,\parr}{\vdash A \parr B, C \parr D, \Gamma}\,\parr$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, B, C, D, \Gamma}{\vdash A \parr B, C, D, \Gamma}\,\parr}{\vdash A \parr B, C \parr D, \Gamma}\,\parr$$

Commutation $\parr/\oplus$:

$$\dfrac{\dfrac{\vdash A, C, D, \Gamma}{\vdash A, C \parr D, \Gamma}\,\parr}{\vdash A \oplus B, C \parr D, \Gamma}\,\oplus$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, C, D, \Gamma}{\vdash A \oplus B, C, D, \Gamma}\,\oplus}{\vdash A \oplus B, C \parr D, \Gamma}\,\parr$$

Commutation $\otimes/\oplus$:

$$\dfrac{\dfrac{\vdash A, C, \Gamma \quad \vdash D, \Delta}{\vdash A, C \otimes D, \Gamma, \Delta}\otimes}{\vdash A \oplus B, C \otimes D, \Gamma, \Delta}\oplus$$

$$\updownarrow$$

$$\dfrac{\dfrac{\vdash A, C, \Gamma}{\vdash A \oplus B, C, \Gamma}\oplus \quad \vdash D, \Delta}{\vdash A \oplus B, C \otimes D, \Gamma, \Delta}\otimes$$