Introduction to Linear Logic

Sixth Lecture

2nd September 2004

Today

- Motivation
- Intuition
- LL Sequent Calculus
- Considerations about the connectives

Motivations

Does Classical Logic allow to model everything?

Let us remove all the structural rules!

The two alternative presentations of disjunction and of conjunction are no more equivalent.

We have to different conjunctions and two different disjunctions!

We need to recover the contraction and disjunction rules, but in a controlled manner.

LL Sequent Calculus

Identity Rules:

$$\frac{}{\vdash A^{\perp}, A}\ ax \qquad \frac{\vdash A, \Gamma \quad \vdash A^{\perp}, \Delta}{\vdash \Gamma, \Delta}\ cut$$

Structural Rule:

$$\frac{\vdash \Gamma, B, A, \Delta}{\vdash \Gamma, A, B, \Delta}\ Ex$$

Logical Rules:

$$\frac{\vdash F, G, \Gamma}{\vdash F \parr G, \Gamma} \parr \qquad \frac{\vdash F, \Gamma \quad \vdash G, \Delta}{\vdash F \otimes G, \Gamma, \Delta} \otimes$$

$$\frac{\vdash F, \Gamma \quad \vdash G, \Gamma}{\vdash F \& G, \Gamma} \& \qquad \frac{\vdash F, \Gamma}{\vdash F \oplus G, \Gamma} \oplus 1 \qquad \frac{\vdash G, \Gamma}{\vdash F \oplus G, \Gamma} \oplus 2$$

$$\frac{}{\vdash 1} 1 \qquad \frac{\vdash \Gamma}{\vdash \bot, \Gamma} \bot \qquad \frac{}{\vdash \top, \Gamma} \top$$

$$\frac{\vdash F, \Gamma}{\vdash ?F, \Gamma} ? \qquad \frac{\vdash F, ?\Gamma}{\vdash !F, ?\Gamma} !$$

$$\frac{\vdash \Gamma}{\vdash ?F, \Gamma} ?W \qquad \frac{\vdash ?F, ?F, \Gamma}{\vdash ?F, \Gamma} ?C$$