On the meaning of logical constants and the justifications of the logical law

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12-13 Étym. "propostion", "jugement"

[The term proposition] has its origin in the Gr. πρότασις, used by Aristotle in the Priori Analytics, the thirs part of the Organon. It was translated, apparently by Cicero, into Lat. Ropositio, ich has is moder counterpartsin It. Proposizine, Eng proposition and Ger. Satz. […] The word proposition, Gr. πρότασις, comes from Aristotle and has domainted the logical tradition, whereas the word theorem, Gr. θεώρημα, is in Euclid, I believe, and has dominated the mathematical tradition.

[…]

The term judgement also has a long history. It is the Gr. κρίσις, translated into Lat judicium, It. Guidizio, Eng. Judgement, and Ger. Urteil.

14 Ambiguïté "proposition", "affirmation"

If you take the word proposition, for instance, it is just as ambiguous between the act of propounding and that which is propounded. Or, if you take the word affirmation, it is ambiguous between the act of affirming and that which is affirmed [*commentaire nôtre* : pas comme "pendaison"]

14 Affirmation, négation

The notions of affirmation and denial have fortunately remained stable, like the notion of proof, and are therefore easy to use without ambiguity. Both derive from Aristotle. Affirmation is Gr. κατάφασις, Lat. Affirmatio, It. Affermazione, and Ger. Bejahung, wheras denial is Gr. ἀπόφασις, Lat. Negatio, It. Negazione, and Ger. Verneinung.

14-15 Forme moderne du jugement

It seems to have Bolzano who took the crucial step of remplacing the Aristotelian forms of judgement by the single form

$$A \text{ is}, A \text{ is true, or } A \text{ holds.}$$

In this, he was followed by Brentano, who also introduced the opposite form

$$A \text{ is not, or } A \text{ is false,}$$

and Frege. And, through Frege's influence, the whole fo modern logic has come to be based on the single form of judgement, or assertition, A is true.

20 Logique = connaissance apodictique

The important thing to realize is of course that to judge and to know, and, correlatively, judgement and knowledge, are essentially the same. And, when the relation between judgement, or assertition, if you prefer, and knowledge is understood in this way, logic itself is naturally understood as the theory of knowledge, that is, of demonstrative knowledge, Aristotle's ἐπιστήμη ἀποδεικτική. Thus logic studies, from an objective point of view, our pieces of knowledge as they are organised in demonstrative science, or, if you think about it from the act point of view, it studies our acts of judging, of knowing, and how they are interrelated.

21 Savoir = avoir vu

Gr. οἶδα, I know, is the perfect form of the verb whose present form is Gr. εἴδω, I see. Thus to know is to have seen merely by the way the verb has been formed in Greek.

[…]

Observe also the two metaphors for the act od understanding which you seem to have in one form or the other in all European languages: the metaphor of seeing, first of all, which was so much used by the Greeks, and which we still use, for instance, when saying that we see that there are infinitely many prime numbers, and, secondly, the metaphor of grasping, which you also find in the verb to comprehend, derived as it is from Lat. prehendere, to seize. The same metaphor is found in Ger. fassen and begreifen

21 Énonciation vs proposition

the distinction [...] between an enunciation and a proposition. Enunciation is not a word of much currency in English, but I think that its Italian counterpart has fared better. The origin is the Gr. ἀπόφανσις as it appears in De Interpretatione, the second part of the Organon. It has been translated into Lat. enuntiatio, It. enunciato, and Ger. Aussage.

23 Affirmer ?

the condition for it to be right of me to affirm a proposition A, that is, to say that A is true, is not that A *is* true, but that *know* that A is true. [*commentaire nôtre* : *cf.* savoir dans *De la certitude*]

23 Évident, vérité, expérience

When you say that a judgement is evident, you merely express that you have undertood, comprehended, grasped, or seen it, that is, that you know it, because to have understood is to know. This is reflected in the etymology of the word evident, which comes from Lat. ex, out of, from, and videre, to see, in the metaphorical senses, of course.

There is absolutely no question of a judgement being evident in itself, independently of us and our cognitive activity. That would be just as absurd as to speak of a judgement as being known, not by somebody, you or me, but in itself. To be evident is to be evident *to* somebody, as inevitably as to be known is to be known *by* somebody. That is what Brouwer meant by saying, in Consciousness, Philosophy, and Mathematics, that there are no nonexperienced truths, a basic intuitionistic tenet. This has been puzzling, because it has been understood as referring to the truth of a proposition, and clearly there are true propositions whose truth has no been experienced, that is, propositions which can be shown to be true in the future, although they have not been proved to be true now. But what Brouwer means here is not that. He does not speak about propositions and truth; he speaks about judgements and evidence, although he uses ther term truth instead of the terme evidence. And what he says is then perfectly right: there is no evident judgement whose evidence has not been experienced, and experience it is what you do when you understand, comprehend, grasp, or see it. There is no evidence outside our actual or possible experience of it. The notion of evidence is by its very nature subject related, relative to the knowing subject, that is, in Kantian terminology.

27 Étym. "démonstration"

Observe that both Lat. demonstratio and the corresponding words in the modern languages, like It. dimonstrazione, Eng. demonstration, and ger. Beweis, are literal translations for Gr. ἀπόδειξις, deriving as it does from Gr. δείκνυμι, I show, which has the same meaning as Lat. monstrare and Ger. Weisen.

28 Preuve & actes

a proof is, not an objet, but an act. This is what Brouwer wanted to stress by saying that a proof is a mental construction, because what is mental, or psychic, is precisely our acts, and the word construction, as used by Brouwer, is but a synonym for proof. Thus he might just as well have said that the proof of a judgement is the act of proving, or grasping it. And the act is **primarily the act as it is being performed**. Only secondarily, and irrevocably, does it become the act that has been performed.

29 Théorie des preuves vs métamathématique ?

if proof theory is construed, not in Hilbert's sense, as metamathematics, but simply as the study of proofs in the original sense of the word, then proof theory is the same as theory of knowledge, which, in turn, is the same as logic in the original sense of the word, as the study of reasoning, or proof, not as metamathematics.

30 Axiomes & théorèmes

an immediately evident judgement is what we call an axiom. […] a mediately evident judgement is what we call a theorem.

34 Vérification = acte de rendre vrai

verification seems to be the perfect term to use together wit proposition, coming as it does from Lat. verus, true, and facere, to make. So to verify is to make true, and verification is the act, or process, of verifying something.

35 Vérité est une *modalité*

to know that a proposition is true, a problem is solvable, an expectation is fulfillable, or an intention is realizable; you must know how to verify, solve, fulfill, or realize it, respectively. Thus this explanation equates truth with verifiability, solvability, fulfillability, or realizability. The important point to observe here is the change from *is* in *A* is true to *can* in *A* can be verified, or *A* is verifable. Thus what is expressed in terms being the first formluation really has the **modal character of possibility**.

[…] to know how to do something is the same as to possess a way, or method, of doing it. This is reflected in the etymology of the word method, which is derived from Gr. Μετά, after and ὁδός, way.

36 Savoir comment & savoir que

knowledge of a judgement is knowledge of a problem, expectation, or intention, which is *knowledge what* to so, simply. […] the distinction between knowledge how and knowledge that evaporates on the intuitionistic analysis of the notion of truth.

40 Étym. "séquent" (Gentzen)

the relation of logical consequence […] must be carefully distinguished from implication. What stands to the left of the consequence sign, we call the hypotheses, in which case what follows the consequence sign is called the thesis, or we call the judgements that precede the consequence sign the antecedents and the judgement that follows the consequence sign the consequent. This is the terminology whiwh Gentzen took over from the scholastics, except that, for some reason, he changed consequent into succedent and consequence into sequence, Ger. Sequenz, usually improperly rendered by sequent in English.

41 Preuve hypothétique/catégorique

The notion of hypothetical proof, [...] which is a primitive notion, is explained by saying that it is a proof which, when supplemented by proofs of the hypotheses, or antecedents, becomes a proof of the thesis, or consequent. Thus the notion of categorical proof precedes the notion of hypothetical poof, or inference, in the order of conceptual priority.

43 Inférence vs conséquence logique ($\vdash \Rightarrow \models$)

an inference is a proof of a logical consequence. Thus an inference is the same as a hypothetical proof.

51 Règle d'introduction pour l'absurde ?

An introduction is an inference in which you conclude that a propositon is true, or can be verifeid, on the ground that you have verified it, that is, that you possess a verification of it. Therefore, $\perp$ being defined by the stipulation that there is nothing that counts as a verification of it, there is no introduction rule for falsehood.

[*commentaire nôtre* : on pourrait toujours en créer mais elles seraient inutiles]

53 Consistance *absolue* !

we convinced ourselves that a proposition is true is and only if the judgement that it is true is provable. Hence, negating both members, a proposition is false if and only if the judgement that it is true cannot be proved., that is, is unprovable. Using this in one direction, we can conclude, from the already established falsity of $\perp$, that the judgement that $\perp$ is true is unprovable. This is, if you want, an absolute consistency proof: it is a proof of consistency with respect to the unlimited notion of provability, of knowability, that pervades theses lectures. And

$$(\perp \text{ is true) is unprovable}$$

is the judgement which expresses the absolute consistency, if I may call it so.

53-54 Consistence = correction des *inférences*

the consistency problem is really the problem of the correctness of the rules of inference, and that, at some stage or another, you cannot avoid having to convince yourself of their correctness. Or course if you take any old formal system, it may be that you can carry out a metamathematical consistency proof for it, but that consistency proof will rely on the intuitive correctness of the principles of reasoning that you use in that proof, which means that you are nevertheless relying on the correctness of certain forms of inference. Thus the consistency problem is really the problem of the correctness of the rules of inference that you follow, consciously or unconsciolsy, in your reasoning.