Logic as Calculus and Logic as Language

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325-326 UN SEUL UNIVERS POUR FREGE

For Frege it cannot be a question of changing universes. One could not even say that he restricts himself to *one* universe. His universe is *the* universe. Not necessarily the physcal universe, of course, because for Frege some objects are not physical. Frege's universe consists of all that there is, and it is fixed.

[…]

Another important consequence of the universality of logic is that nothing can be, or has to be, said outside the system. And, in fact, Frege never raises any metasystematic question (consistency, independence of axioms, completeness.

327 COMPLÉTUDE EXPÉRIMENTALE

The only question of completeness that may arise is, to use an expression of Herbrand's, an *experimental* question. […] each of these works [the Frege-Russell enterprise, Peano's work] can be regarded as a step in an overrenewed attempt at establishing completeness experimentally.

328 LÖWEIHEIM CHANGE D'UNIVERS

With Löwenheim's paper we have a sharp break with the Frege-Russell approach to the foundations of logic and a return to, or at teast a connection with, pre-Fregean or non-Fregean logic. Löwenheim uses Schröder's logical notation, but, whant is more important, with Schröder he also takes the freedom to change the universe of discourse at will and to base considerations on such changes. And just as Frege was ignored for some time because of his break with the tradition established, so Löwenheim too was ignored for some time because of his break with the tradition established.

[…]

Soon, however, the opposition between the two trends in logic dissolved. During the 'twenties the work of Skolem, Herbrand, and Gödel produced an amalgamation and also a *dépassement* of these two trends.

329 BOOLE + PEANO = FREGE

Boole has a propositional calculus but no quantification thoery; Peano has a notation for quantification theory but only a very deficient technique of derivation; Frege has a notation for quantification theory and a technique of derivation.

330 ORIGINE SYSTÈMES FORMELS : FREGE (ET NON HILBERT)

The notion of formal of formal system was again brought into the forefront by Hilbert, in the 'twentites. That is perhaps why the (in our sense) axiomatic systems of logic are called Hilbert-type systems by Kleene (*Introduction to Metamathematics*, p. 441). If the historical priority is to be respected, they should rather be called Frege-type systems.