Meaning and proofs: on the conflict between classical and intuitionistic logic

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The meaning of a sentence is determined by the way it can be proved. Dummett suggest that the knowledge of the meaning of a sentence manifests itself in our capacity to recognise a proof of it when one is presented to us.

It is claimed that nothing less than the total use of the language determines the meaning of an individual sentence. This is to give up the possibility of finding any other fundamental principle of meaning.

A second common kind of meaning theory modifies such a drastically holistic view by singling out a class of sentences that are given individual contents independent of the use of the rest of the language. Typically, there are sentences that are decidable, e.g., by observation, or in mathematics, by calculation, and their meaning can thus be understood as determined by their truth-conditions in the classical sense.

This is of course a common view in the philosophy of science where the meaning of theoretical sentences is reduced in this way to that of observational sentences. In the case of mathematics, this view was held by Hilbert. A view of this kind immediately makes criticism of use possible because the total use of the sentences cannot be allowed to be used to infer privileged sentences that are false according to the meaning given to them. What may make revisions necessary here is that there are two situations in which privileged sentences can be asserted: when they are found to be true according to their individual meaning, and when they can be inferred from other sentences. When there is a conflict between these two founds for asserting a sentence, the first one has the priority; i.e., the first kind of use is the central one that determines meaning.

The required harmony between the rules for inferring a sentence and the rules for drawing consequences from it is clearly related to the inversion principle for introduction and elimination rules in Gentzen's system of natural deduction.

The adherent of the platonistic principle of meaning faces a dilemma [...]: either he concedes that his principle does not yield any consequences beyond the ones obtained by equating meaning with knowing how to use sentences in proofs, in which case his formulation must said to be misleading and to introduce an unnecessary detour, or he himself insists that knowledge of truth-conditions transcends any knowledge about proofs, in which case he may have to admit that this transcendence has no empirical consequences, and if so, his principle again fails to add anything to the elucidation of what it is to know the meaning of a sentence and is now also guilty of introducing assumptions without empirical import.

it seems to me that almost any adequate theory about human actions of some kind will have normative effects, regardless of whether its original aim was only descriptive.

Even when our actions follow a fairly easily seen pattern, before we have reflected about this pattern there are usually some occasions where we are more or less unsure about how to act. A theory successfully describing our actions, filling them into a coherent picture, will then tend to reinforce our customs—after having seen the pattern, we will be more sure how to act. But usually it is not very easy to find a completely coherent picture of our practice and there may simply be none. If we then find general rules that agree well with our actual behaviour in cases where we have fixed opinions about the correct practice, we will tend to trust theses rules in cases where there seems to be no established usage, and the theory will thus establish a (new) usage in
this area where the practice seemed floating. The theory is thus to some extent self-fulfilling: provided it is sufficiently good in its descriptive function, it will affect the very actions that it is studying and will tend to make them better conform with the theory.

We have a more complicated situation when the theory turns out to be in conflict with part of a well established usage. This situation may arise when we seem to follow a certain rule in our actual practice that cannot be incorporated into the theory because it breaks some more fundamental principle of the theory which accounts well for other parts of our practice. In this situation, one of those things can happen: we discard the theory as unable to describe our actions and try to find a better one; or we find the principles of the theory so convincing, so effective in expressing our true intentions, that the conflicting usage becomes interpreted as a mistake and is changed accordingly. The adaptation of the theory and the practice to each other may be a process going through several stages in which both the theory and the practice are successively modified until they agree with each other.

This interweaving of descriptive and normative functions seems to be typical of many theories that describe human actions e.g., linguistic, economic, logical, and moral theories.

The intuitionistic solution of these difficulties [never be in position to say that a sentence was incorrectly asserted on a given occasion] seems obvious: the condition for asserting a sentence is that we either know a proof of the sentence of the kind mentioned in the intuitionistic definition of proof exemplified above or know a procedure for obtaining such a proof. This procedure may also be called a proof of the sentence—to answer the terminological question raised earlier in the section; but it is a proof in a secondary sense. To distinguish the two kinds, we may call the first kind canonical or direct proofs and the second kind indirect proofs.