

Introduction to Mathematical Philosophy

Bertrand RUSSELL
1920

pub. George ALLEN & UNWIN (London)

125-126 SUR L'AXIOME DU CHOIX (« MULTIPLICATIVE AXIOM ») ET LES CHAUSSETTES

But [the equality $A \times A = A$], though we can prove that it is sometimes the case, cannot be proved to happen always unless we assume the multiplicative axiom. This is illustrated by the millionaire who bought a pair of socks whenever he bought a pair of boots, and never at any other time, and who had such a passion for buying both that at last he had $\aleph_0$ pairs of boots and $\aleph_0$ pairs of socks. The problem is: how many boots had he, and how many socks? One would naturally suppose that he had twice as many boots and twice as many socks as he had pairs of each, and that therefore he had $\aleph_0$ of each, since that number is not increased by doubling. But this is an instance of the difficulty, already noted, of connecting the sum of v classes each having μ terms with $\mu \times v$. Sometimes this can be done, sometimes it cannot. In our case it can be done with the boots, but not with the socks, except by some very artificial device. The reason for the differences is this: Among boots we can distinguish right and left, and therefore we can make a selection of one out of each pair, namely, we can choose all the right boots or all the left boots; but with socks no such principle of selection suggests itself, and we cannot be sure, unless we assume the multiplicative axiom, that there is any class consisting of one sock out of each pair. Hence the problem.

172 AMBIGÜITÉ DU VERBE ÊTRE

The proposition "Socrates is a man" is no doubt equivalent to "Socrates is human", but it is not the very same proposition. The *is* of "Socrates is human" expresses the relation of subject and predicate; the *is* of "Socrates is a man" expresses identity. It is a disgrace to the human race that it has chosen to employ the same word "is" for these two entirely different ideas – a disgrace which a symbolic language of course remedies.

205 IMPRÉCISION DU LANGAGE

Because language is misleading, as well as because it is diffuse and inexact when applied to logic (for which it was never intended), logical symbolism is absolutely necessary to any exact or thorough treatment of our subject.