Cours sur les fondements des mathématiques

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23 Sens opératoire des symboles mathématiques

mathematical propositions containing a certain symbol are rules for the use of that symbol, […] these symbols can then be used in non-mathematical statements.

35 Contrée mathématique

I'm trying to conduct you on tours in a certain country. I will try to show that the philosophical difficulties which arise in mathematics as elsewhere arise because we find ourselves in a strange town and do not know our way. So we must learn the topography by going from one place in the town to another, and from there to another, and so on. And one must do this so often that one knows one's way, either immediately or pretty soon after looking around a bit, wherever one may be set down

58 Définitions et analogie

The new meaning [of a word] must be such that we who have had a certain training will find it useful in certain ways.

It is like the case of definitions. Is a definition purely verbal or not? – Definitions do not merely give new meanings to words. We do not accept some definitions; some are uninteresting, some will be entirely mudding, others very useful, etc. And the same with analogy.

59 Sens commun

Don't treat your commonsense like an umbrella. When you come into a room to philosophize, don't leave it outside but bring it in with you.

100 Reconnaissance d'une preuve

The way in which [the multiplication] can be got we accept or acknowledge as a *proof* of it.

44/106/141/145/210 Mythe de la réalité

As soon as I get into mathematics the means and the results become the same. But as soon as I distinguish between means and results, it is not mathematics.

[…]

Mathematical propositions are first of all English sentences; no only English sentences, but each mathematical proposition has a resemblance to certain non-mathematical propositions. – Mathematicians, when they begin to philosophize, always make the mistake of overlooking the difference in function between mathematical propositions and non-mathematical propositions.

[…]

I'm really trying only to examine the difference between counting in mathematics and ordinary counting, and the difference between a mathematical proposition and an experimental one.

[…]

We imagine possible structures and impossible structures, and we distinguish both from real structures. It seems as though in mathematics we showed what structures are conceivable imaginable, not what are real. We'll prove that there *can* be such-and-such a construction.

[…]

A use of language has normally what we might call a *point*. This is immensely important Although it's true this is a matter of degree, and we can't say just where it ends.

107 MATHEMATIQUE DES SIGNES ?

[…] one asks such a thing as what mathematics is about – and someone replies that it is about numbers. Then someone comes along and says that it is not about numbers but about numerals; for numbers seem very mysterious things. And then is seems that mathematical propositions a bout scratches on the blackboard. That must seem ridiculous even to those who hold it, but they hold it because there seems to be no way out. – I am trying to show in a very general way how the misunderstanding of supposing a mathematical proposition to be like an experimental proposition leads to the misunderstanding of supposing that a mathematical proposition is about scratches on the blackboard.

129 PREUVE ET VÉRITÉ

What is the criterion that the mathematical propositions is true? It's not the psychological proof. Is it the watertight proof or what?

Is it like this: God sees it is true. We can get at it in different ways. Some of us are easily persuaded that it is so, others need a long elaborate proof. But it is so.

But what is the criterion for its being so – if not the proof?

The idea seems to be that we get at the truth which was always there apart from the proof. (The proof is a kind of telescope.)

131 PREUVES ET ÉCRITURES

The figure of the Euclidean proof as used in mathematics is just as rigorous as writing – because it has nothing to do with whether it is drawn well or badly. The main difference between a proof by drawing lines and a proof in writing is that is doesn't matter how you draw lines, or whether the *r*'s and *l*'s and *m*'s and *e*'s are written well.

137/249 SUR LE CHOIX D'ALLER DANS LE SENS D'UNE PROPOSITION (AXIOME, CONJECTURE…)

Professor Hardy says, « Goldbach's theorem is either true or false." – We simply say the road hasn't been built yet. At present you have the right to say either; you have the right to *postulate* that it's true or that it's false. – If you look at it this way, the whole idea of mathematics as the physics of the mathematical entities breaks down. For which road you build is not determined by the physics of mathematical entities but by totally different considerations.

The mathematical proposition says : The road goes there. Why we should build a certain road isn't because mathematics says that the road goes there – because the road isn't built until mathematics says it goes there. What determines it is partly practical considerations and partly analogies in the present system of mathematics.

But the fact that a proof of the theorem is *possible* may seem to be a mathematical fact – not a fact of convenience etc.

[…]

Russell says, roughly: "After all, it is not self-evidence which must guide one in the choice of primitive propositions. On the contrary, one is guided sometimes by the results which a given choice produces. Many primitive propositions are shown to be true by what follows from them." – You may choose them because you want to get to a certain point. Not because they are indubitable.

142/150 MATHÉMATIQUE ET JEU

It has been said vey often that mathematics is a game, to be compared with chess. In a sense this is obviously false – it is not a game in the ordinary sense. In a sense it is obviously true – there is some similarity. The thing to do is not to take sides, but to investigate. It is something useful to compare mathematics to a game and sometimes misleading.

[…]

In a sense the pure mathematics of chess makes no predictions. That is one of the important points. The pure mathematics of chess is like the pure mathematics of astronomy. The calculus makes no predictions, but by means of it you can make predictions.

[…]

"The theory of chess is not arbitrary." – It's not arbitrary, mathematics is not arbitrary, only in the sense, that is has an *obvious* applications. Whereas chess hasn't got an obvious application in that way. That's why it is a game.

96/98/186/187/194/196/197/199/241/265-266 SENS = ACCORD DANS L'EMPLOI/L'USAGE

The whole thing is based on the fact that we *don't* all get different results. That's why it was so absurd to say 12 X 12 = 144 may be the wrong result. Because the agreement in getting this result is the justification for this technique. It is one the agreements upon which our mathematical calculations are based.

[…]

"For us human beings, the best thing we can arrive at, the nearest we can get, is that we always get it, or someone who had a lot of experience always got it." […]

It sounds as if your learning how to use [the word 'boo'] were different from your knowing its meaning. *But the point is that we all make the SAME use of it*. To know its meaning is to use it *in the same way* as other people do. "In the right way" means nothing.

[…] the truths of logic are determined by a consensus of opinions. Is this what I am saying? No. There is no *opinion* at all; it is not a question of *opinion*. They are determined by a consensus of *action*: a consensus of doing the same thing, reacting in the same way. There is a consensus but it is not a consensus of opinion. […]

All that I wish to do by this is to show that there are all sorts of different ways in which we could do logic or mathematics. And the fact that we read it out and say every time 'It is true that' makes no difference. What matters is how we later use the things which we read out.

[…]

It is not a question of our first *having* negation, and then asking what logical laws must hold of it in order for us to be able to use it in a certain way. The point is that using it in a certain way is what we mean by negating with it.

[…] let us take another example; the use of "all". "If all the chairs in this room were bought at Eaden Lilley's then this one was. (*x*).*fx* entails *fa*." Suppose I ask, "Are you sure *fa* follows from (*x*).*fx*? Can we assume that it does not follow? What would go wrong if we did assume that? […] What then would go wrong it someone assumes that (*x*).*fx* does not entail *fa*? I would say that all I am assuming is a different use of "all", and there is *nothing* wrong in this.

[…]

I am speaking against the idea of a "logical machinery". I want to say that there is no such thing. […] For us a machinery often stands as a symbol for a certain action.

[…] we might say that the laws of logic show what we *do* with propositions, as opposed to expressing opinions or convictions.

[...]

If you want to understand a word, we always say: "You have to know its use." It is immensely important that to the great majority of words there correspond certain pictures which in some sense or other *represent* for us the meaning of the word. […] But in [some] cases these pictures are more or less misleading or useless. […] There are many such pictures in mathematics.

190/195 Contradiction = blocage psychologique

We most naturally compare a contradiction to something which jams. I would say that anything which we give and conceive to be an explanation of *why* a contradiction does no work is always just another way of saying that we do not want it to work.

[…]

The important point is to see that the meaning of a word can be represented in two different ways: (1) by an image or picture, or something which corresponds to the word, (2) by the use of the word – which also comes to the use of the picture.

Now what is it which is supposed to jam? The picture or the use? Of the *use* you can't say it jams, because you have a right to fix the use as you like. But how could *pictures* jam? There is only one way in which they could, and that is a psychological way.

The phenomenon of jamming *consists* in the fact that we say it jams: that we say, "Oh, it's a contradiction and we cannot do anything with it", etc. The phenomenon is not, as it were, somewhere else and observed by us in some other sphere.

239 Contradiction : renoncement à toute idée de calcul

We have seen that if we didn't recognize a contradiction, or if we allowed a contradiction but, for example, did not draw any further conclusions from it, we could not then say we must come into conflict with any fact. – You *could* say that if we allow a contradiction, in the sense that we allow anything to follow from it, then we have given up any idea of calculus at all.

253-254 Règles et nécessité

What is necessary is determined by the rules. – We might then ask, "Was it necessary or arbitrary to give these rules?" And here we might say that a rule was arbitrary if we made it just for fun and necessary if having this particular rule were a matter of life and death.

[…]

To say, "If you multiply these two, you necessarily get such-and-such number", if it means anything *at all*, must be opposed to a case where there is no necessity. Or else it's a pleonasm to say you *necessarily* get this – why not simply say that you *get* it? – We might speak of getting something but not necessarily, in the case of a calculus in which you could get more than one answer.

291/297 Les tautologies tuent l'information

The point of Russell's sentences is that none of them gives us any information about anything If we substitute propositions of botany for '*p*' and '*q*', then the whole gives us no information about botany; it ceases to be a botanical sentence. This is the point of a tautology: that if any part of it gives information, the rest cancels it out.

[…]

To say that mathematical propositions impart mathematical information is misleading. For the information they impart is different from what is suggested by their structure as sentences.

What is interesting is not that '$p \supset q$' does not impart information: for any nonsense is similar in that respect – but that certain sentences are put together by certain functions *in such a way* that they do not impart information.

That the lever of the balance doesn't move, gives us information about the weights we have placed in it. But if the lever were fixed, it would give none. Only when we know how "⊃" etc. are used otherwise – then it is important that in certain uses they *do not* impart information.

291/299/301 Les tautologies révèlent un usage

"It expresses a law of thought" could be expressed by saying: its being a tautology follows from the way in which we use these signs in thinking, speaking, etc…

[…]

The idea of a law of thought is similar to the idea of a law of measurement. If I describe the rules of measurement you might regard these rules as rules given to someone, so he knows what to do – but all kinds of important things can be inferred from them. That a grocer weighs chees [in such-and-such a way] is interesting, because it shows that the weights don't constantly fluctuate.

You could call the *rules of weighing* laws of thought. In a way, they *define* "*weigth*".

[…]

Now we don't say that if so-and-so is a tautology an inference can be made, but that since an inference can be made in such-and-such a way, this is a tautology.

304 L'induction est un raccourci

You might say:

(1) The real proof would be the whole chain. In some mystical way, I've done all these operations.

(2) In some way what I've done is the *same as* doing all 3000 – I can only say it's not the same.