Calcul de dérivées

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(correction optionnelle)

Solution proposée.

1. On propose ici un calcul détaillé des dérivées avec uniquement des *fonctions* et sans se soucier des problème de sens. Il convient donc de considérer ce qui suit comme un exercice de calcul fonctionnel.

   Rappelons que $|\cdot|$ est dérivable sur $\mathbf{R}_+^*$ (elle y vaut Id de dérivée 1) et sur $\mathbf{R}_-^*$ (elle y vaut $-\mathrm{Id}$ de dérivée -1), donc est dérivable sur $\mathbf{R}^*$ de dérivée

$$|\cdot|' = \frac{\mathrm{Id}}{|\cdot|} = \frac{|\cdot|}{\mathrm{Id}}.$$

   Avec des réels, cela s'écrirait

$$\forall a \neq 0,\ \frac{\partial}{\partial a}|a| = \frac{a}{|a|} = \frac{|a|}{a}.$$

   (a) On dérive $\ln \circ |\cdot|$;

$$\begin{aligned}
[\ln \circ |\cdot|]' &= [\ln' \circ |\cdot|] \times |\cdot|' \\
&= \left[\frac{1}{\mathrm{Id}} \circ |\cdot|\right] \times \frac{|\cdot|}{\mathrm{Id}} \\
&= \frac{1}{|\cdot|}\frac{|\cdot|}{\mathrm{Id}} \\
&= \frac{1}{\mathrm{Id}}.
\end{aligned}$$

   (b) On dérive $[\ln \circ |\cdot| \circ \tan]$, d'où deux possibilités selon le regroupement des trois fonctions composées : ou bien

$$\begin{aligned}
[\ln \circ |\cdot| \circ \tan]' &= [(\ln \circ |\cdot|) \circ \tan]' \\
&= \left([\ln \circ |\cdot|]' \circ \tan\right) \times \tan' \\
&\overset{cf.\ \mathrm{a}}{=} \left(\frac{1}{\mathrm{Id}} \circ \tan\right) \times \left(1 + \tan^2\right) \\
&= \frac{1+\tan^2}{\tan} \\
&= \cot + \tan,
\end{aligned}$$

   ou bien (plus long)

$$\begin{aligned}
[\ln \circ |\cdot| \circ \tan]' &= [\ln \circ (|\cdot| \circ \tan)]' \\
&= [\ln' \circ (|\cdot| \circ \tan)] \times (|\cdot| \circ \tan)' \\
&= \left[\frac{1}{\mathrm{Id}} \circ (|\cdot| \circ \tan)\right] \times \left(|\cdot|' \circ \tan\right) \times \tan' \\
&= \left[\frac{1}{\mathrm{Id}} \circ |\tan|\right]\left(\frac{|\cdot|}{\mathrm{Id}} \circ \tan\right)\left(1+\tan^2\right) \\
&= \frac{1}{|\tan|}\frac{|\tan|}{\tan}\left(1+\tan^2\right) \\
&= \cot + \tan.
\end{aligned}$$

   (c) On dérive $\cos \circ \left(\mathrm{Id}^2 + \mathrm{Id} + 1\right)$:

$$\begin{aligned}
\left[\cos \circ \left(\mathrm{Id}^2 + \mathrm{Id} + 1\right)\right]' &= \left[\cos' \circ \left(\mathrm{Id}^2 + \mathrm{Id} + 1\right)\right] \times \left[\mathrm{Id}^2 + \mathrm{Id} + 1\right]' \\
&= -\sin\left(\mathrm{Id}^2 + \mathrm{Id} + 1\right) \times (2\,\mathrm{Id} + 1).
\end{aligned}$$

(d) On dérive $\sin\circ\frac{3\,\mathrm{Id}-1}{\mathrm{Id}+2}$ en remarquant au préalable que $\frac{3\,\mathrm{Id}-1}{\mathrm{Id}+2}=\frac{3(\mathrm{Id}+2)-7}{\mathrm{Id}+2}=3-\frac{7}{\mathrm{Id}+2}$:

$$\begin{aligned}\left[\sin\circ\frac{3\,\mathrm{Id}-1}{\mathrm{Id}+2}\right]' &= \left(\sin'\circ\frac{3\,\mathrm{Id}-1}{\mathrm{Id}+2}\right)\times\left[3-\frac{7}{\mathrm{Id}+2}\right]' \\ &= \cos\left(\frac{3\,\mathrm{Id}-1}{\mathrm{Id}+2}\right)\times\frac{7}{(\mathrm{Id}+2)^2}.\end{aligned}$$

(e) On dérive $\tan\circ(42\,\mathrm{Id}+18)$:

$$\begin{aligned}[\tan\circ(42\,\mathrm{Id}+18)]' &= [\tan'\circ(42\,\mathrm{Id}+18)]\times(42\,\mathrm{Id}+18)' \\ &= \left[\frac{1}{\cos^2}\circ(42\,\mathrm{Id}+18)\right]\times 42 \\ &= \frac{42}{\cos^2(42\,\mathrm{Id}+18)}.\end{aligned}$$

(f) On dérive $\exp\circ 3\,\mathrm{Id}\circ\cos\circ 2\,\mathrm{Id}$, d'où plusieurs possibilités selon le regroupement des fonctions composées. En voici une :

$$\begin{aligned}[\exp\circ 3\,\mathrm{Id}\circ\cos\circ 2\,\mathrm{Id}]' &= [(\exp\circ 3\,\mathrm{Id})\circ(\cos\circ 2\,\mathrm{Id})]' \\ &= \left([\exp\circ 3\,\mathrm{Id}]'\circ[\cos\circ 2\,\mathrm{Id}]\right)\times[\cos\circ 2\,\mathrm{Id}]' \\ &= \left(\left((\exp'\circ 3\,\mathrm{Id})\times[3\,\mathrm{Id}]'\right)\circ[\cos\circ 2\,\mathrm{Id}]\right)\times(\cos'\circ 2\,\mathrm{Id})\times[2\,\mathrm{Id}]' \\ &= (((\exp\circ 3\,\mathrm{Id})\times 3)\circ[\cos\circ 2\,\mathrm{Id}])\times(-\sin\circ 2\,\mathrm{Id})\times 2 \\ &= -6\,(\exp\circ 3\,\mathrm{Id}\circ\cos\circ 2\,\mathrm{Id})\times(\sin\circ 2\,\mathrm{Id}).\end{aligned}$$

(g) On dérive $\ln\circ(4\,\mathrm{Id}-1)$:

$$\begin{aligned}[\ln\circ(4\,\mathrm{Id}-1)]' &= \left(\ln'\circ(4\,\mathrm{Id}-1)\right)\times[4\,\mathrm{Id}-1]' \\ &= \left[\frac{1}{\mathrm{Id}}\circ(4\,\mathrm{Id}-1)\right]\times 4 \\ &= \frac{4}{4\,\mathrm{Id}-1}.\end{aligned}$$

(h) On dérive $\ln\circ\left(\mathrm{Id}^2-6\,\mathrm{Id}+9\right)=\ln\circ\left[(\mathrm{Id}-3)^2\right]=2\ln\circ(\mathrm{Id}-3)$:

$$\begin{aligned}\left[\ln\circ\left(\mathrm{Id}^2-6\,\mathrm{Id}+9\right)\right]' &= [2\ln\circ(\mathrm{Id}-3)]' \\ &= 2\left(\ln'\circ[\mathrm{Id}-3]\right)\times(\mathrm{Id}-3)' \\ &= 2\left(\frac{1}{\mathrm{Id}}\circ[\mathrm{Id}-3]\right)\times 1 \\ &= \frac{2}{\mathrm{Id}-3}.\end{aligned}$$

(i) On dérive $\mathrm{Id}^3\circ\cos\circ\mathrm{Id}^5$, d'où deux possibilités selon le regroupement des fonctions composées : ou bien

$$\begin{aligned}\left[\mathrm{Id}^3\circ\cos\circ\mathrm{Id}^5\right]' &= \left[\mathrm{Id}^3\circ\left(\cos\circ\mathrm{Id}^5\right)\right]' \\ &= \left(\left[\mathrm{Id}^3\right]'\circ\left(\cos\circ\mathrm{Id}^5\right)\right)\times\left[\cos\circ\mathrm{Id}^5\right]' \\ &= \left(3\,\mathrm{Id}^2\circ\left(\cos\circ\mathrm{Id}^5\right)\right)\times\left(\cos'\circ\mathrm{Id}^5\right)\times\left[\mathrm{Id}^5\right]' \\ &= \left(3\cos^2\circ\mathrm{Id}^5\right)\times\left(-\sin\circ\mathrm{Id}^5\right)\times 5\,\mathrm{Id}^4 \\ &= -15\cos^2\left(\mathrm{Id}^5\right)\times\sin\left(\mathrm{Id}^5\right),\end{aligned}$$

ou bien (plus long)

$$\begin{aligned}
\left[\operatorname{Id}^3\circ\cos\circ\operatorname{Id}^5\right]' &= \left[\left(\operatorname{Id}^3\circ\cos\right)\circ\operatorname{Id}^5\right]' \\
&= \left(\left[\operatorname{Id}^3\circ\cos\right]'\circ\operatorname{Id}^5\right)\times\left[\operatorname{Id}^5\right]' \\
&= \left(\left[\left(\left[\operatorname{Id}^3\right]'\circ\cos\right)\times\cos'\right]\circ\operatorname{Id}^5\right)\times 5\operatorname{Id}^4 \\
&= \left(\left[\left(3\operatorname{Id}^2\circ\cos\right)\times(-\sin)\right]\circ\operatorname{Id}^5\right)\times 5\operatorname{Id}^4 \\
&= \left(\left[-3\cos^2\times\sin\right]\circ\operatorname{Id}^5\right)\times 5\operatorname{Id}^4 \\
&= -15\cos^2\left(\operatorname{Id}^5\right)\times\sin\left(\operatorname{Id}^5\right).
\end{aligned}$$

(j) On dérive $\frac{1}{\operatorname{Id}}\circ\left(\operatorname{Id}^2+2\operatorname{Id}-5\right)$:

$$\begin{aligned}
\left[\frac{1}{\operatorname{Id}}\circ\left(\operatorname{Id}^2+2\operatorname{Id}-5\right)\right]' &= \left(\left[\frac{1}{\operatorname{Id}}\right]'\circ\left(\operatorname{Id}^2+2\operatorname{Id}-5\right)\right)\times\left[\operatorname{Id}^2+2\operatorname{Id}-5\right]' \\
&= \left(-\frac{1}{\operatorname{Id}^2}\circ\left(\operatorname{Id}^2+2\operatorname{Id}-5\right)\right)\times\left[2\operatorname{Id}+2\right] \\
&= -\frac{2\operatorname{Id}+2}{\left(\operatorname{Id}^2+2\operatorname{Id}-5\right)^2}.
\end{aligned}$$

(k) On dérive $\frac{1}{\operatorname{Id}^{18}}\circ\left(\operatorname{Id}^2+2\operatorname{Id}+5\right)$:

$$\begin{aligned}
\left[\frac{1}{\operatorname{Id}^{18}}\circ\left(\operatorname{Id}^2+2\operatorname{Id}+5\right)\right]' &= \left(\left[\frac{1}{\operatorname{Id}^{18}}\right]'\circ\left(\operatorname{Id}^2+2\operatorname{Id}+5\right)\right)\times\left[\operatorname{Id}^2+2\operatorname{Id}+5\right]' \\
&= \left(-\frac{18}{\operatorname{Id}^{19}}\circ\left(\operatorname{Id}^2+2\operatorname{Id}+5\right)\right)\times\left[2\operatorname{Id}+2\right] \\
&= -36\frac{\operatorname{Id}+1}{\left(\operatorname{Id}^2+2\operatorname{Id}+5\right)^{19}}.
\end{aligned}$$

(l) On dérive $\frac{1}{\operatorname{Id}}\circ\cos$:

$$\begin{aligned}
\left[\frac{1}{\operatorname{Id}}\circ\cos\right]' &= \left(\left[\frac{1}{\operatorname{Id}}\right]'\circ\cos\right)\times\cos' \\
&= \left(-\frac{1}{\operatorname{Id}^2}\circ\cos\right)\times(-\sin) \\
&= \frac{\sin}{\cos^2}.
\end{aligned}$$

(m) On dérive $\frac{1}{\operatorname{Id}}\circ\left(\operatorname{Id}^2-4\operatorname{Id}+3\right)\circ\operatorname{Id}^3$:

$$\begin{aligned}
\left[\frac{1}{\operatorname{Id}}\circ\left(\operatorname{Id}^2-4\operatorname{Id}+3\right)\circ\operatorname{Id}^3\right]' &= \left[\frac{1}{\operatorname{Id}}\circ\left(\left(\operatorname{Id}^2-4\operatorname{Id}+3\right)\circ\operatorname{Id}^3\right)\right]' \\
&= \left(\left[\frac{1}{\operatorname{Id}}\right]'\circ\left(\left(\operatorname{Id}^2-4\operatorname{Id}+3\right)\circ\operatorname{Id}^3\right)\right)\times\left[\left(\operatorname{Id}^2-4\operatorname{Id}+3\right)\circ\operatorname{Id}^3\right]' \\
&= \left(-\frac{1}{\operatorname{Id}^2}\circ\left(\left(\operatorname{Id}^6-4\operatorname{Id}^3+3\right)\right)\right)\times\left(\left[\operatorname{Id}^2-4\operatorname{Id}+3\right]'\circ\operatorname{Id}^3\right)\times\left[\operatorname{Id}^3\right]' \\
&= -\frac{1}{\left(\operatorname{Id}^6-4\operatorname{Id}^3+3\right)^2}\times\left(\left[2\operatorname{Id}-4\right]\circ\operatorname{Id}^3\right)\times 3\operatorname{Id}^2 \\
&= 6\operatorname{Id}^2\frac{2-\operatorname{Id}^3}{\left(\operatorname{Id}^6-4\operatorname{Id}^3+3\right)^2}.
\end{aligned}$$

(n) On dérive $\sqrt{\cdot}\circ(3-5\,\mathrm{Id})$:

$$\begin{aligned}\left[\sqrt{\cdot}\circ(3-5\,\mathrm{Id})\right]' &= \left(\sqrt{\cdot}'\circ(3-5\,\mathrm{Id})\right)\times[3-5\,\mathrm{Id}]' \\ &= \left(\frac{1}{2\sqrt{\cdot}}\circ(3-5\,\mathrm{Id})\right)\times(-5) \\ &= -\frac{5}{2\sqrt{3-5\,\mathrm{Id}}}.\end{aligned}$$

(o) On dérive $\sqrt{\cdot}\circ\left(3\,\mathrm{Id}^2+2\,\mathrm{Id}-1\right)$:

$$\begin{aligned}\left[\sqrt{\cdot}\circ\left(3\,\mathrm{Id}^2+2\,\mathrm{Id}-1\right)\right]' &= \left(\sqrt{\cdot}'\circ\left(3\,\mathrm{Id}^2+2\,\mathrm{Id}-1\right)\right)\times\left[3\,\mathrm{Id}^2+2\,\mathrm{Id}-1\right]' \\ &= \left(\frac{1}{2\sqrt{\cdot}}\circ\left(3\,\mathrm{Id}^2+2\,\mathrm{Id}-1\right)\right)\times(6\,\mathrm{Id}+2) \\ &= \frac{3\,\mathrm{Id}+1}{\sqrt{3\,\mathrm{Id}^2+2\,\mathrm{Id}-1}}.\end{aligned}$$

(p) On dérive $\mathrm{Id}^{\frac{1}{2}}\times\mathrm{Id}^{\frac{1}{3}}\times\mathrm{Id}^{\frac{1}{5}}=\mathrm{Id}^{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}}=\mathrm{Id}^{\frac{15+10+6}{30}}=\mathrm{Id}^{\frac{31}{30}}$:

$$\left[\mathrm{Id}^{\frac{1}{2}}\times\mathrm{Id}^{\frac{1}{3}}\times\mathrm{Id}^{\frac{1}{5}}\right]'=\left[\mathrm{Id}^{\frac{31}{30}}\right]'=\frac{31}{30}\,\mathrm{Id}^{\frac{31}{30}-1}=\frac{31}{30}\,\mathrm{Id}^{\frac{1}{30}}.$$

(q) On dérive $\frac{1}{\mathrm{Id}}\circ\left[\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right]$: on a

$$\begin{aligned}\left[\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right]' &= \left[7-2\,\mathrm{Id}^2\right]'\times(\mathrm{Id}+3)+\left(7-2\,\mathrm{Id}^2\right)\times[\mathrm{Id}+3]' \\ &= -4\,\mathrm{Id}\times(\mathrm{Id}+3)+\left(7-2\,\mathrm{Id}^2\right) \\ &= -6\,\mathrm{Id}^2-12\,\mathrm{Id}+7,\end{aligned}$$

d'où l'on tire

$$\begin{aligned}\left[\frac{1}{\mathrm{Id}}\circ\left(\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right)\right]' &= \left(\left[\frac{1}{\mathrm{Id}}\right]'\circ\left(\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right)\right)\times\left[\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right]' \\ &= \left(-\frac{1}{\mathrm{Id}^2}\circ\left(\left(7-2\,\mathrm{Id}^2\right)\times(\mathrm{Id}+3)\right)\right)\times\left(-6\,\mathrm{Id}^2-12\,\mathrm{Id}+7\right) \\ &= \frac{6\,\mathrm{Id}^2+12\,\mathrm{Id}-7}{\left(7-\mathrm{Id}^2\right)^2(\mathrm{Id}+3)^2}\end{aligned}$$

(r) On dérive $\frac{\mathrm{Id}^4\circ\sin\circ(3\,\mathrm{Id})}{\mathrm{Id}^3\circ\cos\circ(2\,\mathrm{Id})}$. On regarde d'une part la dérivée du numérateur

$$\begin{aligned}\left[\mathrm{Id}^4\circ\sin\circ(3\,\mathrm{Id})\right]' &= \left[\mathrm{Id}^4\circ(\sin\circ(3\,\mathrm{Id}))\right]' \\ &= \left(\left[\mathrm{Id}^4\right]'\circ(\sin\circ(3\,\mathrm{Id}))\right)\times[\sin\circ(3\,\mathrm{Id})]' \\ &= \left(4\,\mathrm{Id}^3\circ(\sin\circ(3\,\mathrm{Id}))\right)\times\left(\sin'\circ(3\,\mathrm{Id})\right)\times[3\,\mathrm{Id}]' \\ &= \left(4\sin^3(3\cdot)\right)\times(\cos(3\cdot))\times 3 \\ &= 12\sin^3(3\cdot)\ \cos(3\cdot),\end{aligned}$$

d'autre part celle du dénominateur

$$\begin{aligned}\left[\mathrm{Id}^3\circ\cos\circ(2\,\mathrm{Id})\right]' &= \left[\mathrm{Id}^3\circ(\cos\circ(2\,\mathrm{Id}))\right]' \\ &= \left(\left[\mathrm{Id}^3\right]'\circ(\cos\circ(2\,\mathrm{Id}))\right)\times[\cos\circ(2\,\mathrm{Id})]' \\ &= \left(3\,\mathrm{Id}^2\circ(\cos\circ(2\,\mathrm{Id}))\right)\times(\cos'\circ(2\,\mathrm{Id}))\times[2\,\mathrm{Id}]' \\ &= \left(3\cos^2(2\cdot)\right)\times(-\sin(2\cdot))\times 2 \\ &= -6\cos^2(2\cdot)\ \sin(2\cdot),\end{aligned}$$

d'où l'on tire

$$\begin{aligned}
\left[\frac{\mathrm{Id}^4\circ\sin\circ(3\,\mathrm{Id})}{\mathrm{Id}^3\circ\cos\circ(2\,\mathrm{Id})}\right]' &= \frac{\left[\mathrm{Id}^4\circ\sin\circ(3\,\mathrm{Id})\right]'\times\cos^3(2\cdot)-\sin^4(3\cdot)\times\left[\mathrm{Id}^3\circ\cos\circ(2\,\mathrm{Id})\right]'}{\left(\cos^3(2\cdot)\right)^2}\\
&= \frac{12\sin^3(3\cdot)\ \cos(3\cdot)\times\cos^3(2\cdot)+\sin^4(3\cdot)\times 6\cos^2(2\cdot)\ \sin(2\cdot)}{\cos^6(2\cdot)}\\
&= 6\sin^3(3\cdot)\,\frac{2\cos(3\cdot)\ \cos(2\cdot)+\sin(3\cdot)\ \sin(2\cdot)}{\cos^4(2\cdot)}
\end{aligned}$$

(s) On dérive

$$\begin{aligned}
\exp\circ 42\,\mathrm{Id}\circ\ln\circ\left(\mathrm{Id}^3+1\right) &= \exp\circ\ln\circ\left(\mathrm{Id}^{42}\circ\left(\mathrm{Id}^3+1\right)\right)\\
&= \mathrm{Id}^{42}\circ\left(\mathrm{Id}^3+1\right),
\end{aligned}$$

ce qui donne

$$\begin{aligned}
\left[\mathrm{Id}^{42}\circ\left(\mathrm{Id}^3+1\right)\right]' &= \left(\left[\mathrm{Id}^{42}\right]'\circ\left(\mathrm{Id}^3+1\right)\right)\times\left[\mathrm{Id}^3+1\right]'\\
&= \left(42\,\mathrm{Id}^{41}\circ\left(\mathrm{Id}^3+1\right)\right)\times 3\,\mathrm{Id}^2\\
&= 126\times\left(\mathrm{Id}^3+1\right)^{41}\times\mathrm{Id}^2\,.
\end{aligned}$$

(t) On dérive $\ln\circ\left(\mathrm{Id}+\sqrt{\cdot}\circ\left(\mathrm{Id}^2+1\right)\right)$:

$$\begin{aligned}
\left[\ln\circ\left(\mathrm{Id}+\sqrt{\cdot}\circ\left(\mathrm{Id}^2+1\right)\right)\right]' &= \left(\ln'\circ\left(\mathrm{Id}+\sqrt{\cdot}\circ\left(\mathrm{Id}^2+1\right)\right)\right)\times\left[\mathrm{Id}+\sqrt{\cdot}\circ\left(\mathrm{Id}^2+1\right)\right]'\\
&= \frac{1}{\mathrm{Id}}\circ\left(\mathrm{Id}+\sqrt{\cdot}\circ\left(\mathrm{Id}^2+1\right)\right)\\
&\quad\times\left(1+\left(\sqrt{\cdot}'\circ\left(\mathrm{Id}^2+1\right)\right)\times\left[\mathrm{Id}^2+1\right]'\right)\\
&= \frac{1}{\mathrm{Id}+\sqrt{\mathrm{Id}^2+1}}\times\left[1+\left(\frac{1}{2\sqrt{\cdot}}\circ\left(\mathrm{Id}^2+1\right)\right)\times 2\,\mathrm{Id}\right]\\
&= \frac{1}{\mathrm{Id}+\sqrt{\mathrm{Id}^2+1}}\times\left(1+\frac{\mathrm{Id}}{\sqrt{\mathrm{Id}^2+1}}\right)\\
&= \frac{1}{\mathrm{Id}+\sqrt{\mathrm{Id}^2+1}}\times\left(\frac{\sqrt{\mathrm{Id}^2+1}+\mathrm{Id}}{\sqrt{\mathrm{Id}^2+1}}\right)\\
&= \frac{1}{\sqrt{\mathrm{Id}^2+1}}.
\end{aligned}$$

(u) On dérive $\sqrt{\cdot}\circ\left(3\sin^2-7\right)$:

$$\begin{aligned}
\left[\sqrt{\cdot}\circ\left(3\sin^2-7\right)\right]' &= \left(\sqrt{\cdot}'\circ\left(3\sin^2-7\right)\right)\times\left[3\sin^2-7\right]'\\
&= \left(\frac{1}{2\sqrt{\cdot}}\circ\left(3\sin^2-7\right)\right)\times 3\times 2\sin\times\sin'\\
&= \frac{3\sin(2\cdot)}{2\sqrt{3\sin^2-7}}.
\end{aligned}$$

(v) On dérive $\exp\circ\sqrt{\cdot}\circ\left(\sin^2+3\right)$:

$$\begin{aligned}
\left[\exp\circ\sqrt{\cdot}\circ\left(\sin^2+3\right)\right]' &= \left[\exp\circ\left(\sqrt{\cdot}\circ\left(\sin^2+3\right)\right)\right]'\\
&= \left(\exp'\circ\left(\sqrt{\cdot}\circ\left(\sin^2+3\right)\right)\right)\times\left[\sqrt{\cdot}\circ\left(\sin^2+3\right)\right]'\\
&= \left(\exp\circ\left(\sqrt{\cdot}\circ\left(\sin^2+3\right)\right)\right)\times\left(\left(\frac{1}{2\sqrt{\cdot}}\circ\left(\sin^2+3\right)\right)\times\left[\sin^2+3\right]'\right)\\
&= e^{\sqrt{\sin^2+3}}\times\left(\frac{1}{2\sqrt{\sin^2+3}}\times 2\sin\sin'\right)\\
&= e^{\sqrt{\sin^2+3}}\,\frac{\sin\times\cos}{\sqrt{\sin^2+3}}.
\end{aligned}$$

(w) On dérive $\mathrm{Id}\times\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}$. On peut simplifier $\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}=\frac{2(\mathrm{Id}+3)-7}{\mathrm{Id}+3}=2-\frac{7}{\mathrm{Id}+3}$, ce qui permet d'écrire

$$\begin{aligned}
\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}' &= \left[\sqrt{\cdot}\circ\left(2-\frac{7}{\mathrm{Id}+3}\right)\right]' \\
&= \left(\sqrt{\cdot}'\circ\left(2-\frac{7}{\mathrm{Id}+3}\right)\right)\times\left[2-\frac{7}{\mathrm{Id}+3}\right]' \\
&= \left(\frac{1}{2\sqrt{\cdot}}\circ\left(2-\frac{7}{\mathrm{Id}+3}\right)\right)\times\frac{7}{(\mathrm{Id}+3)^2} \\
&= \frac{1}{2\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}}\times\frac{2}{(\mathrm{Id}+3)^2} \\
&= \frac{7}{2}\frac{1}{\sqrt{2\,\mathrm{Id}-1}}\frac{1}{\sqrt{\mathrm{Id}+3}^3},
\end{aligned}$$

d'où l'on tire

$$\begin{aligned}
\left[\mathrm{Id}\times\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}\right]' &= \mathrm{Id}'\times\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}+\mathrm{Id}\times\sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}' \\
&= \sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}+\frac{7\,\mathrm{Id}}{2}\frac{1}{\sqrt{2\,\mathrm{Id}-1}}\frac{1}{\sqrt{\mathrm{Id}+3}^3} \\
&= \sqrt{\frac{2\,\mathrm{Id}-1}{\mathrm{Id}+3}}\left(1+\frac{\frac{7\,\mathrm{Id}}{2}}{2\,\mathrm{Id}-1}\frac{1}{\mathrm{Id}+3}\right).
\end{aligned}$$