# Lab School Paris 

## Mock Brevet Exam Mathematics

## April 2023

Duration of the test: two hours calculators: allowed

- The wording has four pages ( $1 / 42 / 43 / 4$ : text; $4 / 4$ : figures), in addition to this cover page.

The wording is made up of eight independent exercises, labeled $1,10,15,25,29,36,37 \& 52$. You can process them in any order that suits you.

The test is graded out of one hundred points.
$\rightarrow$ Each answer must be preceeded by a justification.
$\rightarrow$ The English quality and the clarity of reasonning will be decisive for the evaluation.
Any trace of research will be taken into account in the evaluation.

Human activities produce carbon dioxide $\left(\mathrm{CO}_{2}\right)$, which contributes to global warming. The following graph shows the evolution of average atmospheric $\mathrm{CO}_{2}$ concentration (expressed in ppm) as a function of time (expressed in years).
[top caption] Atmospheric $\mathrm{CO}_{2}$ concentration (in ppm)
[right caption] $450 \mathrm{ppm}=$ average level not to be exceeded by 2100
[bottom right caption] (Source: WMO World Data Center)
[bottom left] $1 \mathrm{ppm} \mathrm{CO}=1$ part per million $\mathrm{CO}_{2}=1$ milligram of $\mathrm{CO}_{2}$ per kilogram of air
$1^{\circ}$ Determine graphically the $\mathrm{CO}_{2}$ concentration in ppm in 1995, then in 2005. 2pts
$2^{\circ}$ One wants to model the evolution of $\mathrm{CO}_{2}$ concentration as a function of time using a map $g$, where $g(x)$ is the $\mathrm{CO}_{2}$ concentration in ppm as a function of year $x$.
a. Explain why an affine map seems appropriate to model $\mathrm{CO}_{2}$ concentration as a function of time between 1995 and 2005. 3pts
b. Arnold and Billy each propose an expression for map $g$ :

- Arnold proposes the expression $g(x)=2 x-3630$;
- Billy proposes the expression $g(x)=2 x-2000$.

Which expression best models the evolution of $\mathrm{CO}_{2}$ concentration? $\mathbf{4 p t s}$
c. Using the function you chose in the previous question, indicate the year in which the value of 450 ppm is reached. 4pts
$3^{\circ}$ In France, thanks to photosynthesis, forests capture around 70 megatons of $\mathrm{CO}_{2}$ per year, which amounts to $15 \%$ of 2016 national carbon emissions. Calculate an approximate value, to the nearest megaton, of the mass of $\mathrm{CO}_{2}$ emitted in France in 2016. 4pts

COMPUTING PROGRAMS
8 pts 15 min
Here are two computing programs:

## Program a

Starting number
$\rightarrow$ Add 10
$\rightarrow$ Square
Result

## Program 8

Starting number
$\rightarrow$ Add 20
$\rightarrow$ Multiply by the starting number
$\rightarrow$ Add 100
Result
$1^{\circ}$ Show that, if 5 is chosen as a starting number, then both programs yield the same result.
$2^{\circ}$ With Program A, which starting number must be chosen to get a nil result? Does this number yield a nil result for Program B too? 3pts
$3^{\circ}$ The teacher says both programs always yield the same result, whatever the starting number. Are they right? 3pts

Reminder. Two triangles are qualified similar if their angles have same measures. When this is the case, the lengths of one triangle can be obtained by multiplying those of the other triangle by the same number, called the reduction or enlargement coefficient.

10 COMPUTING PROGRAMS 12pts 15 min
The following two computing programs are given:

## Program a

## Program 8

Choose a number.
Subtract 5 from this number.
Multiply the result by the starting number

Choose a number.
Square this number. Subtract 4 from the result.

| $1^{\circ}$ Alice chooses number 4 and applies program A. Show that she will obtain -4. | $\mathbf{1 p t}$ |
| :--- | :--- | :--- |
| $2^{\circ}$ Lucie chooses number -3 and applies program B. What result will she obtain? | $\mathbf{2 p t s}$ |

Tom wants to find a number for which both programs will give the same result. He chooses $x$ as the starting number for both programs.
$3^{\circ}$ Show that the result of program $A$ can be written as $x^{2}-5 x$.
4pts
$4^{\circ}$ Express the result obtained with program B as a function of $x$.
$5^{\circ}$ What number is Tom looking for? 3pts

25 Filling \& Painting a round tank 9pts 15 min
A tank is made up of two identical half-spheres linked by a 3 m long cylindrical part, all three of them being 1.8 m in diameter.
$1^{\circ}$ What is the total capacity of this tank?
4pts
The outer surface of the tank is to be painted twice. The paint used is sold in 3 L buckets, each of which costs $€ 60$. The instructions state that one liter covers about $5 \mathrm{~m}^{2}$.
$2^{\circ}$ How much paint do I need to paint this tank?
5pts

36 HoIST THE SAIL! 12 pts 15 min
For her own comfort, Lisa wants to install a triangular shade sail in her garden.
The area of the sail must be at least $8 \mathrm{~m}^{2}$.
For each of the following three models, indicate whether it is suitable.
$1^{\circ}$ 2pts
$2^{\circ}$
4pts
$3^{\circ} \quad \mathbf{6 p t s}$

Mrs Smith wishes to create a concrete terrace opposite her bay window. She makes the following drawing.

| [caption in left parallelogram] | Bay window |
| :--- | :---: |
| [caption in right rectangle] | Terrace |

To facilitate rainwater run-off, the terrace floor must be sloped.
The terrace is shaped like a right prism whose base is quadrilateral ABCD and height is segment [CG].
$P$ is the point on segment $[A D]$ such that $B C D P$ is a rectangle.
$1^{\circ}$ Angle $\measuredangle \mathrm{ABP}$ must measure between $1^{\circ}$ and $1.5^{\circ}$. Does Mrs Smith's project satisfy this condition? 6pts
$2^{\circ} \mathrm{Mrs}$ Smith would like to have the concrete required for her terrace delivered to her home. She calls in a specialist company. Using the information below, determine the amount of the invoice issued by the company. 10pts

Information 1 Distance between the company and Mrs. Smith's house: 23 km .
INFORMATION 2 Volume of a right prism $=$ Area of prism base $\times$ Height of prism.
Information 3-Specialize company's pricing conoitions Delivery charge: $€ 5$ per km covered by the truck. Maximum capacity of truck-mixer: $6 \mathrm{~m}^{3}$. Price per $\mathrm{m}^{3}$ of concrete: $€ 95$. The company charges for round-trip distances (company $\leftrightarrow$ delivery site) covered by the truck-mixer.

37 Similar triangles 16pts 15 min
Consider triangle ABC above, right-angled at A , with $\measuredangle \mathrm{ABC}=30^{\circ}$ and $\mathrm{AB}=7 \mathrm{~cm}$. Point H is the foot of the height from A .
$1^{\circ}$ Draw the full-scale figure. Leave the construction lines visible. 3pts
$2^{\circ}$ Prove that $\mathrm{AH}=3.5 \mathrm{~cm} . \quad$ 5pts
$3^{\circ}$ Show that triangles $A B C$ and HAC are similar. 4pts
$4^{\circ}$ Determine the reduction coefficient from triangle $A B C$ to triangle $H A C$. 4pts


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\begin{aligned}
& \text { pictures } \\
& \text { \& figures }
\end{aligned}
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