Literal computation test

Friday 22d March 24

adapted from *Annales abc* p. 58 & 64

Suggested correction

Wording 11.

1. Let us apply program 1 on number 5: tripling it gives $3 \cdot 5 = 15$, adding 1 gives $15 + 1 = 16$.

   Let us apply program 2 on number 5: taking 1 from it (resp. adding 2 to it) gives $5 - 1 = 4$ (resp. $5 + 2 = 7$), multiplying the difference and sum obtained gives $4 \cdot 7 = 28$.

   (a) Image $A(r)$ is the outcome of program 1 applied on number r. Let us therefore apply that program on that number: tripling r gives $3r$, adding 1 gives $3r + 1$. One consequently has equality

   $$A(r) = 3r + 1.$$

   (This equality holding for *each* number r, map $A : t \mapsto \boxed{3}\, t + \underline{\underline{1}}$ is affine, its slope is $\boxed{3}$ and its y-intercept $\underline{\underline{1}}$.)

   (b) *Analysis.* Let d be such a number. One has then equality $A(d) = 0$, *i. e.* $3d+1 = 0$. subtracting 1 gives $3d = -1$, then dividing by 3 gives $d = -\frac{1}{3}$. The only possible candidate is therefore $-\frac{1}{3}$.

   Synthesis. Let us check is the latter is suitable: one has equalities

   $$A\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0.$$

   Conclusion: number $-\frac{1}{3}$, and only it, meets the given conditions.

2. It is custom to imply that x is *any* number, *i. e.* on which everything we will say later will apply *whatever its numerical value*. The one way – which has already been used above – to EXPLICIT this is the evocation[1]

   « Let x be number ».

   Let us then ("then" = "following the previous evocation"), so as to obtain image $B(x)$, apply program 2 on evoked number x: subtracting 1 from it (resp. adding 2 to it) gives $x - 1$ (resp. $x + 2$), multiplying the difference and sum obtained yields $(x-1)(x+2)$. One consequently has equalities

   $$B(x) = (x-1)(x+2) = x^2 + 2x - x - 2 = x^2 + x - 2.$$

   (a) One has on the one hand equalities

   $$\begin{aligned} B(s) - A(s) &= s^2 + s - 2 - (3s + 1) \\ &= s^2 + s - 2 - 3s - 1 \\ &= s^2 - 2s - 3, \end{aligned}$$

   on the other hand

   $$\begin{aligned} (s+1)(s-3) &= s^2 - 3s + s - 3 \\ &= s^2 - 2s - 3. \end{aligned}$$

   Since we get the same thing, both starting numbers are equal, *QED*.

[1] *to evoke* here is *to create by way of magic* (it does not mean *to mention*), as though we "ordered" a number to be (the verb "let... be" is an imperative!)

(b) *Analysis*. Let m be such a number: images $A(m)$ and $B(m)$ are then equal, so their difference is nil. But that difference equals $(m+1)(m-3)$: one of the factors of this nil-product is therefore zero, which writes $\left\{\begin{array}{c} m+1=0 \\ \text{or} \\ m-3=0 \end{array}\right.$, *i. e.* $\left\{\begin{array}{c} m=-1 \\ \text{or} \\ m=3 \end{array}\right.$. The only possible candidates are consequently -1 and 3.

Synthesis. Let us show that these two numbers are suitable. On the one hand, equalities

$$\left\{\begin{array}{l} A(-1)=3(-1)+1=-3+1=-2 \\ B(-1)=(-1-1)(-1+2)=-2\cdot 1 \end{array}\right. \text{ show that } -1 \text{ is suitable,}$$

on the other hand equalities

$$\left\{\begin{array}{l} A(3)=3\cdot 3+1=9+1=10 \\ B(3)=(3-1)(3+2)=2\cdot 5 \end{array}\right. \text{ show that 3 is suitable.}$$

Conclusion: both programs yield the same result when we choose as starting number -1 or 3, and only these two numbers.

Wording 13.

1. One has equalities

$$\begin{array}{rcl} E &=& (\square-2)(2\square+3)-3(\square-2) \\ &=& \begin{array}{c} 2\square^2+3\square-4\square-6 \\ -3\square+6 \end{array} \\ &=& 2\square^2-4\square. \end{array}$$

2. One has equalities

$$\begin{array}{rcl} E &=& 2\square^2-4\square \\ &=& 2\square(\square-2) \\ &=& n\square(\square-2) \text{ where we defined } n:=2. \end{array}$$

Alternative solution: start directly from the non-developed expression at the beginning and see in it $\square-2$ as a common factor:

$$\begin{array}{rcl} E &=& \underline{(\square-2)}(2\square+3)-3\underline{(\square-2)} \\ &=& \underline{(\square-2)}(2\square+3\quad-3) \\ &=& (\square-2)(2\square) \\ &=& 2\square(\square-2). \end{array}$$

3. The hypothesis means product $2\square(\square-2)$ is nil, *i. e.* that its half $\square(\square-2)$ is zero: one at least of its factors is therefore nil, which writes $\left\{\begin{array}{c} \square=0 \\ \text{or} \\ \square-2=0 \end{array}\right.$, *i. e.* $\square=0$ or $\square=2$.

4. The condition on sought-after numbers a translates (subtract the right-hand side) as expression E being zero after each symbol $\square$ has been replaced by a, *i. e.* as product $a(a-2)$ being nil, *i. e.* (we just said it above) as elementhood[2] $a\in\{0,2\}$. The sought-after numbers are therefore 0 and 2.

Remark: instead of reasoning (as we did above) by *analysis-synthesis*, we have here reasoned directed by *equivalences* (hidden in the "*i. e.*").

[2] "$e\in E$" reads "e **is an element of** E" or "e **belongs to** E"