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# Spontaneous Symmetry-Breaking in the 3-D Wake of a Long Cylinder Simulated by the Lattice Gas Method and Drag Coefficient Measurements

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**Abstract.** We present a direct numerical simulation by the lattice gas method of the three-dimensional non-stationary incompressible flow at a Reynolds number of 74, past a circular cylinder, with a uniform incident flow. We describe the three-dimensional structure and the time- evolution of the wake, which leads to an oblique vortex shedding situation.

We also present early results on measurements of drag coefficients for spheres and cylinders at a Reynolds number of 20.

**Key words:** lattice gas simulations – external flows – drag coefficients

## 1. The Physical Problem to Solve

Numerous experimental studies have revealed that the *two- dimensional* wake of a circular cylinder presents a Hopf bifurcation when the Reynolds number crosses a critical value $Re_c$ which is close to 47. This Hopf bifurcation leads from a stationary velocity field with a symmetrical recirculation zone behind the obstacle, to an oscillating wake known as "Bénard-von Kármán vortex street". The problem of the incompressible flow past a circular cylinder can be viewed as a three-dimensional extension of the previous problem by just adding a spatial coordinate $Z$, in the axial direction.

This additional space coordinate raises the question of the three-dimensional structure of the wake. Experimental results [1] show that there still exists a critical value of the Reynolds number, at which a bifurcation occurs, leading to a non-stationary oscillatory flow. Near that critical Reynolds number, the bifurcated flow can be viewed in first approximation as a "stack" of oscillators corresponding to the two-dimensional Bénard-von Kármán street, with a phase which may or may not vary with the axial coordinate.

Although many laboratory experiments have already been performed, some disagreement between experiments still remains regarding the three-dimensional structure of the wake, especially at moderated Reynold numbers between 35 and 120. Some authors (e.g. [2]) find parallel vortex shedding, in other words, all the oscillators are phase-synchronized. This is observed at Reynolds numbers, quite far above the critical value, such as 117. Other authors (e.g. [3]) find oblique vortex shedding situations already at a Reynolds number of 75. End-effects and input flow inhomogeneities may be major causes of the reported discrepancies.

## 2. The Numerical Simulation Method

We have investigated this problem at a Reynolds number of 74, by a direct numerical simulation of the incompressible Navier-Stokes equations using a 3-D lattice gas method FCHC-8 implemented on a CRAY-2 [4, 5, 6]. In order to be free from end-effects, we have chosen periodic boundary conditions for all hydrodynamical fields in the axial direction. In the flow direction, transverse to the cylinder axis, we have imposed wind tunnel- like conditions to maintain a constant mean flow rate. We have used a $512 \times 128 \times 512$-node lattice. The cylinder, on which rigid boundary conditions are assumed, has a circular cross-section with a diameter of 20 nodes. The axis of the cylinder spans the full Z-width of the lattice (512 nodes).

We have explored the velocity field extracted by cell- averaging every 200 time steps (that is, roughly every circulation time, based on the diameter and on the up-stream mean velocity).

## 3. The Results

This study has revealed the following scenario for the time-evolution of the wake:

- Immediately after the impulsive start of the flow, an oscillating vortex street appears with a phase having a disordered axial structure (fig. 1).
- Due to diffusive viscous coupling, the early disordered structure decays in favour of an ordered mode involving wavy vortices (figs. 2 and 3).
- A dislocation occurs in the wavy structure (fig. 4). Eventually, an oblique vortex pattern with slanted rolls emerges. The slanting angle is quantified by the periodicity condition (figs. 5 and 6). This oblique shedding remains stable at least for the entire time-span covered by the simulation. This does not rule out the appearance, on an even longer time scale, of another dislocation leading to a different vortex pattern.

There remain many aspects of the three-dimensional vortex shedding behind a circular cylinder which are not addressed by our simulation. It would be of interest for example to know the lowest Reynolds number at which parallel shedding is stable, if such a threshold exists. Here, we have provided evidence that oblique shedding is already possible at Reynolds numbers as low as 74.

## 4. Drag Coefficient Measurements

We have designed and implemented a strategy to extract drag coefficients from a lattice gas simulation. We present here the principles of these measurements and some early results obtained with spheres and cylinders.

3-D wake of a circular cylinder: velocity field with its mean value subtracted shown in a plane tangent to the cylinder and parallel to the mean flow.

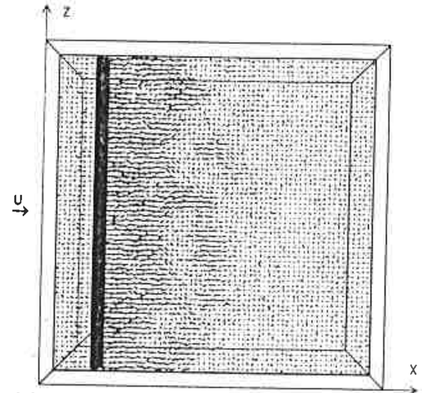


Fig. 1. 4000 steps (18.3 circulation times).

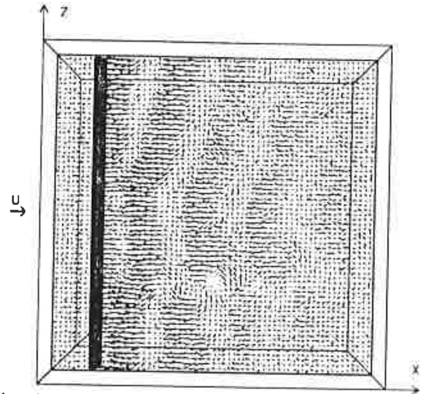


Fig. 4. 16000 steps (74.4 circulation times).

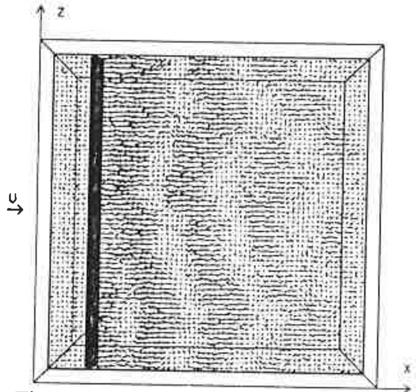


Fig. 2. 8000 steps (37.0 circulation times).

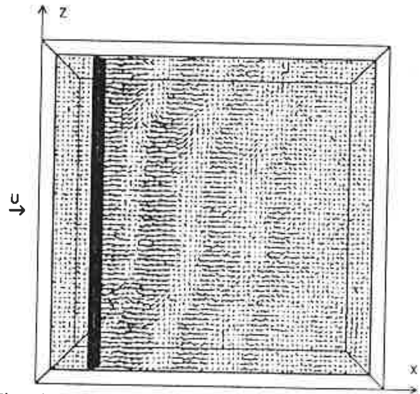


Fig. 5. 20000 steps (93.1 circulation times).

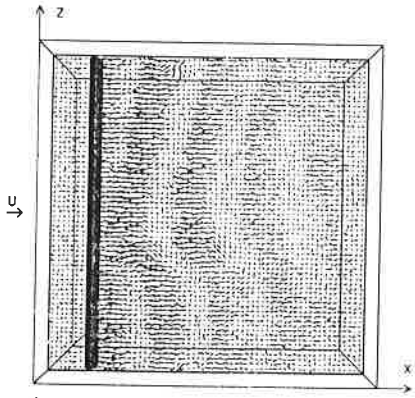


Fig. 3. 12000 steps (55.7 circulation times).

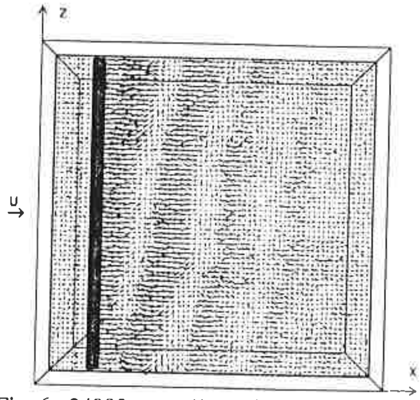


Fig. 6. 24000 steps (112. circulation times).

The drag coefficient $C_D$ of a solid object is defined by $C_D = F_s/(S \cdot \frac{1}{2}\rho U^2)$, where $F_s$ is the streamwize component of the force experienced by the object immersed in a fluid with density $\rho$, flowing with an up-stream uniform velocity $U$. In the above formula, $S$ is the area of the object projected on a plane transverse to the stream direction.

The measurement is done by computing the microscopic momentum $\Delta \mathbf{P}$ transfered per time step from the fluid to the object by particles bouncing back on it. This quantity, after projection on the streamwize direction, time averaging and proper rescaling gives the drag coefficient:

$$C_D = \frac{1}{a.g(\rho_0)} \frac{\Delta P_s}{S.\frac{1}{2}\rho_0 U^2},$$

where $a$ stands for the volume of the unitary mesh of the lattice, and $g(\rho_0)$ for the galilean factor [4] of the model, which depends on the mean density per node $\rho_0$. The accuracy obtained with the early experiments presented here is essentially limited by the statistical noise level on the quantities involved in the above formula, and by the unperfect (noisy) knowledge of the galilean factor.

As a validation test, we have measured the drag coefficient for a sphere at $R_e = 20 \pm 5\%$ in a relatively small "universe" ($128 \times 128 \times 256$ nodes). The result $C_D = 2.4 \pm 8\%$ is in good agreement with the experimental value of $2.5 \pm 14\%$ reported in [7].

The drag measurement for a cylinder at $R_e = 20 \pm 5\%$ used a simulation on a $512 \times 512 \times 128$-node "universe", with time-averaging over 10000 time-steps. The time-averaging process was started after the 5000 steps needed to reach a stationary régime. This gave $C_D = 2.7 \pm 5\%$, which is consistent with experimental results, but near the high end of the fairly large scatter of presently reported experimental values.

## References

1. C. Mathis, *"Propriétés des composantes de vitesse transverse dans l'écoulement de Benard-von Kármán aux faibles nombres de Reynolds"*, Thesis, Université de Provence, (1983).
2. F. Hama, *J. Aero. Sci.*, **24**, (1957).
3. S. Taneda, *Rep. Res. Inst. Appl. Mech.*, **1**, (1952).
4. U. Frisch, D. d'Humières, B. Hasslacher, P. Lallemand, Y. Pomeau, J.P. Rivet, *Complex Systems*, **1**, p. 649-707, (1987).
5. D. d'Humières, P. Lallemand, U. Frisch, *Europhys. Lett.*, **2**, p. 291-297, (1986).
6. B. Dubrulle, M. Hénon, U. Frisch, J.P. Rivet, *J. Stat. Phys.*, **59**, p. 1186-1226, (1990).
7. American Institute of Physics Handbook, p. 253, *Eds. Mc Graw Hill*, (1957).