MPRI M1 Internship

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# *Mondex*, an Electronic Purse : Specification & Refinement Checks with the *Alloy* Model-Finding method

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# Outline

- The Mondex Project
- Alloy Principles
- Technical Issues
- Results
- Using FOL theorem provers
- Conclusions

# The Mondex Project

- **Grand Challenges in Computer Science**

  UK Computer Research Committee

  - **Dependable Systems Evolution**

    Jim Woodcock, University of York

    - **Verified Software Repository**

      several formal methods for machine-aided verification

      - **Mondex Case Study**

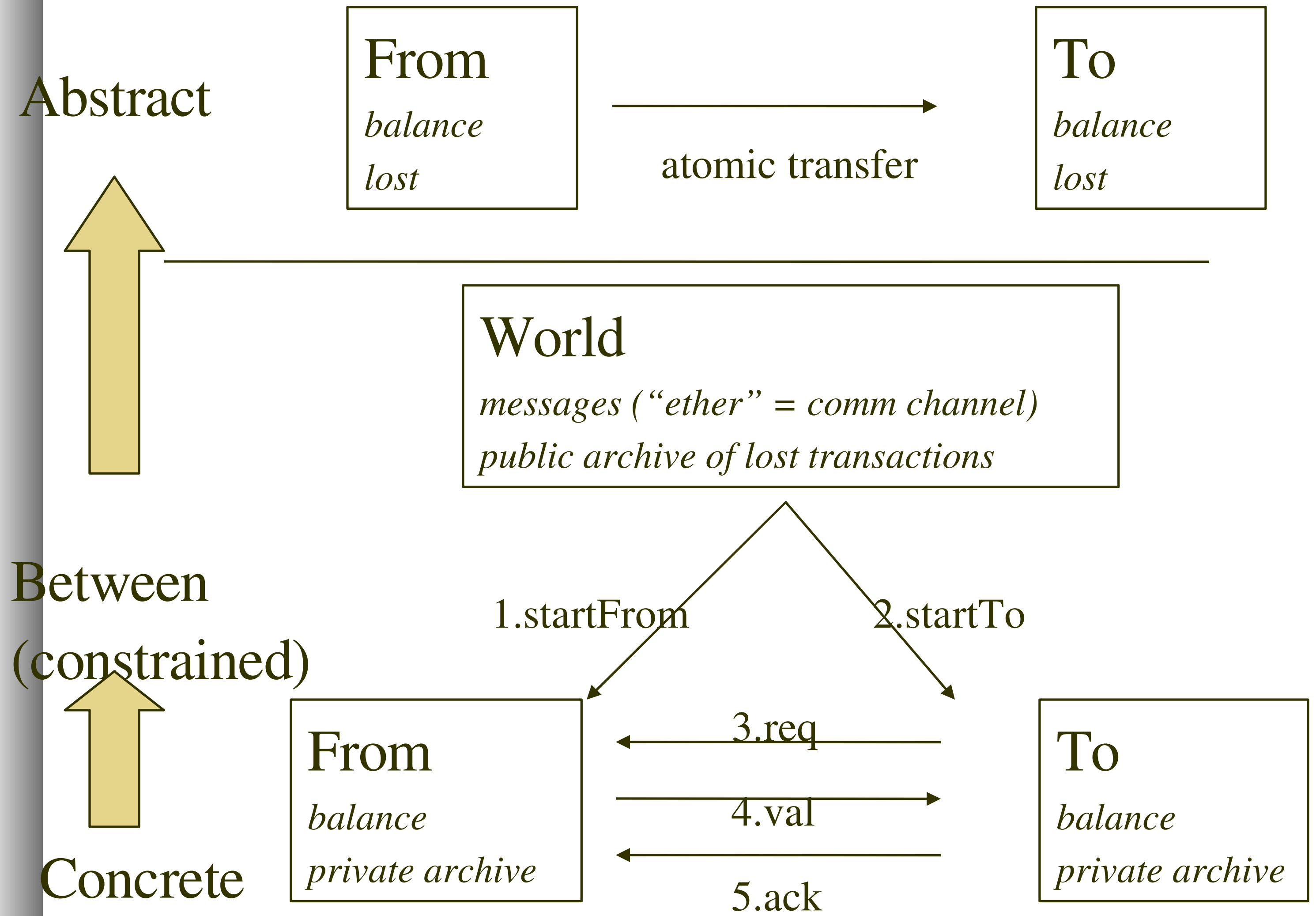
# The Mondex Case Study

- An electronic purse (smart card) system
  – Replace physical coins with values stored in the card (not remotely : not a credit card)
- Highly critical security issues for banks
- Specified by hand in Z (Stepney, Cooper, Woodcock 2000)
  – Granted ITSEC security level 6 out of 6 (2001)
- Aim : machine-check this specification with **automated** formal methods

# Mondex

Total balances not increasing

Total balances and lost constant



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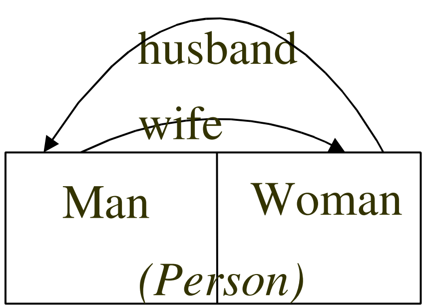
# Alloy Spec Language & Logic

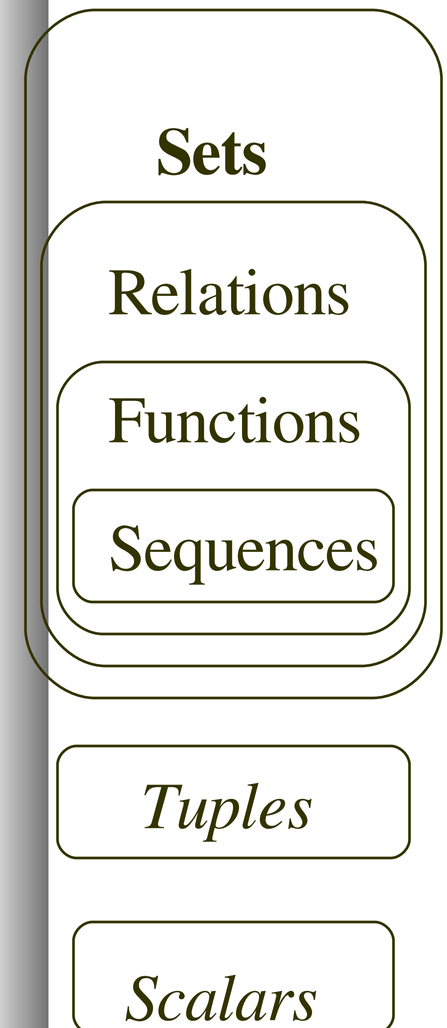
- Typed and modular specification language
- Sets and relations
  - Signatures define “basic” sets and relations
    - Can be abstract, extended (“inheritance” as in Java)
      - Typing, overloading, modularity
      - quite like Z schema extensions
  - Specification can be constrained
- Relational first-order logic + transitive closure



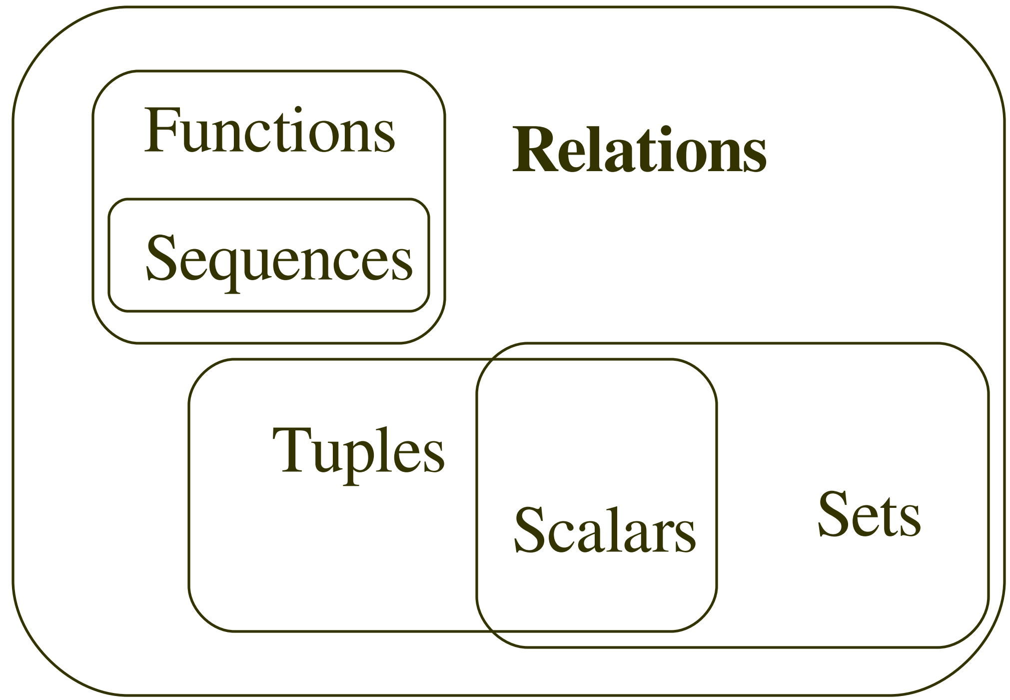

```
abstract sig Person {}
sig Man extends Person {wife:set Woman}
sig Woman extends Person {husband:set Man}

fact Constraint {
   all m:Man |
   some m.wife implies m.wife.husband = m
   all w:Woman |
   some w.husband implies w.husband.wife = w
}
```

# Alloy relations vs. Z sets




**Z**

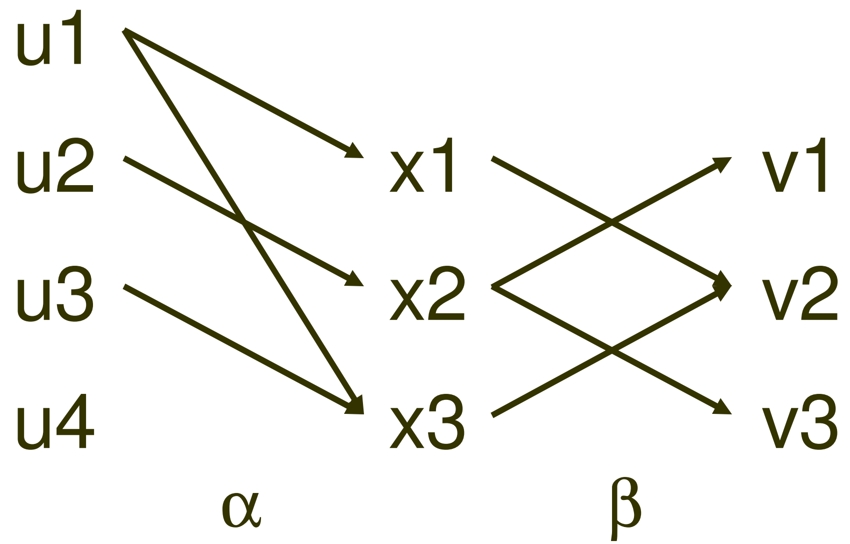



- sets are unary relations
- scalars are singletons

**Alloy**

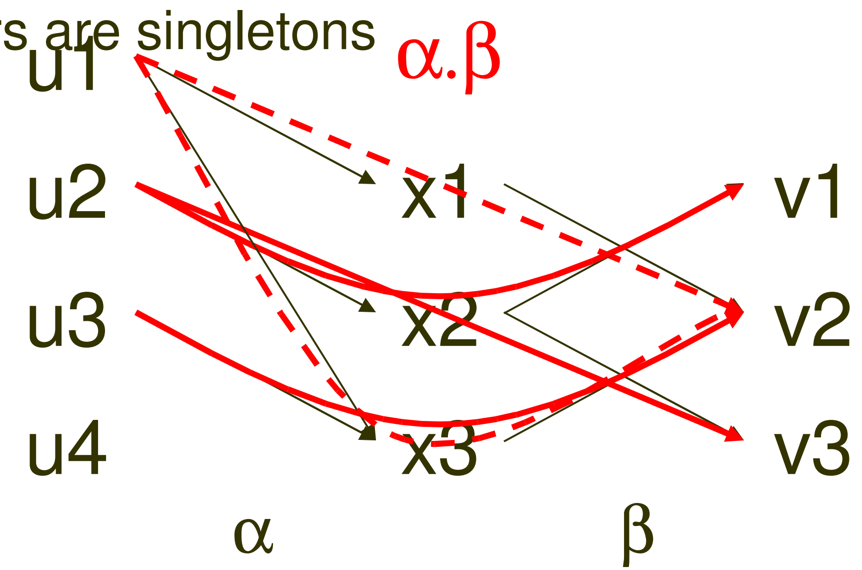
# Joining relations (.)

- Let α and β be two relations
  - `sig U {alpha : set X}`
  - `sig X {beta : set V}`
  - `sig V`



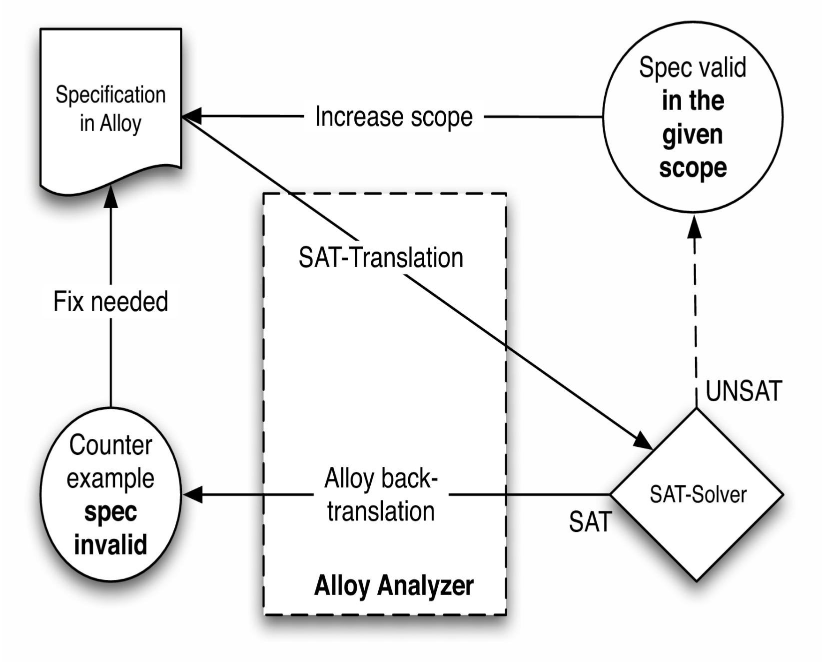
# Joining relations (.)

- Let α and β be two relations
- so we define α.β the *joined relation*
  - Cf. database ▷◁
- We may write `u2.(alpha.beta)=v1+v3` , it is the same join operator because :
  - sets are unary relations
  - scalars are singletons



# Alloy Analyzer, a Model Finder

- Specification Analysis by Model Finding
  - "Run" predicate: find example
  - Check assertion: find counterexample
- "Scope" required : bounded **finite** models
  - Number of objects for each signature
  - Can show theorems hold in *specified scope*




```
pred Married (p:Person) {some p.(wife+husband)}

pred Simulation () {some p:Person|Married(p)}
run Simulation for 18 Man, 1 Woman

assert Theorem {
   all p:Person|lone p.(wife+husband)
   all p,q:Person|p.husband=q iff q.wife=p }
check Theorem for 7
```

# A naive attempt

```
sig NAME {}
```

[ NAME ]

# A naive attempt

[ NAME ]

AbPurse

| balance, lost : N |
| --- |

```
sig NAME {}

sig AbPurse {balance,lost: Int}
```

# A naive attempt

[ NAME ]

AbPurse

| |
|---|
| balance, lost : N |

AbWorld

| |
|---|
| abAuthPurse : NAME ⇸ AbPurse |

```
sig NAME {}

sig AbPurse {balance,lost: Int}

pred Abstract (abAuthPurse:NAME->Purse) {
 -- functional
 all n:NAME | lone n.abAuthPurse
}

sig AbWorld {abAuthPurse: NAME -> AbPurse}

fact AbWorldConstr {
  all a : AbWorld | Abstract (a.abAuthPurse)
}
```

# A naive attempt

[ NAME ]

AbPurse

| |
|---|
| balance, lost : N |

AbWorld

| |
|---|
| abAuthPurse : NAME ⇻ AbPurse |

Unable to express finiteness : ignore

```
sig NAME {}

sig AbPurse {balance,lost: Int}

pred Abstract (abAuthPurse:NAME->Purse) {
 -- functional
 all n:NAME | lone n.abAuthPurse
}

sig AbWorld {abAuthPurse: NAME -> AbPurse}

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# A naive attempt

[ NAME ]

AbPurse

| |
|---|
| balance, lost : N |

AbWorld

| |
|---|
| abAuthPurse : NAME ⇻ AbPurse |

AbIgnore

| |
|---|
| ΔAbWorld |
| abAuthPurse’ = abAuthPurse |

```
sig NAME {}

sig AbPurse {balance,lost: Int}

pred Abstract (abAuthPurse:NAME->Purse) {
 -- functional
 all n:NAME | lone n.abAuthPurse
}

sig AbWorld {abAuthPurse: NAME -> AbPurse}

fact AbWorldConstr {
  all a : AbWorld | Abstract (a.abAuthPurse)
}

pred AbIgnore (a,a’:AbWorld) {
 a’.abAuthPurse = a.abAuthPurse
}
```

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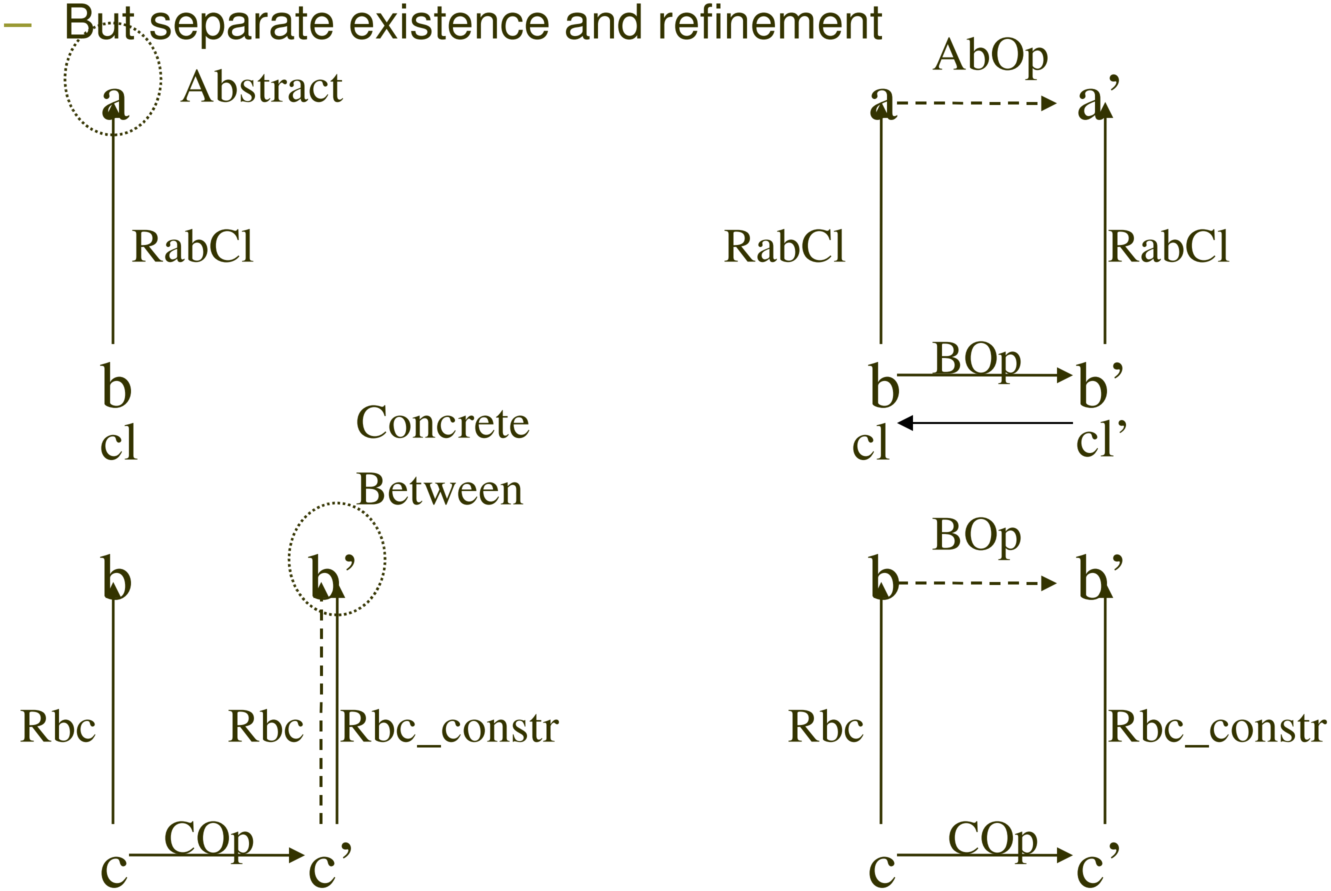
# Refinements : checking method

- Follow Z spec strategy (A/B backwards, B/C forwards)
  - But separate existence and refinement




- Rbc_constr : equality predicates (explicit "construction")
  - Not necessary for RabCl (already in this form)

# Integers in Alloy

- Integers in Alloy are heavy
  - Builds boolean circuits for +, <
  - Expensive operations
- So, avoid them
  - Not all properties of N used
  - Determine which
  - Pick most lightweight repr that works

# Representing SEQNO

- Sequence numbers just require total order
  - No operations
  - Even no successor
- Simply use Alloy's ordering module

# Representing amounts

- Sets of coins

| Z | Alloy |
| --- | --- |
| Integers | Sets of coins |
| Equality | Set equality |
| Ordering | Set inclusion |
| Sum | Set union |
| Difference | Set difference |

- OK, because no comparison between purses
  - Globally : coins between whole worlds
  - Locally : between a purse balance & a payment
- Add constraints to avoid coin sharing

# Existential issue

- Can't guarantee object exists for every combination of field values
  - The empty model
  - To enforce existence with algebraic constraints would dramatically increase scope
- Solution :
  - Instead of ∃, construct *explicit witness* :
    ```
    all c, c', a | some a' | P (c, c', a, a')
    ```
    becomes
    ```
    all c, c', a |
    let a' = F(c, c', a) | P(c, c', a, a')
    ```
  - Requires to get rid of global constraints
    - Integrate them into theorems

# The identity of objects

- Z : schemas define records
- Alloy : signatures define atomic objects
  - Objects have an *identity*
    - Notion does not exist in Z
  - Suitable for names, coins
- Two objects with same field values may be distinct
  - Naive solution : impose equality constraint

```
fact {
    no disj a1,a2:AbPurse {
        a1.balance=a2.balance
        a1.lost=a2.lost
    }
}
```

# The identity of objects

- Smoother solution : represent purses and states as standalone objects rather than records
  - No names

```
[ NAME ]

AbPurse
 balance, lost : N

AbWorld
 abAuthPurse : NAME ⇸ AbPurse

AbIgnore
 ΔAbWorld
 ────────
 abAuthPurse' = abAuthPurse
```

```
sig Coin

sig AbPurse {balance,lost: Coin->AbWorld}

sig AbWorld {abAuthPurse : set AbPurse}

pred AbIgnore (a,a':AbWorld) {
 a'.abAuthPurse = a.abAuthPurse
 all p : AbPurse | p in a.abAuthPurse implies {
   p.balance.a' = p.balance.a
   p.lost.a' = p.lost.a
 }
}
```

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# Bugs found in Z Specification

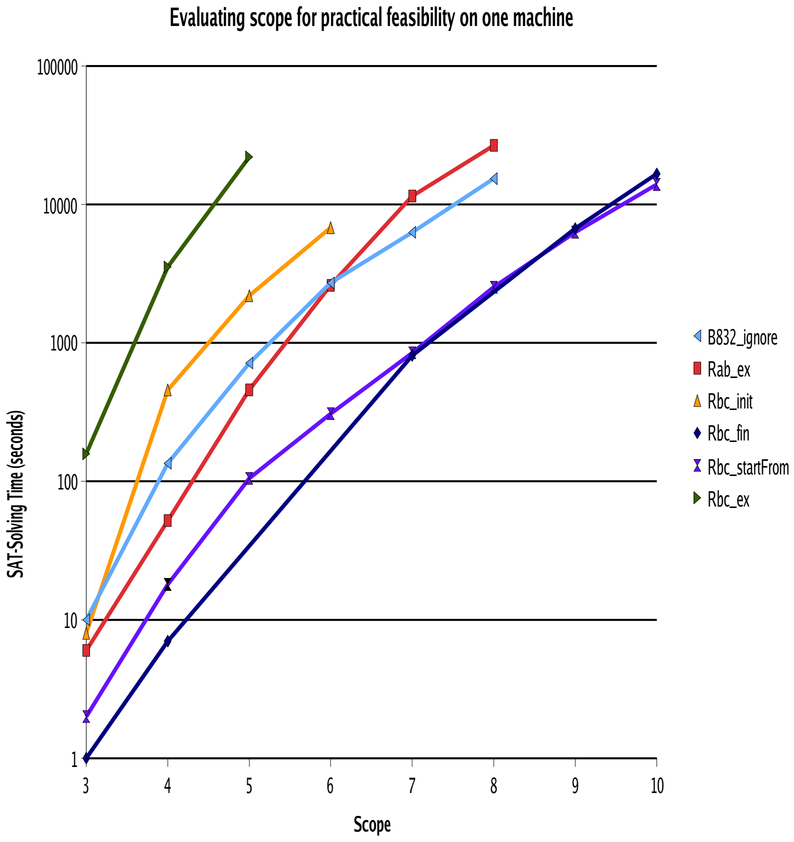
- Missing authenticity constraints
  - Spurious cases where purses deal with irrelevant transactions are not eliminated
- Wrong proof steps
  - Wrong assumption made by informal comments
  - 2 bugs with this form

# Alloy's Approach Summary

- Refinement checks with model finding
  - Try to find c, c', a, a' such that Rac(a, c) & Rac(a', c') & COp(c, c') hold but not AOp(a, a')
- Original approach
  - Quite high confidence level
  - Not as high as theorem proving
  - but much cheaper !

# Choosing scopes

- Must be enough for quantifications
- Started with 10
  - worked fine with Abstract theorems
  - too long for more complex theorems
    - SAT-solving time exponentially grows with the scope
    - SAT solver crashed for refinement checks
  - so grow scope incrementally
- Achieved scope of 8 for most theorems eventually
  - restricted scope for Worlds is complete



# Almost everything represented

- Alloy modules close to Z specification
  - *Representation* size is comparable
  - Alloy Proof size is negligible
    - Actually no proof details in Alloy modules
- Only changes :
  - Integer representation
  - Unable to express infiniteness in Alloy
    - finiteness properties ignored
- Fits first order logic
  - No transitive closures needed

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# The direct attempt

- FOL atoms are Alloy atoms
  - But Alloy predicates take arbitrary relations as arguments
  - So they have to be inlined
  - Formulae become huge
- Simplifications to decrease formula size
  - Eliminate redundancy with subsumption tests
  - Split theorems through
  - Attempt to reach a normal form
    - Does not terminate
- Very few results :
  - Proved theorems relative to the abstract world (atomic transactions) alone

# The “lifted” attempt

- FOL atoms are Alloy relations
- Axiomatize relational algebra
  - Bound arities according to spec in Alloy
- Problems :
  - Trouble to prove obvious-looking general theorems such as :
    - The Cartesian product of two atoms is a singleton of arity 2
  - Would have to prove intermediate lemmas
  - Loss of automation
- No significant results

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# General observations

- High level checking
  - Proof structure not needed: automated
  - But need to provide explicit witness for ∃
- SAT-Solving duration varies
  - From seconds to hours (even days!)
  - Time correlated with theorem importance?

# Alloy Limitations

- FOL and Finiteness
  - Cannot express infiniteness
  - But in practice, world of purses finite
- Alloy Analyzer's analysis is bounded
  - Results valid only on given scope
  - Is scope of 8 enough?
- Enough for industry?
  - Much less effort than theorem proving
  - But problems with critical security issues need a proof

# Personal Experience

- Learn Z and Alloy *from scratch*
- Nice :
  - Language easy to understand
    - no Δ/Ξ/graphical issues
  - Though quite close to Z
  - Expressive & smooth relational logic
- Nasty :
  - Signatures are not records
    - Equality & Existential theorems
  - Resource- and time-consuming SAT-Solving
    - Very long time for obvious-looking theorems (easily provable by hand, e.g. Ignore refinements)
    - Perhaps syntactic pre-analysis would help?

# Lessons

- Learn another verification approach
  - Automation does not exclude proof formalism
- Even though not theorem proving
  - But allows also checking informal comments
- Discover problems more quickly
  - Alloy Analyzer allows finding several bugs
  - Counterexample gives useful information when bug found

# Future work

- Argue small model theorem (Momtahan 2004) ?
- Improve checking with FOL theorem provers
  - To expect better FOL theorem provers is quite hopeless : undecidable
  - Better model Alloy into FOL
  - Fit into decidable sublogic ?
- Tackle finiteness
  - HOL necessary at first sight
  - Use incomplete FOL theories ?
- Interface Alloy method with others

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- At MIT :
  - The SDG group, in particular Daniel Jackson
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- At RAL :
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# Any questions ?

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- Alloy Website :
  - http://alloy.mit.edu